QUASILINEAR CONTROL

Performance Analysis and Design of Feedback Systems with Nonlinear Sensors and Actuators

This is a textbook on quasilinear control (QLC). QLC is a set of methods for performance analysis and design of linear plant/nonlinear instrumentation (LPNI) systems. The approach of QLC is based on the method of stochastic linearization, which reduces the nonlinearities of actuators and sensors to quasilinear gains. Unlike the usual – Jacobian linearization – stochastic linearization is global. Using this approximation, QLC extends most of the linear control theory techniques to LPNI systems. In addition, QLC includes new problems, specific for the LPNI scenario. Examples include instrumented LQR/LQG, in which the controller is designed simultaneously with the actuator and sensor, and partial and complete performance recovery, in which the degradation of linear performance is either contained by selecting the right instrumentation or completely eliminated by the controller boosting.

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To my parents, with love,

SHINUNG CHING

To my wife Haengju, my son David, and my mother Ahn Young,
with love and gratitude,

YONGSOON EUN

To my family, with love and gratitude,

PIERRE T. KABAMBA

To my dear wife Terry and to our children, Meera, Meir, Leah,
and Rachel, with deepest love and admiration,

SEMYON M. MEERKOV

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Preface

Purpose: This volume is devoted to the study of feedback control of so-called *linear plant/nonlinear instrumentation* (LPNI) systems. Such systems appear naturally in situations where the plant can be viewed as linear but the instrumentation, that is, actuators and sensors, can not. For instance, when a feedback system operates effectively and maintains the plant close to a desired operating point, the plant may be linearized, but the instrumentation may not, because to counteract large perturbations or to track large reference signals, the actuator may saturate and the nonlinearities in sensors, for example, quantization and dead zones, may be activated.

The problems of stability and oscillations in LPNI systems have been studied for a long time. Indeed, the theory of absolute stability and the harmonic balance method are among the best known topics of control theory. More recent literature has also addressed LPNI scenarios, largely from the point of view of stability and anti-windup. However, the problems of performance analysis and design, for example, reference tracking and disturbance rejection, have not been investigated in sufficient detail. This volume is intended to contribute to this end by providing methods for designing *linear controllers* that ensure the desired *performance* of closed loop LPNI systems.

The methods developed in this work are similar to the usual linear system techniques, for example, root locus, LQR, and LQG, modified appropriately to account for instrumentation nonlinearities. Therefore, we refer to these methods as *quasilinear* and to the resulting area of control as *quasilinear control*.

Intent and prerequisites: This volume is intended as a textbook for a graduate course on quasilinear control or as a supplementary textbook for standard graduate courses on linear and nonlinear control. In addition, it can be used for self-study by practicing engineers involved in the analysis and design of control systems with nonlinear instrumentation.

The prerequisites include material on linear and nonlinear systems and control. Some familiarity with elementary probability theory and random processes may also be useful.

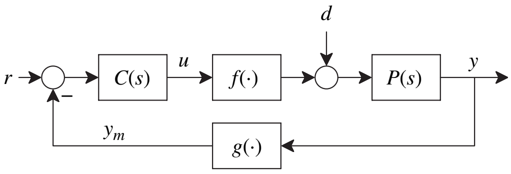

Figure 0.1. Linear plant/nonlinear instrumentation control system

Problems addressed: Consider the single-input single-output (SISO) system shown in Figure 0.1, where $P(s)$ and $C(s)$ are the transfer functions of the plant and the controller; $f(\cdot)$, $g(\cdot)$ are static odd nonlinearities characterizing the actuator and the sensor; and r, d, u, y, and y_m are the reference, disturbance, control, plant output, and sensor output, respectively. In the framework of this system and its multiple-input multiple-output (MIMO) generalizations, this volume considers the following problems:

- P1. *Performance analysis:* Given $P(s)$, $C(s)$, $f(\cdot)$, and $g(\cdot)$, quantify the quality of reference tracking and disturbance rejection.
- P2. *Narrow sense design:* Given $P(s)$, $f(\cdot)$, and $g(\cdot)$, design a controller $C(s)$ so that the quality of reference tracking and disturbance rejection meets specifications.
- P3. *Wide sense design:* Given $P(s)$, design a controller $C(s)$ and select instrumentation $f(\cdot)$ and $g(\cdot)$ so that the quality of reference tracking and disturbance rejection meets specifications.
- P4. *Partial performance recovery:* Let $C_\ell(s)$ be a controller, which is designed under the assumption that the actuator and the sensor are linear and which meets reference tracking and disturbance rejection specifications. Given $C_\ell(s)$, select $f(\cdot)$ and $g(\cdot)$ so that the performance degradation is guaranteed to be less than a given bound.
- P5. *Complete performance recovery:* Given $f(\cdot)$ and $g(\cdot)$, modify, if possible, $C_\ell(s)$ so that performance degradation does not take place.

This volume provides conditions under which solutions of these problems exist and derives equations and algorithms that can be used to calculate these solutions.

Nonlinearities considered: We consider actuators and sensors characterized by piecewise continuous odd scalar functions. For example, we address:

- saturating actuators,

$$f(u) = \mathrm{sat}_\alpha(u) := \begin{cases} \alpha, & u > +\alpha, \\ u, & -\alpha \le u \le \alpha, \\ -\alpha, & u < -\alpha, \end{cases} \tag{0.1}$$

where α is the actuator authority;

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- quantized sensors,

$$g(y) = \mathrm{qn}_\Delta(y) := \begin{cases} +\Delta\lfloor +y/\Delta \rfloor, & y \geq 0, \\ -\Delta\lfloor -y/\Delta \rfloor, & y < 0, \end{cases} \tag{0.2}$$

  where Δ is the quantization interval and $\lfloor u \rfloor$ denotes the largest integer less than or equal to y;
- sensors with a deadzone,

$$g(y) = \mathrm{dz}_\Delta(y) := \begin{cases} y-\Delta, & y > +\Delta, \\ 0, & -\Delta \leq u \leq +\Delta, \\ y+\Delta, & y < -\Delta, \end{cases} \tag{0.3}$$

  where 2Δ is the deadzone width.

The methods developed here are *modular* in the sense that they can be modified to account for any odd instrumentation nonlinearity just by replacing the general function representing the nonlinearity by a specific one corresponding to the actuator or sensor in question.

Main difficulty: LPNI systems are described by relatively complex nonlinear differential equations. Unfortunately, these equations cannot be treated by the methods of modern nonlinear control theory since the latter assumes that the control signal enters the state space equations in a linear manner and, thus, saturation and other nonlinearities are excluded. Therefore, a different approach to treat LPNI control systems is necessary.

Approach: The approach of this volume is based on the method of *stochastic linearization*, which is applicable to dynamical systems with random exogenous signals. Thus, we assume throughout this volume that both references and disturbances are random. However, several results on tracking deterministic references (e.g., step, ramp) are also included.

According to stochastic linearization, the static nonlinearities are replaced by *equivalent* or *quasilinear* gains N_a and N_s (see Figure 0.2, where $\hat{u}$, $\hat{y}$, and $\hat{y}_m$ replace u, y, and y_m). Unlike the usual Jacobian linearization, the resulting approximation is global, that is, it approximates the original system not only for small but for large signals as well. The price to pay is that the gains N_a and N_s depend not only on the nonlinearities $f(\cdot)$ and $g(\cdot)$, but also on all other elements of Figure 0.1, including the transfer functions and the exogenous signals, since, as it turns out, N_a and N_s are functions of the standard deviations, $\sigma_{\hat{u}}$ and $\sigma_{\hat{y}}$, of $\hat{u}$ and $\hat{y}$, respectively, that is, $N_a = N_a(\sigma_{\hat{u}})$ and $N_s = N_s(\sigma_{\hat{y}})$. Therefore, we refer to the system of Figure 0.2 as a *quasilinear control system*. Systems of this type are the main topic of study in this volume.

Thus, instead of assuming that a linear system represents the reality, as in linear control, we assume that a quasilinear system represents the reality and carry out

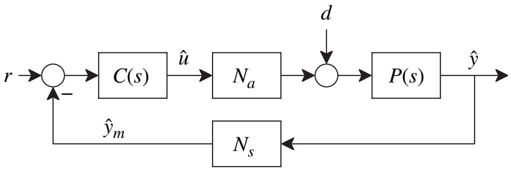

Figure 0.2. Quasilinear control system

control-theoretic developments of problems P1–P5, which parallel those of linear control theory, leading to what we call *quasilinear control* (QLC) *theory*.

The question of accuracy of stochastic linearization, that is, the precision with which the system of Figure 0.2 approximates that of Figure 0.1, is clearly of importance. Unfortunately, no general results in this area are available. However, various numerical and analytical studies indicate that if the plant, $P(s)$, is low-pass filtering, the approximation is well within 10% in terms of the variances of y and $\hat{y}$ and u and $\hat{u}$. More details on stochastic linearization and its accuracy are included in Chapter 2. It should be noted that stochastic linearization is somewhat similar to the method of harmonic balance, with $N_a(\sigma_{\hat{u}})$ and $N_s(\sigma_{\hat{y}})$ playing the roles of describing functions.

Book organization: The book consists of eight chapters. Chapter 1 places LPNI systems and quasilinear control in the general field of control theory. Chapter 2 describes the method of stochastic linearization as it applies to LPNI systems and derives equations for quasilinear gains in the problems of reference tracking and disturbance rejection. Chapters 3 and 4 are devoted to analysis of quasilinear control systems from the point of view of reference tracking and disturbance rejection, respectively (problem P1). Chapters 5 and 6 also address tracking and disturbance rejection problems, but from the point of view of design; both wide and narrow sense design problems are considered (problems P2 and P3). Chapter 7 addresses the issues of performance recovery (problems P4 and P5). Finally, Chapter 8 includes the proofs of all formal statements included in the book.

Each chapter begins with a short motivation and overview and concludes with a summary and annotated bibliography. Chapters 2–7 also include homework problems.

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Needless to say, however, all errors, which are undoubtedly present in the book, are due to the authors alone. The list of corrections is maintained at http://www.eecs.umich.edu/∼smm/monographs/QLC/.

Last, but not least, we are indebted to our families for their love and support, which made this book a reality.