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To my love, Bertram

Contents

Preface

The purpose of this book is to provide an introduction to the use of partial differential equations (PDEs) for digital image restoration. It is a way of sharing what I have learned while studying these methods for about ten years, defending a Ph.D. thesis on PDE inpainting, teaching courses in Göttingen and Cambridge on the topic and writing a couple of research papers on the use of PDEs in image inpainting along the way. Let me say what this book is and what it is not. It is:

1. An introduction to inpainting methods that use PDEs and *local* variational approaches to restore lost image contents;
2. An account from an enthusiast on some state-of-the-art inpainting methods from their abilities to their limitations and reference for informed researchers in the field of digital image processing; and
3. A work targeted at readers with basic knowledge in functional analysis, PDEs, measure theory and convex optimisation.
   Therefore, I recommend this textbook only for students from the graduate level onwards.

This book is not:

1. A book that gives credit to the whole wealth of inpainting methods (in particular, this book will focus only on local inpainting methods and will scratch only non-local inpainting methods such as exemplar-based inpainting); or
2. A book for undergraduate students.

One final comment before we go *in medias res*: the more I learn and understand about image inpainting, the more the complexity of the matter and

the variety of different problems, each requiring specialised methods, become apparent. Therefore, I feel a bit like Goethe's Faust, who exclaims:

Habe nun, ach! Philosophie,
Juristerei und Medizin,
Und leider auch Theologie
Durchaus studiert, mit heißem Bemühn.
Da steh ich nun, ich armer Tor!
Und bin so klug als wie zuvor;
Heiße Magister, heiße Doktor gar
Und ziehe schon an die zehen Jahr
Herauf, herab und quer und krumm
Meine Schüler an der Nase herum –
Und sehe, daß wir nichts wissen können!
Das will mir schier das Herz verbrennen.

Let me finally mention some important people who have influenced me and this work. This book could not have been written without the support of many of my colleagues and friends.

First of all, I thank Bertram Düring, who accompanied and supported me thoughout the work on this book. I also thank Peter Markowich – without him, I would never have started to write this book, nor finish it without his regular enquiries, 'Und, wie gehts dem Buch?' And I thank Arieh Iserles for inspiring me with his renowned book on the numerical solution of partial differential equations.

Many sections of this book are based on joint work with various co-authors: Wolfgang Baatz, Martin Benning, Andrea Bertozzi, Christoph Brune, Martin Burger, Luca Calatroni, Bertram Düring, Massimo Fornasier, Carsten Gottschlich, Lin He, Rob Hocking, Rien Lagerwerf, Andreas Langer, Jan Lellmann, Peter Markowich, Jean-Michel Morel, Stanley Osher, Kostas Papafitsoros and Bati Sengul. These people have made working on the topic of image inpainting interesting, exciting and fun.

I also owe many thanks to my image-analysis group for proofreading and invaluable comments on the presentation of this book. I especially thank Luca Calatroni, Rob Hocking, Jan Lellmann and Kostas Papafitsoros.