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978-1-009-36124-8 — The Materiality of Numbers
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Excerpt
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1 Numbers in a Nutshell

This is a book about what numbers are and where they come from, as understood through their *materiality*, the material devices used to represent and manipulate them: things like fingers, tallies, tokens, and symbolic notations. This book is concerned with the *natural* or *counting numbers* – the sequence *one*, *two*, *three*, *four*, and so on, and maybe as high as *ten* or *twenty* or *hundred* – that are the basis of arithmetic and mathematics. While the book focuses on how concepts of number emerge and ultimately become elaborated as arithmetic and mathematics through the use of material devices, it will also examine related phenomena, like the way numbers vary cross-culturally.

This book examines numbers through the lens of archaeology. Why *archaeology*, of all things, is a reasonable question, since numbers are not the sort of thing that can be dug up from the ground or analyzed in the lab, the activities typically performed by most archaeologists. However, archaeology is also the science of material objects, and here we are looking at numbers through their material component, the counting devices used to represent and manipulate them. These devices include *distributed exemplars* (these are objects like the arms or the hand, whose dependable quantity is used to express quantities like *two* and *five*); the fingers used in counting; tallies and other devices that accumulate quantity; tokens and forms like the abacus that accumulate, group, and permit the manipulation of quantity; and numerical notations. As noted in the preface, some of these forms are unconventional as material devices, but will be treated as such for the purposes of this analysis.

We are also taking a *cognitive* approach to material objects. Accordingly, we will consider how and why material objects contribute to numerical concepts and numerical thinking, past and present. This will require us to consider the material devices used in numbers as having a role in conceptualizing and thinking about numbers. We will consider material devices to be an implicit part of the cognitive system for numbers, and this approach and the theoretical framework used are explained in later chapters.

To understand what material devices do in numerical conceptualization and thinking, we will also need to look beyond the archaeological data and consider data from other disciplines, particularly psychology and neuroscience, paleoneurology, biological anthropology and zoology, linguistics, and ethnography. The interdisciplinary data provide information that is useful for attesting or explaining how material forms function in numbers. For example, contemporary languages often attest to ancient finger-counting in forms like *six* that mean *five and one* and in productive terms that show counting structured by the number of fingers, like *ten* (the number of the fingers) and *hundred* (the number of the fingers counted by the same amount). Similarly, neuropsychology provides insight into neural interconnections within the brain that explain why finger-counting is ubiquitous and cross-culturally prevalent. Such data are also useful for understanding what numbers are as concepts. This understanding is vital when investigating the questions of how, when, and why numbers began, as it necessarily informs what we look for in the archaeological record and how we interpret what we find there. Thus, we will begin by looking at what numbers are as concepts.

WHAT NUMBERS ARE AS CONCEPTS

> *Number* is formally defined as "a unit belonging to an abstract mathematical system and subject to specified laws of succession, addition, and multiplication; an element (such as π) of any of many mathematical systems obtained by extension of or analogy with the natural number system."
>
> (Merriam-Webster, *2014, def. 1c2 and 1c3*)

As formally if somewhat circularly defined, a *number* is an element of a mathematical system obtained by extending or analogizing the *natural numbers*,[1] which are also known as the *counting numbers*, the *whole numbers*, or the *integers* – *one*, *two*, *three*, and so on. Numbers are the basic elements of a mathematical system, so all of the things that we think of as arithmetic and mathematics develop – or have the potential to develop – once a basic counting sequence is available.

As stated, the formal definition is arguably an unsatisfactory basis for our stated goal, which is understanding numerical emergence and elaboration through the material devices used for representing and manipulating numbers. We need a definition that specifies numbers in terms of their properties – particularly those properties that can be associated with and explained by the material devices used, and which can be empirically established through the devices and properties of different cultural number systems.

We will start by considering the old and deeply philosophical questions of what numbers *are* as concepts – what the Greek philosopher Aristotle might have called their *essence*, the properties that give an entity or a substance its identity and nature. Here we will examine what numbers are as concepts by specifying their properties.

A number, first and foremost, is the idea of how many of something there are, a distinct or *discrete* amount. This is *cardinality*, or how many of something there are in a group of objects. For example, a trio has three members, a property of *threeness*, and the number *three* is how many members all trios have in common.[2] In offering this definition, the philosopher and mathematician Bertrand Russell distinguished a property of a particular trio (*threeness*) from a property shared and instantiated by all trios (the number *three*). The former is the property of having three members and is applicable to a particular trio. The latter is a number, a property of all the sets with that many members. The distinction between the quantity of a particular set and

[1] *Merriam-Webster*, 2014. [2] Russell, 1920.

the idea that a number is a quantity shared by two or more sets is consistent with the idea that number begins as the perception of quantity: The first is something we can appreciate through the perceptual system for quantity when there are no more than three or four members, while the second is the conceptualization of that quantity as a number. We will look at how material forms are used as a second (or "reference") set to express perceptible quantity, which helps us visualize, understand, and express quantities that lie beyond the perceptible range of about three or four.

A number also has a specific place in a counting sequence. This is *ordinality*, numbers in order. For example, *six* is the number between *five* and *seven*. In any counting sequence, numerical order is based on increasing size: It is *five*, *six*, *seven*, *eight*, and never *six*, *five*, *eight*, *seven* or any of the other permutations possible – though granted, the sequence *eight*, *seven*, *six*, *five* might preface an annual cheering of *Happy New Year!* in Times Square or follow the phrase "ten seconds to liftoff" at NASA. When whole numbers or integers are counted in sequence, each number is *one* more than the one it follows. In the sequence *one*, *two*, *three*, *three* follows *two* and is *one* more than *two*, and *two* follows *one* and is *one* more than *one*. While the relation of *one-more* is implicit to an ordinal sequence of counting numbers, it is not necessarily explicit. After all, ordinality is no more than ordering, and as such, is as equally applicable to sequences like the letters of the alphabet or the days of the week as it is to a sequence of counting numbers. Ordinality does not fix the interval between any of the members of any sequence. Discovering that the interval between counting numbers is *one* is a matter of using material devices, where each new notch on a tally, for example, can be visually discerned as *one-more* than the previous notch in the process of making them.

Numbers have the potential for many more relations between them than just *one-more*. For example, *six* is the result of adding *four* and *two*, one of the many additive combinations that produce this number; others are *three* plus *three*, *five* plus *one*, *eight* minus *two*, and *thirteen* minus *seven*. Even *one*, *two*, and *three* are potentially related to each other in more ways than just the *one-more* of an ordinal counting sequence, since for example, *three* is *two* more than *one*. Just like the explicit *one-more* relation between sequential numbers was a matter of elaboration, so too are any other explicit relations between numbers. What is required for such elaboration is a manipulable technology like pebbles or tokens, objects that can be rearranged into different subgroups.

Numbers – or rather, the relations between them – have the potential to be manipulated by means of *operations* like addition and subtraction. Operations can involve explicit relations between numbers. For example, knowing the relations between *two*, *four*, and *six* permits the addition of *two* and *four* to obtain *six*, the subtraction of *two* from *six* to obtain *four*, and the subtraction of *four* from *six* to obtain *two*. It is also possible to add and subtract without explicit relations. For example, two groups of like objects can be commingled, and the whole counted to obtain the total without knowing any relations between numbers. This is true of numerical counters as well, since the beads on an abacus can be moved without the numerical relations being explicit. In any case, when relations are explicit, they facilitate the ability to compute mentally, rather than mechanically. Such relations are essential to mental – or, more accurately, knowledge-based – calculation. The corollary to that thought is this: When such relations do not yet exist, knowledge-based calculation is not yet possible. We will look at how material forms support the emergence of mechanical and knowledge-based calculation.

Not all numbers have attributes like the meshwork of potential relations – for example, *two* being the square root of *four* and the difference between 1,245,762 and both 1,245,760 and 1,245,764 – that characterize Western numbers. These are numbers in a decimal or base 10 system typically written in the familiar Hindu–Arabic notations (0 through 9). We are particularly interested in the differences between cultural number systems, not only because they are

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fascinating, but also because they are potential clues to where numbers come from and how they become elaborated over time.

THE WORKING DEFINITION OF NUMBER

The working definition of number used here is this: *Numbers* are concepts of discrete quantity, arranged in magnitude order, with relations between them, and operations that manipulate the relations (Fig. 1.1). As a system of numbers elaborates, it will also acquire a *productive base*, a number upon which other numbers are built. For example, in Western numbers, the number *ten* serves as the

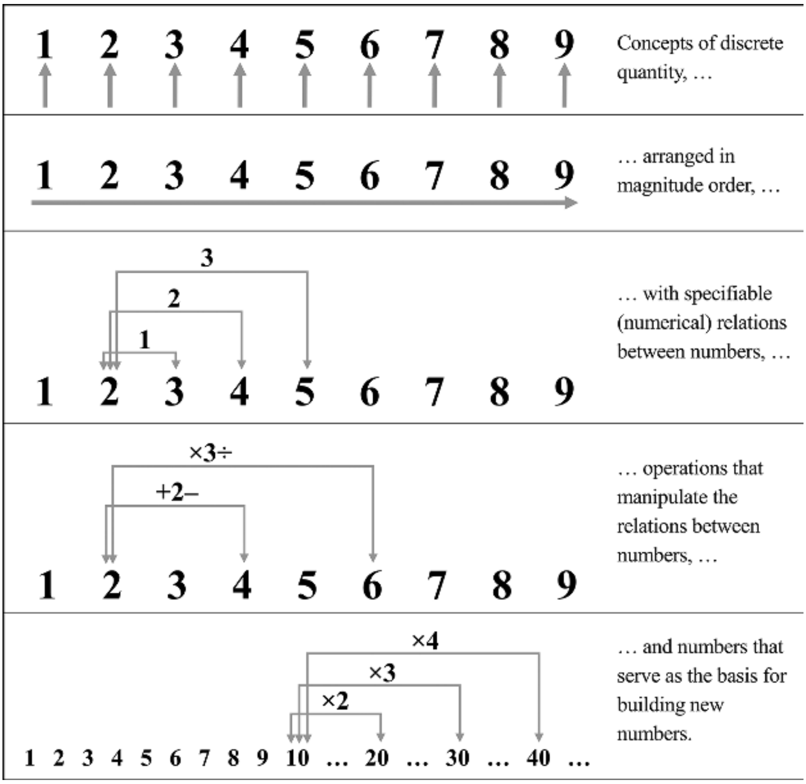

FIG. 1.1 The working definition of number. The definition focuses on five key properties: discreteness, magnitude ordering, relatedness, operational manipulability, and productive grouping. Image by the author.

productive base, as it is repeatedly added or multiplied to produce values like *twenty* (either 10 + 10 or 10 × 2), *thirty* (10 + 10 + 10 or 10 × 3), *hundred* (ten tens or 10 × 10), and *thousand* (ten hundreds or 10 × 10 × 10).

These qualities are simply, no more and no less, what Western readers will already know about numbers from what they have been exposed to through culture and language and have learned through formal education. Granted, many readers may not have thought explicitly about numbers in terms of such properties before. Readers have also learned algorithms, or sequences of operations, that enable them to do things like add columns of numbers, divide one number into another regardless of which one is larger, and convert fractions from ratios to decimal format. While algorithmic insight will not be much called upon here – since our interest lies more in how such computations are performed, rather than performing such computations – readers can nonetheless use their existing knowledge of numbers and computations as a basis for gaining new insights into how such things become elaborated from a sequence of counting numbers, say, the numbers *one* through *ten*.

ANALYZING NUMBERS THROUGH AN EXISTING FAMILIARITY

People are enculturated into the numbers of their society from day one. For example, people in the Western tradition are exposed to objects that have quantity and can be counted; social behaviors like counting and finger-counting; social purposes like inventorying that involve numbers; material representations of numbers like written symbols and tally marks; and different forms of numbers in language. This means that most readers will have a considerable knowledge of numbers, whether or not that knowledge is explicit in the particular ways used here.

Something to keep in mind about our familiar Western numbers is that the Western numerical tradition is quite old. Its roots lie deep in the world's ancient mathematical traditions, those of Rome,

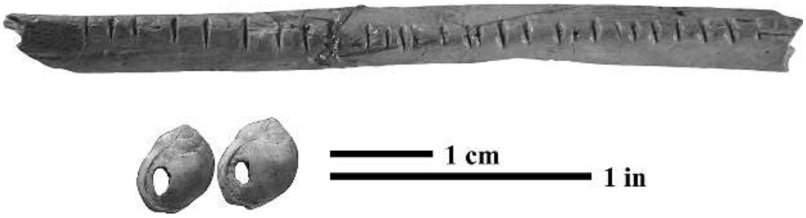

FIG. 1.2 Possible prehistoric counting devices. (Top) Notched bone from Border Cave, South Africa, dated to approximately 42,000 years ago. (Bottom) Shells punched to be strung from Blombos Cave, South Africa, dated to approximately 77,000 years ago.
Top image adapted from d'Errico et al. (2012, Supporting Information, Fig. 9, top image). Image © *PNAS* and used with permission. Bottom image adapted from one by Christopher Henshilwood and Francesco d'Errico, distributed under a Creative Commons license.

Greece, India, Egypt, and Babylon, traditions with even deeper temporal roots in counting sequences and practices that would have developed during the Neolithic and Upper Palaeolithic. The world's earliest known unambiguous numbers are numerical impressions in clay found in Mesopotamia in the mid-fourth millennium BCE.[3] Since Mesopotamian numbers are one of their roots, this makes Western numbers at least 5000 or 6000 years old. Undoubtedly, Western numbers are considerably older – perhaps 20,000 or 30,000 years old – given that the Mesopotamian numbers were already significantly elaborated by the time they first appear in the archaeological record. As if this timespan were not already impressive enough, Western numbers are likely to be older still, if archaeologists are correct in interpreting 42,000-year-old notched bones as tallies[4] (Fig. 1.2, top) and 77,000-year-old shell beads as rosaries[5] (Fig. 1.2, bottom). This impressive lifespan means that Western numbers have had a lot of time to change, and indeed, they have become highly elaborated, acquiring properties that are not necessarily shared by numbers in other cultural traditions.

3 Schmandt-Besserat, 1992a; Nissen et al., 1993; Overmann, 2016b, 2019b.
4 Beaumont, 1973; d'Errico et al., 2012.
5 Henshilwood et al., 2004; d'Errico et al., 2005.

Readers' knowledge of the highly elaborated Western numerical concepts produced by this lengthy history and prehistory is a valuable resource for understanding the numbers of other cultural traditions. The key is thinking analytically about what is already known: This can help in understanding the ways in which other cultural number systems differ from the Western tradition, and in appreciating the principles of content, organization, and structure illuminated by the differences.

WHO HAS NUMBERS? AND DO WE ALL HAVE THE SAME NUMBERS?

Most, but not all, human societies have numbers. And while all societies that have numbers develop ones that are highly similar in their content, structure, and organization, no two societies develop identical number systems. We will look at differences and similarities between numerical traditions and the reasons for these differences and similarities. A major reason for similarity is that numbers emerge from the same perceptual experience of quantity and are represented with the same devices, things like the hands. Another reason, one that complicates the attempt to understand numerical emergence and elaboration, is that societies often borrow the numbers developed by another. Today, many societies have adopted Western numbers, just as the West once adopted the Hindu–Arabic notations and used them alongside the Roman numerals that have since become an archaic system retained for its prestige value.[6] The current prevalence of Western numbers reflects cultural contact, exposure, borrowing, and transfer through mechanisms like trade, conquest, and education. In many cases, the societies borrowing Western numbers had numbers that were similar to them; in other cases, the numbers differed, and this is one of the things that would have influenced the ease and speed with which the Western numbers were adopted. These matters would

6 Chrisomalis, 2020.

have been true of number systems coming into contact in ancient times as well.

Western numbers differ from other cultural systems of number. As noted, they have become adopted by many contemporary societies on the planet, and they are quite old, so they have become highly elaborated, the basis for the complex mathematics that have developed in the West.[7] They also tend to be what we think of when we think about what a number is. Unfortunately, we also tend to superimpose this Western idea of what a number is onto all the other numbers we encounter, regardless of whether they are Western or not, contemporary or ancient, or elaborated or not.

One of the reasons for this "backward appropriation"[8] – our superimposing our idea of number onto all numbers, regardless of place or time – is that we have been taught to think of number as a thing that is well defined, fixed, and timeless. This idea goes back to another of the Greek philosophers, Plato. He thought numbers were real, by which he meant abstract, immaterial, invisible, intangible, nonmental, external, and eternal entities of the same kind as those designated by words like "beauty," "truth," and "justice." While no one, including Plato himself, has ever convincingly explained how we might come into contact with entities we can neither see nor touch, the idea that we somehow did has seemed to explain one of the most interesting qualities of numbers, their universality. That is, everyone has the same numbers that everyone else does, not personal or idiosyncratic systems of numbers. This is even true cross-culturally, despite the variability that is to be found there. While number is not a monolithic construct, a number is still recognizably a number, no matter how the details of its properties might differ.

Numbers also work the same for everyone. If we were to add several numbers together, we would get the same results that everyone else does: 2 + 2 equals 4, assuming that everyone performs the calculation correctly. If we were to prove that an equation or

[7] Gowers, 2008. [8] Rotman, 2000, p. 40.