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CHAPTER 1

Introduction. Gödel and analytic philosophy: how did we get here?

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1 Introduction

It is often said about Kurt Gödel that he was the greatest logician of the twentieth century. His work in mathematical logic, when it does not constitute the very ground out of which its various subfields grew and developed, made the continuation of the subject possible at a time when fundamental concepts had not even been identified, and proofs of key theorems – in those cases when they had been stated – had not materialized in anything like their final form. This is not to say that Gödel was intellectually infallible; one could also point to the richness of Gödel's logical milieu. But there is no doubt that a gigantic intelligence had turned to the field of mathematical logic – and how much better off the subject was for it!

Gödel's philosophical work on the other hand, work to which he devoted himself almost exclusively from the mid-1940s until his death in 1978, has not been as well received. Put another way, any praise of Gödel's contributions to the foundations of mathematics has largely been limited to his theorems.[1] Gödel the *philosopher* – and indeed even today it is a matter of debate, whether Gödel can be regarded as a philosopher at all – has traditionally been seen as advocating a crude form of Platonism in his philosophical writings, one entangled with the views of Kant and Leibniz in a way which was seen as philosophically naive and primarily historical; and one which, anachronistically, seemed to give no quarter to what turned out to be the single most important development in twentieth

[1] See for example Boolos's introduction (Gödel 1995, pp. 290–304) to Gödel's posthumously published 1951 Gibbs Lecture ("Some basic theorems of the foundations of mathematics and their philosophical implications," reprinted in Gödel (1995), pp. 304–323):

> What may be found problematic in Gödel's judgement that his conclusion is of philosophical interest is that it is certainly not obvious what it means to say that the human mind... is a Turing machine.

century (analytic) philosophy, namely the so-called linguistic turn inaugurated by Frege, Russell and Moore. To the contrary, Gödel's Platonism, that is to say his various formulations of the view that mathematics is contentual, or in other versions that mathematical truth is bivalent, or in still other versions that mathematical objects enjoy some positive sense of existence, were seen by philosophers – when they did not simply bypass his work – as the antiquarian views of an old-fashioned, albeit great mathematician, untrained in philosophy and nostalgic for the days when the concept of mathematical truth was considered to be beyond criticism – an ironic development in the light of Gödel's actual discoveries.

With this volume we wish to effect a change in the philosophical body politic; to call attention to threads in Gödel's thinking which have turned out to be, in light of the directions in which philosophy has developed since Gödel's time, either newly or persistently important. We wish to reassess Gödel's practice of *philosophy as mathematics*; in a word, to reassess his philosophical work in the light of possibly favorable developments. Recent excursions into mathematical naturalism, for example, to be found in works by Penelope Maddy and others, have brought into the philosophy of mathematics a newly invigorated focus on mathematical practice – a nonnegotiable, core commitment for Gödel. Of course, much of the writing on Gödel's philosophical work has focused on his avowed Platonism. And while there is every reason to expect that Gödel will continue to be a canonical representative of that view in the minds of many philosophers, others have gained philosophical traction in areas of Gödel's writings which are less overtly metaphysical and more oriented toward actual mathematics, set theory in particular, but also other material which is "closer to the ground" mathematically and logically.

Of Gödel's philosophically informed logical work, his Completeness Theorem is a fundamental technical result. But the resurgence of interest in logical consequence places it at the center of contemporary philosophical focus. As Curtis Franks puts it in Chapter 5,

> While the theorem contained in Gödel's thesis is a cornerstone of modern logic, its far more sweeping and significant impact is the fact that, through its position in a network of technical results and applications, the way of thinking underlying the result has come to seem definitive and necessary, to the extent that we have managed to forget that it has not always been with us.

Franks's observation that as far as the concept of logical consequence goes, our world is Gödelian through and through, could equally well apply to other projects within the contemporary philosophy of mathematics enterprise. Gödel's trademark as a philosopher, his *modus operandi* as it were, was to practice philosophy as if it were mathematics; to conjure sharp, mathematical conjectures out of inchoate philosophical material, refashioning that material so as to be subject to proof. It was a radical approach to philosophical practice, harkening back to Leibniz's *calculemus*, if not to the calculating machines of Ramon Lull, as well as to the Husserlian project of *Philosophie als Strenge Wissenschaft*. It is ironic that if one scrutinizes Gödel's philosophical writings in the light of his own standards, this renders much of it ungrounded; and indeed, Gödel often remarked of his writings that short of a more exact treatment, much of what had been laid out there was not to be taken as definitive. On the other hand – and this is the subtlety here – Gödel had a very broad notion of proof.[2]

The method was found uncongenial at the time. For example George Boolos's 1995 introduction to Gödel's posthumously published 1951 Gibbs Lecture, about the philosophical consequences of the Incompleteness Theorems, results which, according to Gödel, "... have not been adequately discussed, or have only just been taken notice of" (Gödel 1995, p. 305), contains the following statement:

> Gödel's idea that we shall one day achieve sufficient clarity about the concepts involved in the philosophical discussion to be able to prove, mathematically, the truth of some position in the philosophy of mathematics, however, appears significantly less credible at present than his Platonism. (Gödel 1995, p. 290)

We referred above to favorable developments. In the years since Boolos wrote his introduction, the field of philosophy of mathematics has begun to shift toward, one might say, the concrete. Episodes in the history of mathematics – odd accidents, moments of perplexity, turning points and the like – whose philosophical significance was previously overlooked, are now given sustained and detailed treatment, "under a microscope" so to speak, and "pushed to the limit."[3] "Conceptual fine-structure" is a term of art; and while the a priorist tradition – championed, of course, by Gödel, but from a standpoint that was centered within the practice – is still very dominant, for a significant percentage of philosophers of mathematics, the importance of a priorism has begun to recede. Franks described this

[2] See, for example, Gödel (1946), "Remarks before the Princeton bicentennial conference of problems in mathematics," reprinted in Gödel (1990).

[3] See, for example, Arana and Mancosu (2012), Wilson's massive book (2008) and Brandom (2011), a review of Wilson (2008).

shift away from a priori or *second-order* philosophical discourse (in his terminology) as follows:

> Recent philosophical writing about mathematics has largely abandoned the *a priorist* tradition and its accompanying interest in grounding mathematical activity. The foundational schools of the early twentieth century are now treated more like historical attractions than like viable ways to enrich our understanding of mathematics. This shift in attitudes has resulted not so much from a piecemeal refutation of the various foundational programs, but from the gradual erosion of interest in laying foundations, from our culture's disenchantment with the idea that a philosophical grounding may put mathematical activity in plainer view, make more evident its rationality, or explain its ability to generate a special sort of knowledge about the world. (Franks 2009, p. 169)

Our point is this: the role of the *practitioner*, in philosophy of mathematics but also in other philosophical subfields, has now become central. The consequence for set theory in particular is that this wholly mathematical project is now beginning to be perceived as a wholly philosophical one as well – surely a mark of our Gödelian inheritance, and one we take particular note of in this volume.

Rather than a comprehensive survey, our volume is more of a snapshot of the contemporary take on Gödel's work. We suspend judgment on the taxonomy of subjects, appropriating for *philosophy* issues like the decidability of diophantine equations, the generic multiverse in set theory and Shelah's Main Gap program in model theory. Chapters on these topics by Poonen, Steel, Väänänen and Shelah, respectively, are set side by side and on an equal basis (in terms of philosophical interest) with chapters on intuition by Folina and Burgess, on logical consequence by Detlefsen and Franks, and on analyticity by Parsons.

Practical considerations may undermine comprehensiveness in an editorial volume, and ours is no exception. (For example, Gödel's massive contribution to intuitionism is only touched on in some of the essays here, and only in passing.) We thus take this opportunity to refer the reader to the recent work of a few important commentators whose work does not appear here: in addition to W. W. Tait's work on Gödel and intuitionism, D. A. Martin's papers on Gödel's conceptual realism; P. Maddy's work on perceptual realism and more recently on naturalism in set theory; H. Woodin's work on the multiverse and on the decidability of the continuum problem generally; finally, R. Tieszen and M. van Atten's work on Gödel and phenomenology.

2 Gödel's Platonism: a case study in method

The term Platonism as it is used in the current context of philosophy of mathematics seems to have been coined by Bernays in his 1934 lecture "On Platonism in mathematics":

> . . . the objects of a theory are viewed as elements of a totality such that one can reason as follows: For each property expressible using the notions of the theory, it is [an] objectively determinate [fact] whether there is or there is not an element of the totality which possesses this property . . . the tendency of which we are speaking consists in viewing the objects as cut off from all links to the reflecting subject. Since this tendency asserted itself especially in the philosophy of Plato, allow me to call it "Platonism." (Reprinted in Benacerraf and Putnam 1983, p. 259)

But mathematicians have always been drawn to the idea that mathematics is a fundamentally *descriptive science*. The set theorist Mirna Džamonja recently stated the view this way:

> I think that the observable reality is only a small part of the actual reality. This view is supported by developments in various physical sciences, of course, where one can talk about many objects that cannot or have not been observed so far. In mathematics, specifically in set theory, this means that for me objects such as ω_1 or $\aleph_\omega$ are in no way less real than the number 3, which is in turn no less real than a table or a chair. I am therefore a very strong platonist, to the point that I cannot even entertain having a different view. The axioms I view as an approximation of reality, and the fact that they do not (provably) describe the whole of the reality, is not surprising to me. The opposite would have been a serious surprise. (Personal communication.)

It is possible that Gödel's remarks and writings on Platonism attracted more attention of (analytic) philosophers than on anything else he ever wrote. His Platonism takes various forms and indeed Gödel formulated his own position differently at different times. He often expressed this as the view that mathematics is contentual:

> Logic and mathematics—like physics—are built up on axioms with a real content and cannot be explained away. The presence of this real content is seen by studying number theory. We encounter facts which are independent of arbitrary conventions. These facts must have a content because the consistency cannot be based on trivial facts. . . (Wang 1996, remark 7.1.4)

On other occasions Gödel meant Platonism to refer to the idea that precisely stated mathematical conjectures, such as the continuum hypothesis,[4] are either true or false. He sometimes used the term Platonism (or, interchangeably, realism) to indicate the idea that mathematical truth is itself objective; and on still other occasions he expressed himself in an ontological vein:

> [By the Platonistic view I mean the view that] mathematics describes a non-sensual reality, which exists independently both of the acts and the dispositions of the human mind and is only perceived, and probably perceived very incompletely, by the human mind. (Gibbs Lecture, Gödel 1995, pp. 304–323)

Somewhere toward the mid-1950s Gödel's Platonism became qualified and complicated by a regulative principle called *epistemological parity*, a fact that has gone largely unnoticed in the literature on Gödel's Platonism (but see van Atten and Kennedy 2003). This is the idea that, regarding physical objects on the one hand and abstract or mathematical objects on the other, from the point of view of what we know about them, there is no reason to be more (or less) committed to the existence of one than of the other:

> It seems arbitrary to me to consider the proposition "This is red" an immediate datum, but not so to consider the proposition stating modus ponens. (Gödel 1990, p. 347)[5]

The interpretive predicament posed by epistemological parity aside, it is well known that philosophers mounted a sustained attack on Platonism throughout the twentieth century, if not earlier,[6] the discussion becoming acute during the so-called Grundlagenstreit of the early twentieth century and the ensuing rise of the various foundational schools. Of course those discussions were not about Platonism explicitly; but Platonism, as Bernays and subsequent generations of philosophers called it, was, for the most part, the underlying issue at stake in many of those discussions.

[4] The continuum hypothesis states that the cardinality of any infinite set of real numbers is either that of the natural numbers, or that of the full set of real numbers. It is independent of the axioms of set theory: its consistency with those axioms was shown by Gödel in 1934; the consistency of its negation was shown by P. J. Cohen in 1963.

[5] The principle, in another form, is stated already in Gödel's 1944 essay on Russell: "It seems to me that the assumption of such objects is quite as legitimate as the assumption of physical bodies and there is quite as much reason to believe in their existence" (Gödel 1990, p. 128). The principle preoccupied Gödel through the 1960s, for example, the idea occurs in some form in a note to himself in the folder titled Phil[osophische] Varia (mostly after 1961): "It should also be noted that even a statement like 'this is red' *if there is to be a valid motive for making it* presupposes that something besides the independent sense experience is given." [GN folder 12/43, 060572; emphasis ours.] A view very similar to epistemological parity surfaces also in Tait's "On the Platonism of mathematics," in Tait (2005) and in some form in Burgess's essay "Why I am not a nominalist," in Burgess (2008).

[6] For example in some form in debates between Kronecker and Cantor.

As of this writing, nominalistic reconstructions of mathematical theories remain very attractive to philosophers. And if the motivation behind such reconstructions runs counter to the mathematician's understanding of her own practice, or if those reconstructions suffer from *awkwardness*, that is if they reconfigure the enterprise to such an extent as to be unrecognizable to its practitioners, then so be it. The mathematician is left with an uninterpreted world view, and the nominalist is charged with irrelevance by the very community whose scientific practice he set out to explain.

Now nominalism, as John Burgess notes,[7] is a big subject, but it is not ours. We wish to say something else here; in particular, we wish to ask the question, what does Gödel's philosophical legacy amount to?

3 Gödel's legacy: decidability in set theory

To assert, as Gödel did, that problems like the continuum hypothesis are solvable, is to embroil oneself in one of the main, if not the main, philosophical dispute of the field.[8] For now we only observe that a nontrivial number of set theorists are by and large somewhat reluctant to give up bivalence, and indeed there is at present a vibrant philosophical literature in this area of foundations of set theory, tracking what can only be described as a spectacular collection of results which have been obtained by set theorists over the last few decades.[9]

Set forcing is the main cause of variability in set theory,[10] but it turns out that the understanding of the set-theoretic universe has advanced to such a point that we now know that some of this variability can be disabled. For example, an axiom like Projective Uniformization is true "across the multiverse" (see Chapters 8 and 9). That is, it is true in "all" set-theoretic universes (i.e., those universes which are either (set) forcing extensions of V, or those of which V is a forcing extension), assuming that one, and hence all of these universes, have enough large cardinals.[11] In fact, by a result of

7 "Being explained away," in Burgess (2008).

8 For example, Tait has recently said about this project (see Tait 2008):

> For me, the most important open problem in philosophy of mathematics is in foundations of mathematics, and that is the search for new axioms of set theory, which means, too, the search for grounds for accepting them.

9 See for example the contributions of Feferman, Friedman, Maddy and Steel (2000). See also Maddy (1998a) as well as the more recent Maddy (2009), Woodin (2001a, 2001b, 2002), Koellner (2009) and Bagaria (2006).

10 Or at least one of them, depending on one's point of view.

11 This is a series of results due to Hugh Woodin, D. A. Martin and John Steel.

Hugh Woodin and assuming large cardinals again, this also holds true for *all* Σ^2_1 statements, that is, these are generically absolute,[12] assuming the continuum hypothesis. Finally, and again under the assumption of enough large cardinals, the theory of $L[\mathbb{R}]$ is generically absolute.[13]

How far this form of decidability will extend is not known. As of this writing, the continuum problem remains undecided in this sense, that is, it is not necessarily true across the multiverse, and cannot be if the multiverse is based on set forcing.

In addition to the generic absoluteness results, forcing axioms in the form of maximality principles, which Gödel advocated, actually decide the value of the continuum in the direction of what was for a time Gödel's suggested value, $\aleph_2$. The so-called *core model program* searches for canonical L-like inner models, that is, models built up from small "manageable" pieces, which decide the continuum hypothesis and many other canonical statements. Other results fix the theory of certain canonical structures in the presence of large cardinals. Indeed, the large cardinals not only decide individual statements, they introduce a conceptual coherence into the whole set-theoretic universe, just as Gödel predicted they would.

Gödel's careful pursuit of his program in the foundations of mathematics and set theory was driven by decidability, as we have said. What is striking about contemporary set-theoretic practice, as it happens – whether this is due to Gödel's advocacy or whether this happened on its own – is that much work in set theory nowadays also revolves around, if it is not explicitly driven by, decidability. Gödel's unbending commitment to decidability did involve a wider, elevated view of human rationality – as did Hilbert's, for example, whose concept even of human dignity itself, was linked to decidability in mathematics (Hilbert 1930a).

What one can say is this: the simple commitment to "keeping mathematics as it is," as we have called it; to preserving mathematical practice *in its original form* by extending to certain natural set-theoretic questions the kind of decidability one is able to take for granted in the rest of mathematics, was, and is still, the goal. And whether or not it will be conclusively established that set-theoretic variability is here to stay; whether or not it will become clear that the mathematician really does live in a "multiverse" rather than a universe, Gödel's program for large cardinals, and much of the philosophical analysis that was activated by and around it, remains the essential inheritance of every working set theorist.

[12] Meaning that their truth value cannot be changed by set forcing.
[13] This is also due to Woodin. For proofs of this and of Σ^2_1-absoluteness, see Larson (2004).

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PART I

Gödel on intuition

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