

Cambridge University Press & Assessment

978-1-009-19339-9 — The Calabi Problem for Fano Threefolds
C. Araujo , A.-M. Castravet , I. Cheltsov , K. Fujita , A.-S. Kaloghiros
J. Martinez-Garcia , C. Shramov , H. Süß , N. Viswanathan

Table of Contents

[More Information](#)

Contents

	<i>Introduction</i>	<i>page 1</i>
1	K-Stability	7
1.1	What is K-stability?	7
1.2	Valuative criterion	12
1.3	Complexity one \mathbb{T} -varieties	14
1.4	Tian's criterion	22
1.5	Stability threshold	31
1.6	Equivariant Stibitz–Zhuang theorem	46
1.7	Abban–Zhuang theory	54
2	Warm-Up: Smooth del Pezzo Surfaces	80
3	Proof of Main Theorem: Known Cases	117
3.1	Direct products	117
3.2	Homogeneous spaces	117
3.3	Fano threefolds with torus action	117
3.4	Del Pezzo threefolds	120
3.5	K-stable cyclic covers	122
3.6	Threefolds with infinite automorphism groups	124
3.7	Divisorially unstable threefolds	126
4	Proof of Main Theorem: Special Cases	140
4.1	Prime Fano threefolds	140
4.2	Fano threefolds with $\mathrm{PGL}_2(\mathbb{C})$ -action	147
4.3	Blow ups of del Pezzo threefolds in elliptic curves	156
4.4	Blow up of \mathbb{P}^3 in curves lying on quadric surfaces	166
4.5	Threefolds fibered into del Pezzo surfaces	184
4.6	Blow ups of Veronese and quadric cones	193
4.7	Ruled Fano threefolds	205

Cambridge University Press & Assessment

978-1-009-19339-9 — The Calabi Problem for Fano Threefolds
C. Araujo , A.-M. Castravet , I. Cheltsov , K. Fujita , A.-S. Kaloghiros
J. Martinez-Garcia , C. Shramov , H. Süß , N. Viswanathan

Table of Contents

[More Information](#)

5	Proof of Main Theorem: Remaining Cases	210
5.1	Family №2.8	210
5.2	Family №2.9	214
5.3	Family №2.11	233
5.4	Family №2.12	241
5.5	Family №2.13	246
5.6	Family №2.16	249
5.7	Family №2.17	252
5.8	Family №2.20	255
5.9	Family №2.21	269
5.10	Family №2.26	275
5.11	Family №3.2	280
5.12	Family №3.3	282
5.13	Family №3.4	284
5.14	Family №3.5	288
5.15	Family №3.6	295
5.16	Family №3.8	296
5.17	Family №3.10	303
5.18	Family №3.12	312
5.19	Family №3.13	319
5.20	Family №3.15	334
5.21	Family №4.3	342
5.22	Family №4.13	345
5.23	Family №5.1	350
6	The Big Table	354
7	Conclusion	365
7.1	Family №1.10	365
7.2	Family №2.20	367
7.3	Family №2.21	373
7.4	Family №2.22	375
7.5	Family №3.5	376
7.6	Family №3.8	379
7.7	Family №3.12	380
Appendix	Technical Results Used in Proof of Main Theorem	382
A.1	Nadel's vanishing and Kollar-Shokurov connectedness	382
A.2	Inversion of adjunction and Kawamata's subadjunction	388
A.3	Mobile log pairs and Corti's inequality	390
A.4	Equivariant tie breaking and convexity trick	392
A.5	α -invariants of del Pezzo surfaces over non-closed fields	401

Cambridge University Press & Assessment

978-1-009-19339-9 — The Calabi Problem for Fano Threefolds
C. Araujo , A.-M. Castravet , I. Cheltsov , K. Fujita , A.-S. Kaloghiros
J. Martinez-Garcia , C. Shramov , H. Süß , N. Viswanathan

Table of Contents

[More Information](#)

Contents

vii

A.6	Groups acting on Hirzebruch surfaces	412
A.7	Auxiliary results	418
<i>References</i>		430
<i>Index</i>		440