Chapter 1: Review of number concepts

1.1 Different types of numbers

KEY LEARNING STATEMENTS

- Real numbers are either rational or irrational.
- You can write rational numbers as fractions in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. (Integers are negative and positive whole numbers, and zero.)
- Rational numbers include integers, fractions, recurring and terminating decimals and percentages.

KEY CONCEPTS

- Classifying and using different types of numbers.
- Interpreting and using the symbols $=$, $\neq$, $<$, $>$, $\leqslant$ and $\geqslant$.

1 Use the numbers in the box. List the numbers that are:

a natural **b** integers **c** prime **d** fractions.

-0.2	-57	3.142	0	$0.\dot{3}$	1	51	$10\,270$	$-\frac{1}{4}$	$\frac{2}{7}$	11	$\sqrt[3]{512}$

2 List:

a four square numbers greater than 100.

b four rational numbers smaller than $\frac{1}{3}$.

c two prime numbers that are > 80.

d the prime numbers < 10.

3 What number is halfway between:

a 6.2 and 6.5 **b** 4 and 1.2 **c** -3 and 7 **d** 39 and 40.1

4 Two of the highest earning films of all time are *Avatar*, with gross earnings of two billion, eight hundred and forty-seven million, three hundred and seventy-nine thousand, seven hundred and ninety-four dollars and *Avengers: Endgame* with gross earnings of two billion, seven hundred and ninety-seven million, five hundred and one thousand, three hundred and twenty-eight dollars.

a Write each amount in digits.

b What is the difference between the two amounts? Give your answer in digits and in words.

1.2 Multiples and factors

KEY LEARNING STATEMENTS

- When you multiply a number by another number you get a multiple of the original number.
- The lowest common multiple (LCM) of two or more numbers is the lowest number that is a multiple of both (or all) of the numbers.
- Any number that will divide into a number exactly is a factor of that number.
- The highest common factor (HCF) of two or more numbers is the highest number that is a factor of all the given numbers.

KEY CONCEPT

Finding the highest common factor and lowest common multiple of two or more numbers.

1 Find the LCM of the given numbers.

a 9 and 18 **b** 12 and 18 **c** 15 and 18

d 24 and 12 **e** 36 and 9 **f** 12 and 8

2 Find the HCF of the given numbers.

a 12 and 18 **b** 18 and 36 **c** 27 and 90

d 12 and 15 **e** 20 and 30 **f** 19 and 45

3 Amira has two rolls of cotton fabric. One roll has 72 metres on it and the other has 90 metres on it. She wants to cut the fabric to make as many equal length pieces as possible of the longest possible length. How long should each piece be?

4 In a shopping mall promotion every 30th shopper gets a $10 voucher and every 120th shopper gets a free meal. How many shoppers must enter the mall before one receives both a voucher and a free meal?

5 Amanda has 40 pieces of fruit and 100 sweets to share with the students in her class. She is able to give each student an equal number of pieces of fruit and an equal number of sweets. What is the largest possible number of students in her class?

6 The Smit family want to tile their rectangular veranda with dimensions 3.2 metres × 6.72 metres with a whole number of identical square tiles. They want the tiles to be as large as possible.

a Find the area of the largest possible tiles in cm^2.

b How many of these tiles will they need to tile the veranda?

TIP

To find the LCM of a set of numbers, you can list the multiples of each number until you find the first multiple that is in the lists for all of the numbers in the set.

TIP

You need to work out whether to use LCM or HCF to find the answers. Problems involving LCM usually include repeating events. Problems involving HCF usually involve splitting things into smaller pieces or arranging things in equal groups or rows.

REFLECTION

Read through the problems in the exercise carefully.

How can they help you to recognise similar problems in future, even if you are not told to use HCF and LCM?

1.3 Prime numbers

KEY LEARNING STATEMENTS

- Prime numbers only have two factors: 1 and the number itself.
- Factors of a number that are also prime numbers are called prime factors.
- You can write any number as a product of prime factors. Remember the number 1 itself is *not* a prime number, so you cannot use it to write a number as the product of its prime factors.
- You can use the product of prime factors to find the HCF or LCM of two numbers.

KEY CONCEPT

Prime numbers and prime factors.

1 Identify the prime numbers in each set.

a 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

b 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60

c 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105

2 Express the following numbers as a product of their prime factors.

a 36 **b** 65 **c** 64 **d** 84

e 80 **f** 1000 **g** 1270 **h** 1963

TIP

You can use a tree diagram or division to find the prime factors of a composite whole number.

3 Find the LCM and the HCF of the following numbers by using prime factors.

a 27 and 14 **b** 85 and 15 **c** 96 and 27

d 53 and 16 **e** 674 and 72 **f** 270 and 234

1.4 Working with directed numbers

KEY LEARNING STATEMENTS

- Integers are directed whole numbers.
- You write negative integers with a minus (−) sign. Positive integers may be written with a plus (+) sign, but usually they are not. Zero (0) is an integer because it is a whole number but it is neither negative nor positive.
- In real life, negative numbers are used to represent temperatures below zero, movements downwards or left, depths, distances below sea level, bank withdrawals and overdrawn amounts, and many more things.

KEY CONCEPTS

- Using directed numbers in practical situations.
- Basic calculations with positive and negative numbers.

1 If the temperature is 4 °C in the evening and it drops 7 °C overnight, what will the temperature be in the morning?

2 Which is colder in each pair of temperatures?

a 0 °C or −2 °C **b** 9 °C or −9 °C **c** −4 °C or −12 °C

3 An office block has three basement levels (−1, −2 and −3), a ground floor (0) and 15 floors above the ground floor (1 to 15). Where will the lift be in the following situations?

a Starts on the ground floor and goes down one floor then up five?

b Starts on level −3 and goes up ten floors?

c Starts on floor 12 and goes down 13 floors?

d Starts on floor 15 and goes down 17 floors?

e Starts on level −2, goes up seven floors and then down eight?

4 Write the number that is 12 less than:

a 9 **b** −14 **c** −2 **d** 12

5 Calculate:

a $-400 \div 80$ **b** $-54 + 120 + (-25)$

c $-3 \times (14 - (-12))$ **d** $\frac{-18}{6} \times 3$

e $13 + (-7) + 25 + (-15)$

6 The table shows how much the value of a rupee changed in comparison to the euro over a period of five days. The rate was 80.72 rupees : 1 euro before any changes were recorded.

Day	1	2	3	4	5
Change	−0.25	+0.14	−0.27	−2.08	−3.04

a What was the value of the rupee compared to the euro and the end of day 3?

b What was the total change over the period of five days? Give your answer as a directed number.

1.5 Powers, roots and laws of indices

KEY LEARNING STATEMENTS

- Index notation is a way of writing repeated multiplication. For example, you can write $2 \times 2 \times 2$ as 2^3. 2 is the base and 3 is the index that tells you how many times 2 is multiplied by itself.
- The $\sqrt[x]{n}$ of a number is the value that is multiplied by itself x times to reach that number.
- Any number to the power of 0 is equal to 1: $a^0 = 1$.
- Negative indices are used to write reciprocals. a^{-m} is the reciprocal of a^m because $a^{-m} \times a^m = 1$.
- You can use fractional indices to express the roots of numbers. $\sqrt{a} = a^{\frac{1}{2}}$, $\sqrt[3]{a} = a^{\frac{1}{3}}$ and $\sqrt[n]{a} = a^{\frac{1}{n}}$. For non-unit fractions, $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$.
- To multiply numbers with the same base you add the indices. In general terms $a^m \times a^n = a^{m+n}$.
- To divide numbers with the same base you subtract the indices. In general terms $\frac{a^m}{a^n} = a^{m-n}$.
- To raise a power to another power you multiply the indices. In general terms $(a^m)^n = a^{mn}$.

KEY CONCEPTS

- Calculating with squares, square roots, cubes, cube roots and other powers and roots of numbers.
- The meaning of zero, negative and fractional indices.
- The laws of indices.

1 Find all the square and cube numbers between 100 and 300.

2 Simplify.

a $\sqrt{9} + \sqrt{16}$ **b** $\sqrt{9 + 16}$ **c** $\sqrt{64} + \sqrt{36}$

d $\sqrt{64 + 36}$ **e** $\sqrt{\frac{36}{4}}$ **f** $(\sqrt{25})^2$

g $\frac{\sqrt{9}}{\sqrt{16}}$ **h** $\sqrt{169 - 144}$ **i** $\sqrt[3]{27} - \sqrt[3]{1}$

j $\sqrt{100 \div 4}$ **k** $\sqrt{1} + \sqrt{\frac{9}{16}}$ **l** $\sqrt{16} \times \sqrt[3]{27}$

m $\sqrt{(-5)^2} \times \sqrt[3]{-1}$ **n** $\sqrt{\frac{1}{4}} + \sqrt{\left(\frac{1}{3}\right)^2}$ **o** $\sqrt[3]{1} - \sqrt[3]{-125}$

3 Find the value of the following.

a $13^3 - 3^5$ **b** $3^3 + 2^7$ **c** $\sqrt[3]{64} + 4^5$

d $(2^4)^3$ **e** $5^4 \times \sqrt[5]{32}$ **f** $\sqrt[6]{729} \times 5^4$

g $\sqrt[4]{625} + 5^5$

4 A cube has a volume of 12 167 cm^3. Calculate:

a the height of the cube **b** the area of one face of the cube.

5 Rewrite each of the following using only positive indices.

a 4^{-1} **b** 5^{-1} **c** 8^{-1} **d** 5^{-2} **e** 3^{-3}

f 2^{-5} **g** 3^{-4} **h** 8^{-6} **i** 23^{-3} **j** 12^{-4}

6 Express each term using a negative index.

a $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{1}{3^2}$ **d** $\frac{1}{2^3}$ **e** $\frac{1}{3^3}$

f $\frac{1}{2^4}$ **g** $\frac{1}{11^2}$ **h** $\frac{1}{4^3}$ **i** $\frac{2}{10}$ **j** $\frac{3}{9}$

7 Simplify. Leave your answers in index form.

a $3^2 \times 3^6$ **b** $10^{-2} \times 10^4$ **c** $3^8 \times 3^{-5}$ **d** $5^0 \times 3^2$

e $2^{-3} \times 2^{-4}$ **f** $3 \times 3^2 \times 3^{-2}$ **g** $4^0 \times 4^{-2} \times 4$

h $10^2 \times 10^3 \times 10^{-2}$

i $(3^2)^0$ **j** $(4^3)^4$ **k** $(3^{-2})^{-3}$ **l** $(4^{-3} \times 4^2)^{-2}$

m $\frac{10^6}{10^{-3}}$ **n** $\frac{10^0}{10^4}$ **o** $\frac{2^{-4}}{2^{-5}}$ **p** $\frac{4^3}{4^{-3}}$

8 Write each value using a root sign.

a $3^{\frac{1}{2}}$ **b** $4^{\frac{1}{3}}$ **c** $5^{\frac{1}{9}}$ **d** $4^{\frac{3}{8}}$ **e** $6^{\frac{4}{9}}$

9 Write in index notation.

a $\sqrt{7}$ **b** $\sqrt[3]{6}$ **c** $\left(\sqrt[3]{8}\right)^5$ **d** $\left(\sqrt[4]{9}\right)^3$ **e** $\left(\sqrt[6]{5}\right)^5$

TIP

Apply the index laws and work in this order:

- simplify any terms in brackets
- apply the multiplication law to numerators and then to denominators
- cancel numbers if you can
- apply the division law if the same letter appears in the numerator and denominator
- express your answer using positive indices.

10 Evaluate.

a 5^{-2} **b** $81^{\frac{1}{2}}$ **c** $\left(\frac{2}{3}\right)^{-1}$ **d** $7^{-\frac{2}{3}}$ **e** $\left(\frac{5}{2}\right)^{-2}$

f $64^{\frac{1}{6}}$ **g** $81^{\frac{3}{4}}$ **h** $\left(0.64^{\frac{1}{2}}\right)^2$ **i** $3 \times 36^{\frac{1}{2}}$ **j** $\left(3^{\frac{1}{2}}\right)^{-4}$

11 Evaluate.

a $(-3^4) \times (-4)^2$ **b** $\frac{-2^4}{(-2)^4}$ **c** $\frac{6^3}{(-3)^4}$ **d** $8^{\frac{1}{3}}$

e $256^{-\frac{1}{4}}$ **f** $125^{-\frac{4}{3}}$ **g** $\left(\frac{1}{4}\right)^{-\frac{5}{2}}$ **h** $\left(\frac{1}{8}\right)^{-\frac{2}{3}}$

i $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$ **j** $\left(\frac{8}{18}\right)^{-\frac{1}{2}}$

12 Calculate.

a $5 - 7(23 - 5^2) - 16 \div 2^3$ **b** $3(5^2) - 6(-3^2 - 4^2) \div -15$

c $-2(-3^2) + 24 \div (-2)^3$ **d** $-2(3)^4 - (6 - 7)^6$

1.6 Order of operations

KEY LEARNING STATEMENTS

- When there is more than one operation to be done in a calculation you must work out the parts in brackets first. Then do any division or multiplication (from left to right) before adding and subtracting (from left to right).
- Long fraction lines and square or cube root signs act like brackets, indicating parts of the calculation that have to be done first.
- Scientific calculators apply the rules for order of operations automatically. If there are brackets, fractions or roots in your calculation you need to enter these correctly on the calculator. When there is more than one term in the denominator, the calculator will divide by the first term only unless you enter brackets.

KEY CONCEPT

Calculating using the correct order of operations.

1 Calculate and give your answer correct to two decimal places.

a $8 + 3 \times 6$ **b** $(8 + 3) \times 6$ **c** $8 \times 3 - 4 \div 5$

d $12.64 + 2.32 \times 1.3$ **e** $6.5 \times 1.3 - 5.06$ **f** $(6.7 \div 8) + 1.6$

g $1.453 + \frac{7.6}{3.2}$ **h** $\frac{5.34 + 3.315}{4.03}$ **i** $\frac{6.54}{2.3} - 1.08$

j $\frac{5.27}{1.4 \times 1.35}$ **k** $\frac{11.5}{2.9 - 1.43}$ **l** $\frac{0.23 \times 4.26}{1.32 + 3.43}$

m $8.9 - \frac{8.9}{10.4}$ **n** $\frac{12.6}{8.3} - \frac{1.98}{4.62}$ **o** $12.9 - 2.03^2$

p $(9.4 - 2.67)^3$ **q** $12.02^2 - 7.05^2$ **r** $\left(\frac{16.8}{9.3} - 1.01\right)^2$

s $\frac{4.07^2}{8.2 - 4.09}$ **t** $6.8 + \frac{1.4}{6.9} - \frac{1.2}{9.3}$ **u** $4.3 + \left(1.2 + \frac{1.6}{5}\right)^2$

v $\frac{6.1}{2.8} + \left(\frac{2.1}{1.6}\right)^2$ **w** $6.4 - (1.2^2 + 1.9^2)^2$ **x** $\left(4.8 - \frac{1}{9.6}\right) \times 4.3$

TIP

Remember the order of operations using BODMAS:

Brackets
Orders
Divide
Multiply
Add
Subtract

Some people remember the order of operations as BIDMAS – I stands for indices.

1.7 Rounding and estimating

KEY LEARNING STATEMENTS

- You may be asked to round numbers to a given number of decimal places or to a given number of significant figures.
- To round to a decimal place:
  - look at the value of the digit to the right of the place you are rounding to
  - if this value is ⩾5 then you round up (add 1 to the digit you are rounding to)
  - if this value is ⩽4 then leave the digit you are rounding to as it is.
- To round to a significant figure:
  - the first non-zero digit (before or after the decimal place in a number) is the first significant figure
  - find the correct digit and then round off from that digit using the rules above.
- Estimating involves rounding values in a calculation to numbers that are easy to work with (usually without the need for a calculator).
- An estimate allows you to check that your calculations make sense.

KEY CONCEPTS

- Rounding numbers to a given number of decimal places or significant figures.
- Estimating an approximate answer.

1 Round these numbers to:

i two decimal places

ii one decimal place

iii the nearest whole number.

a 5.6543 **b** 9.8774 **c** 12.8706 **d** 0.0098

e 10.099 **f** 45.439 **g** 13.999 **h** 26.001

2 Round each of these numbers to three significant figures.

a 53 217 **b** 712 984 **c** 17.364 **d** 0.007279

3 Round the following numbers to two significant figures.

a 35.8 **b** 5.234 **c** 12 345 **d** 0.00875

e 432 128 **f** 120.09 **g** 0.00456 **h** 10.002

4 Use whole numbers to show why these estimates are correct.

a 3.9×5.1 is approximately equal to 20

b 68×5.03 is approximately equal to 350

c 999×6.9 is approximately equal to 7000

d $42.02 \div 5.96$ is approximately equal to 7

TIP

If you are told what degree of accuracy to use, it is important to round to that degree. If you are not told, you can round to 3 significant figures.

5 Estimate the answers to each of these calculations to the nearest whole number.

a $5.2 + 16.9 - 8.9 + 7.1$ **b** $(23.86 + 9.07) \div (15.99 - 4.59)$

c $\dfrac{9.3 \times 7.6}{5.9 \times 0.95}$ **d** $8.9^2 \times \sqrt{8.98}$

REVIEW EXERCISE

1 State whether each number is natural, rational, an integer and/or a prime number.

$-\frac{3}{4}$	24	0.65	−12	$3\frac{1}{2}$	0	0.66	17

2 **a** List the factors of 36.

b How many of these factors are prime numbers?

c Express 36 as the product of its prime factors.

d List two numbers that are factors of both 36 and 72.

e What is the highest number that is a factor of both 36 and 72?

3 Write each number as a product of its prime factors.

a 196 **b** 1845 **c** 8820

4 Amira starts a new exercise routine on 3 March. She decides she will swim every three days and cycle every four days. On which dates in March will she swim and cycle on the same day?

5 State whether each equation is true or false.

a $18 \div 6 + (5 + 3 \times 4) = 20$ **b** $6 \times (5 - 4) + 3 = 9$

c $\dfrac{30 + 10}{30} - 10 = 1$ **d** $(6 + 3)^2 = 45$

6 Simplify:

a $\sqrt{100} \div \sqrt{4}$ **b** $\sqrt{100 \div 4}$

c $(\sqrt[3]{64})^3$ **d** $4^3 + 9^2$

e $2^3 \times \sqrt[4]{1296}$ **f** $(-2)^4 \times \sqrt[3]{343}$

g $\left(\frac{1}{2}\right)^{-2} + \sqrt[5]{1}$ **h** $\left(\frac{1}{2}\right)^{-4} - \sqrt[6]{46\,656}$

CONTINUED

7 Calculate. Give your answer correct to two decimal places.

a $\frac{5.4 \times 12.2}{4.1}$ **b** $\frac{12.2^2}{3.9^2}$ **c** $\frac{12.65}{2.04} + 1.7 \times 4.3$

d $\frac{3.8 \times 12.6}{4.35}$ **e** $\frac{2.8 \times 4.2^2}{3.3^2 \times 6.2^2}$ **f** $2.5 - \left(3.1 + \frac{0.5}{5}\right)^2$

8 Write each of the following in the form of 3^x.

a 1 **b** 27 **c** $\frac{1}{9}$ **d** $\frac{1}{3}$

e $\sqrt{27}$ **f** $3^4 \times 3^{-2}$ **g** $\frac{3^8}{3^8}$ **h** $\frac{3^2}{3^4}$

i $(3^2)^4$ **j** $(3^{-2})^2$

9 Simplify. Leave your answers in index notation.

a $\frac{3^4 \times 3^7}{3^4}$ **b** $\frac{2^5 \times 2^4}{2^3}$ **c** $\frac{2^3 \times 2^{-4}}{2^2 \times 2^{-2}}$ **d** $\frac{4 \times 4^{-3}}{4^{-2} \times 4^0}$

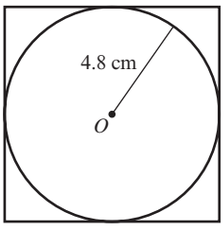
10 Determine the value of x in each equation.

a $\frac{2^2}{2^5} = 2^x$ **b** $2 \times 2^x = \frac{2^3}{2^5}$ **c** $\frac{3^x}{3} = \frac{3^2}{3^5}$ **d** $2^2 \times 2^{-x} = \frac{2^2}{2^6}$

11 Round each number to three significant figures.

a 1235.6 **b** 0.76513 **c** 0.0237548 **d** 31.4596

12 The diagram shows a design for a square tile with a circle printed on it. The circle has a radius of 4.8 cm.



a What is the area of the square tile?

b How much of the square is not covered by the circle? Give your answer correct to two decimal places.

13 Ziggy has a square sheet of fabric with sides 120 cm long. Is this big enough to cover a square table of area 1.4 m^2? Explain your answer.

14 A cube has a volume of 3.375 m^3. How high is it?

15 Estimate the answer to each of these calculations to the nearest whole number.

a 9.75×4.108 **b** $0.0387 \div 0.00732$

c $\frac{36.4 \times 6.32}{9.987}$ **d** $\sqrt{64.25} \times 3.098^2$

TIP

You can find the area of a circle using the formula $A = \pi r^2$.