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### **Principles of Automated Negotiation**

With an increasing number of applications in the context of multi-agent systems, automated negotiation is a rapidly growing area. Written by top researchers in the field, this state-of-the-art treatment of the subject explores key issues involved in the design of negotiating agents, covering strategic, heuristic, and axiomatic approaches. The authors discuss the potential benefits of automated negotiation as well as the unique challenges it poses for computer scientists and for researchers in artificial intelligence. They also consider possible applications and give readers a feel for the types of domain where automated negotiation is already being deployed.

This book is ideal for graduate students and researchers in computer science who are interested in multi-agent systems. It will also appeal to negotiation researchers from disciplines such as management and business studies, psychology, and economics.

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# Principles of Automated Negotiation

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Shaheen:

*To my parents*

Sarit:

*To my family*

Michael:

*To Georg Gottlob, Peter Millican, and Bill Roscoe, with thanks*

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## Preface

We all of us have to negotiate – whether formally, as part of our jobs, or informally, as part of our everyday lives – and the outcomes of our negotiations have direct and often dramatic consequences, for us and others. However, it is surely a safe bet that most of us wish we were better negotiators. There are many reasons why we might not be as good at negotiating as we would wish. For one thing, it is often hard for us to really understand the issues at stake and the consequences of various potential settlements, and for this reason we can end up with outcomes that are not as good as those that we might in fact have been able to obtain. Moreover, in many cultures, negotiation is regarded as greedy or impolite, and as a consequence, some people may find it socially awkward or stressful to negotiate. Cultural inhibitions like these can prevent us from obtaining the best outcome even when the topic of negotiation is of great importance to us. Wouldn't it be wonderful, then, if we had *computers that could effectively negotiate on our behalf*...? In short, the main aim of this book is to investigate this idea.

The idea of computers negotiating on our behalf may sound like science fiction, but we hope to demonstrate in this book that the theory and software technology underpinning automated negotiation is sufficiently advanced and sufficiently robust that it is entirely plausible that some form of automated negotiation technologies could soon be deployed in large-scale real-world (i.e., non-laboratory) applications. However, we would be failing in our duty if we left you with the impression that you will be handing over negotiations for your next pay rise to a computer program. Reliable practical experience with automated negotiating systems is currently limited to a few restricted domains. As with any new technology, we should be appropriately cautious about the claims we make for its potential value and applications. Towards the end of the book, we give a feel for the types of domains where automated negotiation has been and may be deployed.

There are many reasons why it would be desirable to have computers that can negotiate for us. Perhaps most importantly, computers can explore systematically and dispassionately possible outcomes that human negotiators might overlook, thereby reaching more efficient outcomes than might be obtained by humans. We discuss the potential advantages and disadvantages of automated negotiation in the main text of the book.

While the desire to ultimately have computers that can negotiate on our behalf is the primary motivation for this book, we should point out that there are many other good reasons for studying automated negotiation. First, negotiation is clearly an important activity in our lives, and indeed in the global economy. Any such activity is surely worthy of serious academic study. Second, as we hope to demonstrate, the mathematical formalisation and analysis of negotiation raises many interesting and deep scientific problems. And third, the automation of negotiation poses unique challenges for computer scientists and researchers in artificial intelligence; in addressing these challenges, we extend the range of problems that are amenable to solution by computerised techniques.

In some more detail, the aims of this book are threefold:

1. Our first aim is to show how negotiation can be modelled and analysed mathematically. The main mathematical framework we use is *game theory* – the theory of interactions between self-interested economic agents. Although usually thought of as a branch of micro-economics, game theory has recently been found to be of great relevance and value in the analysis of distributed computer systems in which the participant nodes cannot be assumed to share common goals, as is the case in negotiation settings.
2. Second, as the name of the book and the discussion above indicate, we aim to consider how negotiation can be automated within computer programs. One key issue here is that while we ideally seek computer programs that can determine *optimal* solutions to negotiation problems, in practice, it may be beyond the scope of conventional computing techniques to compute such optimal solutions (i.e., the problem may be NP-hard, or worse). In such cases, our task is to map out the border between computationally tractable negotiation problems and computationally intractable ones, and to develop alternative techniques (heuristics or approximations) that can be used to find nearly optimal solutions, or at least satisfactory solutions, for cases where they cannot be resolved using conventional techniques.
3. Third, we aim to give an indication of the kinds of applications for which automated negotiation solutions are being considered, the kinds of issues that arise when automated negotiating systems interact with people, and

finally, how negotiation stands in relation to other related approaches to reaching agreements, such as argumentation, social choice, and fair division.

*Intended readership.* The main intended audience for this book is graduate computer science students and researchers in computer science/artificial intelligence who wish to understand the main challenges and state of the art in automated negotiation. However, we hope that the book will also be of interest and value to negotiation researchers from disciplines such as management and business studies, psychology, and economics: much of the most fruitful research is carried out in the intersections between different research disciplines, and automated negotiation seems to us to be a wonderful and fascinating exemplar of such interdisciplinary research.

*Prerequisites.* The book assumes a knowledge of computer science that would be obtained from a typical undergraduate degree in computer science, and a level of mathematical sophistication that would be consistent with such a course. With respect to the latter, the main requirements are familiarity with the basic structures of discrete mathematics, the notion of formal proof (and in particular proof by induction), the basic concepts of probability, some familiarity with series, sequences, and their sums, and ideally some understanding of basic concepts in operations research and optimisation (e.g., linear programming and related concepts).

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## Summary of key notation

### Sets

- $\mathbb{N}$  the natural numbers: 0, 1, 2, 3, ...  
 $\mathbb{R}$  the real numbers  
 $\mathbb{R}_+$  the positive real numbers:  $\mathbb{R}_+ = \{x \in \mathbb{R} \mid x \geq 0\}$   
 $\mathbb{R}_+^m$  the set of  $m$ -element vectors of positive real numbers  
 $[0, 1]$  the closed interval from 0 to 1:  $\{x \in \mathbb{R} \mid 0 \leq x \leq 1\}$   
 $(0, 1)$  the open interval from 0 to 1:  $\{x \in \mathbb{R} \mid 0 < x < 1\}$   
 $|S|$  cardinality of set  $S$   
 $2^S$  power set of set  $S$

### Games in normal form (Chapter 2)

- $P$  the set of players (i.e., the agent set)  
 $S_i$  strategy set for Player  $i \in P$   
 $S = S_1 \times \dots \times S_{|P|}$  set of strategy profiles  
 $s \in S$  a strategy profile  
 $U_i : S \rightarrow \mathbb{R}$  utility function for Player  $i \in P$   
 $G = (P, (S_i)_{i \in P}, (U_i)_{i \in P})$  normal form game  
 $MS_i \in [0, 1]^{|P|}$  mixed (randomised) strategy set for Player  $i \in P$   
 $W(s)$  social welfare of strategy profile  $s \in S$

### Games in extensive form (Chapter 3)

- $G = (X, E, \Theta, T, W, \mathcal{A}, p, U)$  extensive form game  
 $X$  set of nodes in extensive form game  
 $Z$  set of leaf nodes in game tree  
 $T : X \setminus Z \rightarrow P \cup P_0$  function that assigns to each non-leaf node a player or chance  
 $P_0$  chance move

$\Theta = \{\Theta_0, \Theta_1, \dots, \Theta_{|P|}\}$  a player partition of  $X \setminus Z$  that indicates which player has to make the decision at each non-leaf node

$W_i$  Player  $i$ 's information set

$W = \{W_1, W_2, \dots, W_{|P|}\}$  information partition: for each Player  $i \in P$ ,  $W_i$  is a partition of  $\Theta_i$

$\mathcal{A}$  choice partition, that is, a set of all possible choices of different players at different information sets

$p$  probability distribution function: assigns to each  $x \in \Theta_0$  a probability distribution  $p_x$  over the edges in  $x$

$S_i$  strategy set for Player  $i \in P$

$S = S_1 \times \dots \times S_{|P|}$  set of strategy profiles

$s \in S$  a strategy profile

$U_i : S \rightarrow \mathbb{R}$  utility function for Player  $i \in P$

$\mu$  belief system

#### Task-oriented domains (Chapter 4)

$Tsk$  set of all tasks

$c : 2^{Tsk} \rightarrow \mathbb{R}_+$  cost function

$(\mathcal{T}_1, \dots, \mathcal{T}_{|P|})$  encounter in a TOD

$D = (\mathcal{T}_1, \mathcal{T}_2)$  a deal ( $\mathcal{T}_i \subseteq Tsk$ )

$\chi$  the conflict deal

#### State-oriented domains (Chapter 4)

$St$  set of states of the environment

$Pln$  set of joint plans

$c : Pln \times P \rightarrow \mathbb{R}_+$  cost function for SODs

$st_0 \in \mathcal{S}$  initial state of environment

$\gamma_i \subseteq \mathcal{S}$  goal of Agent  $i$

$(s_0, \gamma_1, \dots, \gamma_{|P|})$  an encounter in a SOD

$J : s_1 \rightsquigarrow s_2$  plan  $J \in \mathcal{J}$  transforms  $s_1$  to  $s_2$

$worst_i$  cost of most expensive plan for  $i$

$U_i(J, s)$  utility  $i$  gets from executing plan  $J$  in state  $s$

#### Worth-oriented domains (Chapter 4)

$\mathcal{W} : St \times P \rightarrow \mathbb{R}$  worth function

#### The delivery domain (Chapter 4)

$(Loc, Links)$  undirected graph with vertices  $Loc$  and edges  $Links$

$distance : Links \rightarrow \mathbb{R}_+$  distance function

$\ell_0 \in Loc$  location of distribution warehouse

$(Loc, Links, distance, \ell_0)$  delivery domain instance  
 $\mathcal{T}_i \subseteq Loc$  Agent  $i$ 's tasks in delivery domain

#### Data allocation domain (Chapter 4)

$DS$  data sets  
 $Serv$  servers  
 $alloc : DS \rightarrow Serv$  allocation of data sets to servers  
 $usage : DS \times Serv \rightarrow \mathbb{R}_+$  usage of data sets by clients of servers  
 $c(i, j)$  cost that would be incurred by  $i$  in serving data set to  $j$

#### Production sequencing domain (Chapter 4)

$Prod$  set of products  
 $o : Prod \rightarrow \mathbb{N}$  an order  
 $Ord$  set of orders  
 $PS$  set of production sequences  
 $sat(o)$  set of production sequences satisfying order  $o$   
 $c_i : PS \rightarrow \mathbb{R}_+$  cost function for production cells

#### Single-issue negotiation (Chapter 5)

$RP_i$  reservation price of Agent  $i$   
 $\chi$  conflict deal  
 $c_i \in \mathbb{R}_+$  cost incurred by Agent  $i$  for a one-time-step delay  
 $\delta_i \in [0, 1]$  discount factor for Agent  $i$  ( $0 \leq \delta_i \leq 1$ )  
 $n \in \mathbb{N}$  negotiation deadline  
 $u^i : [0, 1] \times \mathbb{N} \rightarrow \mathbb{R}_+$  Agent  $i$ 's utility for single-issue negotiation

#### Multi-issue negotiation (Chapter 6) and the negotiation agenda (Chapter 7)

$I = \{1, \dots, m\}$  set of  $m$  issues  
 $n \in \mathbb{N}$  deadline for completing negotiation on all the issues  
 $\bar{n} \in \mathbb{N}$  deadline for a single stage  
 $\delta \in [0, 1]$  common discount factor  
 $O$  set of possible outcomes/deals  
 $o = (o_1, \dots, o_m)$  an outcome where  $o \in O$  and  $o_i \in \mathbb{R}_+^m$   
 $w_j^i \in \mathbb{R}_+$  Player  $i$ 's weight for issue  $j$   
 $u_j^i : [0, 1] \times \mathbb{N} \rightarrow \mathbb{R}_+$  Agent  $i$ 's utility for issue  $j$   
 $U_i : O \times \mathbb{N} \rightarrow \mathbb{R}_+$  Agent  $i$ 's cumulative utility for all issues  
 $EU_i : O \times \mathbb{N} \rightarrow \mathbb{R}_+$  Agent  $i$ 's expected cumulative utility for all issues  
 $s_i(t)$  Agent  $i$ 's ( $i \in \{a, b\}$ ) strategy for time period  $t$   
 $UA_i(t)$  Agent  $a$ 's equilibrium utility for issue  $i$  and time period  $t$

$\mathbf{UA}(t)$  Agent  $a$ 's cumulative equilibrium utility for time period  $t$   
 $\mathbf{UB}_i(t)$  Agent  $b$ 's equilibrium utility for issue  $i$  and time period  $t$   
 $\mathbf{UB}(t)$  Agent  $b$ 's cumulative equilibrium utility for time period  $t$   
 $\{\delta_1, \dots, \delta_r\}$  set of possible values for  $\delta$   
 $P_i$  probability that  $\delta = \delta_i$   
 $A^g \subseteq I$  an agenda of size  $g$   
 $AG^g$  set of all possible agendas of size  $g$   
 $OA^g$  Agent  $a$ 's optimal agenda of size  $g$   
 $OB^g$  Agent  $b$ 's optimal agenda of size  $g$   
 $f_j : \{1, \dots, g\} \rightarrow \{a, b\}$  first mover function for issue  $j$

### Multilateral negotiations (Chapter 8)

$P = \{1, \dots, |P|\}$  set of  $|P|$  agents  
 $\mathcal{Z} = \{z_1, z_2, \dots, z_m\}$  set of resources  
 $v_i : 2^{\mathcal{Z}} \rightarrow \mathbb{R}$  valuation function for Agent  $i$   
 $p_i$  payment to Agent  $i$   
 $\bar{p} = (1, \dots, p_{|P|})$  payment vector

### Heuristic approaches for automated negotiation (Chapter 9)

$m \in \mathbb{N}$  the number of negotiation issues  
 $x_{a \rightarrow b}^j(t)$  price offered by Agent  $a$  to Agent  $b$  at time  $t$   
 $\min_i^j$  the reserve price for Agent  $i$  ( $i \in \{a, b\}$ )  
 $\alpha_j^a : \mathbb{N} \times \mathbb{R} \rightarrow [0, 1]$  negotiation decision function for Agent  $a$   
 $t_{max}^i$  Agent  $i$ 's negotiation deadline ( $i \in \{a, b\}$ )  
 $k_j^i \in \mathbb{R}_+$  concession made by Agent  $i$  ( $i \in \{a, b\}$ ) in the first time period  
 $NA(t)$  set of agents negotiating with Agent  $a$  at time  $t$   
 $\mathcal{Z}_{i \leftrightarrow a}^t$   
 $\tau^a$  time Agent  $a$  negotiates with a single agent  
 $|\mathcal{Z}_{i \leftrightarrow a}^t|$  length of negotiation between  $i$  and  $a$   
 $\mathcal{M} \in \mathbb{N}$  maximum amount by which an agent can change her imitative behaviour  
 $\mathcal{R}(\mathcal{M})$  a random number in the range  $[0, \mathcal{M}]$

### Axiomatic models of negotiation (Chapter 11)

$\mathcal{A}$  set of possible agreements  
 $D$  disagreement point  
 $U_i : \mathcal{A} \cup \{D\} \rightarrow \mathbb{R}_+$  utility function for Agent  $i$   
 $\mathcal{S}$  the bargaining set, a compact convex subset of  $\mathbb{R}_+^2$   
 $d \in \mathbb{R}_+^2$  utility pair for the disagreement outcome  
 $d_i \in \mathbb{R}_+$  Agent  $i$ 's utility for the disagreement outcome

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*Summary of key notation*

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$\mathcal{B}$  set of all bargaining problems of the form  $(\mathcal{S}, d)$

$f: \mathcal{B} \rightarrow \mathbb{R}_+^2$  bargaining function

$x^i$  Agent  $i$ 's utility from an agreement