A GENERAL RELATIVITY COURSEBOOK

General relativity is a subject that most undergraduates in physics are particularly curious about, but it has a reputation for being very difficult. This book provides as gentle an introduction to general relativity as possible, leading you through the necessary mathematics in order to arrive at important results. Of course, you cannot avoid the mathematics of general relativity altogether, but, using this book, you can gain access to and appreciation of tensors and differential geometry at a pace you can keep up with. Early chapters build up to a complete derivation of Einstein's equations, while the final chapters cover the key applications on black holes, cosmology and gravitational waves. It is designed as a coursebook with just enough material to cover in a one-semester undergraduate class, but it is also accessible to any numerate readers who wish to appreciate the power and beauty of Einstein's creation for themselves.

ED DAW is Professor of Particle Astrophysics at the University of Sheffield. He has worked as an experimental physicist since 1998, on searches for dark matter and gravitational waves. His work on gravity led him to volunteer to teach general relativity at Sheffield, which he has continued to do from 2003 until the present. He considers general relativity a hobby, albeit one that is crucial to underpin his understanding of his own research. He also enjoys trying to explain hard things in simple terms, a very good habit for a professor.

The approach in the book is unique, and especially valuable for the student first encountering general relativity. It shows in detail the computational steps involved in gaining the main results.

RAINER WEISS, Nobel laureate; Professor Emeritus at MIT

I think this is an excellent introduction to general relativity, and its important applications to cosmology and gravitational wave astrophysics, for the serious student who has not experienced the necessary mathematical formalism before and is willing to follow the text and attempt the many examples. It is an ideal lead-in to many of the more sophisticated modern textbooks which are now available.

SIR JAMES HOUGH, OBE FRS FRSE; University of Glasgow

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Dedicated to my father, Stephen Francis Daw, 1944–2012

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Preface

General relativity (GR) is one of the most fascinating areas of physics. It is therefore naturally an attractor for undergraduates, often given as a reason for choosing the subject. It is not, however, always offered as an undergraduate course because it is hard to teach. I have taught an undergraduate GR course at Sheffield since 2004 (with a few years off at one point). I have developed some strategies that have allowed the course to continue to be successful. My notes are now mature enough that I offer them up as a coursebook, hoping it will be of service to others faced with the same task.

I try to adhere to the maxim of uncovering some of the material rather can covering all of it. A detailed description of the tensor formalism is unavoidable. I use what some would consider an old fashioned approach, defining tensors in terms of the transformation properties of their components. The book contains all the mathematical detail students need to arrive at important results. In this sense, it differs markedly from other books, which tend to leave the lengthier components of derivations as stated without proof or for the students to derive. Experience has taught me that students are simply not ready for such long tracts of algebra, or they don't have the time for it. So, almost everything is there, even though the algebra is sometimes tedious. I also leave in all the factors of c and G, so the whole book is in SI (MKS) units. These constants also allow you to maintain constant contact with the experimental world, to assess the relative magnitudes of terms, and to check your algebra with dimensional analysis.

There are three chapters on applications, on the Schwarzschild solution, the Friedman–Lemaître–Robertson–Walker cosmology, and gravitational waves. The chapters contain the GR at the core of these areas; the problems at the end contain selected applications. Many of the problems are therefore foundational general relativity. I find it pedagogically far better to have the students go through this work for themselves than to teach it as bookwork, as in my experience students fail to digest the latter and it is almost immediately forgotten. The problems are at a spectrum of levels of difficulty from extremely easy to quite long and involved. The idea is that there is something there for students having a wide range of abilities and levels of preparedness. There is also an example of what I refer to as a 'diagnostic quiz' in Section 1.8. I set a quiz like this to students starting the course well before the drop date, as a refresher course and a friendly warning shot across the bows for those who have embarked upon the course not understanding the required level of mathematical competency. I recommend this practice to my fellow teachers.

I find that I can cover the material up to the end of Section 8.8 in 18–20 50 minute lectures and that students who spend perhaps 6 hours a week studying for themselves succeed in learning the material. If you only have 18 lectures, then you can move quite quickly through the calculations leading to the Einstein tensors in Chapters 6 and 7, though your course will be improved if you can dwell in these areas and give students a strong feel for the weight and work represented in these calculations. At the end of the book, starting in Section 8.9.1, I have included some 'optional extras'. Firstly, there is a look at some of the technical aspects of practical experimental gravitational wave interferometry. This is not general relativity, but I believe that the subject should not live in isolation from the rest of science. It can be by all means left out of a one semester course. The final chapter is a guide to further reading, a summary of conventions in differential geometry that differ from mine, and a short review of the more advanced literature in various research fields to which general relativity is connected.

I do not know if I have succeeded in walking the balance beam between the abyss of a book that is too advanced and the disappointment a book that is too superficial. Now that I consider the book in its entirety, I see that it might be of interest beyond the original target audience. Time will tell. I hope at least to convey the love of the subject that I have gained through familiarity and that you enjoy what is between these covers.

Acknowledgments

When my colleagues at Sheffield found out that I had written this book, many of them asked how I had found the time. The book evolved as I addressed areas of confusion that I perceived amongst the students. So I thank the students for their many questions, comments and corrections. I am sure that many times at home I have been staring at my computer or into space when I should have been with my family. I thank my wife Anne and my children Georgia and Eli for their patience, encouragement and love during this journey. I would also like to thank Pieter Kok for a careful reading and my colleagues at LIGO for numerous invigorating discussions, which informed my thinking in many areas, most significantly for Chapter 8.