

MAKING SENSE OF NUMBER

Improving Personal Numeracy

Many adults feel that they lack the necessary foundational knowledge in mathematics required to confidently use mathematics in daily life and in their careers. *Making Sense of Number* is a concise introduction to personal and professional numeracy skills, helping readers to become more mathematically competent. It includes relevant content to assist pre-service teachers to improve numeracy for the classroom or to prepare for the LANTITE, as well as support for practising teachers to develop their understanding and skills in numeracy.

Making Sense of Number focuses on number sense as a conceptual framework for understanding mathematics, covering foundational areas of mathematics that often cause concern such as multiplication, fractions, ratio, rate and scale. The authors use real-world examples to explain mathematical concepts in an accessible and engaging way. Learning activities throughout the book help readers self-assess their understanding of the mathematical concepts discussed, and answers to activities are included.

Written by authors with over 30 years' experience teaching mathematics at primary, secondary and tertiary levels, *Making Sense of Number* is an essential guide for both pre-service teachers and those looking to improve their understanding of numeracy.

Annette Hilton is an Industry Fellow in the School of International Studies and Education at the University of Technology Sydney.

Geoff Hilton is an Honorary Research Fellow at the University of Queensland School of Education.

Cambridge University Press
978-1-009-00992-8 — Making Sense of Number
Annette Hilton , Geoff Hilton
Frontmatter
[More Information](#)

Cambridge University Press acknowledges the Australian Aboriginal and Torres Strait Islander peoples of this nation. We acknowledge the traditional custodians of the lands on which our company is located and where we conduct our business. We pay our respects to ancestors and Elders, past and present. Cambridge University Press is committed to honouring Australian Aboriginal and Torres Strait Islander peoples' unique cultural and spiritual relationships to the land, waters and seas and their rich contribution to society.

CAMBRIDGE

Cambridge University Press
978-1-009-00992-8 — Making Sense of Number
Annette Hilton , Geoff Hilton
Frontmatter

[More Information](#)

MAKING SENSE OF NUMBER

Improving Personal Numeracy

ANNETTE HILTON &
GEOFF HILTON



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-1-009-00992-8 — Making Sense of Number
Annette Hilton, Geoff Hilton
Frontmatter
[More Information](#)

CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781009009928

© Cambridge University Press 2021

This publication is copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2021

Cover designed by Anne-Marie Reeves
Typeset by Integra Software Services Pvt. Ltd
Printed in China by C & C Offset Printing Co., Ltd, July 2021

A catalogue record for this publication is available from the British Library

A catalogue record for this book is available from the National Library of Australia

ISBN 978-1-009-00992-8 Paperback
ISBN 978-1-009-03049-6 Paperback and LANTITE bundle

Reproduction and communication for educational purposes

The Australian *Copyright Act 1968* (the Act) allows a maximum of one chapter or 10% of the pages of this work, whichever is the greater, to be reproduced and/or communicated by any educational institution for its educational purposes provided that the educational institution (or the body that administers it) has given a remuneration notice to Copyright Agency Limited (CAL) under the Act.

For details of the CAL licence for educational institutions contact:

Copyright Agency Limited
Level 12, 66 Goulburn Street
Sydney NSW 2000
Telephone: (02) 9394 7600
Facsimile: (02) 9394 7601
E-mail: memberservices@copyright.com.au

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

CONTENTS

Preface	ix
Chapter 1 Reflection on personal mathematics experiences and abilities	1
Introduction	2
What is my mathematical circumstance?	3
Understanding the problem: a personal perspective	4
Been there, done that	7
Understanding the problem: a research perspective	7
What is mathematics anxiety?	8
What is the impact of mathematics anxiety?	8
Why is it important that teachers address their mathematics knowledge and dispositions?	9
Addressing mathematics difficulties	10
Number sense	10
Lifelong learning	12
Growth mindset	13
Conclusion	15
Chapter 2 Mathematics and numeracy: the role of number sense	17
Introduction	18
Numeracy: a real-world reason for learning mathematics	18
What is numeracy?	19
Some everyday examples	20
The central role of context in numeracy	22
Why is numeracy so important?	23
The key ideas of number sense	25
Mental computation	26
Estimation	26
Judging magnitudes	27
Place value	28
Numerical relationships	29
Representational fluency	29
Problem solving	30
Conclusion	32
Chapter 3 The Hindu-Arabic number system	33
Introduction	34
A brief history of the Hindu-Arabic number system	34
Feature number one: place value	36
Placement of digits in place value columns	37

Naming numbers	40
Decimal place value	42
Feature number two: Base 10	44
Feature number three: zero	45
Some things we should not have learned	45
Multiplying and dividing by powers of 10	46
Conclusion	48
Chapter 4 Number facts	50
Introduction	51
Learning the number facts	51
Addition facts	54
Multiplication facts	54
Approaches to learning number facts	57
Why learn number facts?	58
Properties to assist the learning of number facts	59
Working with zero and one	59
Commutativity	60
Auditing number fact knowledge	61
The addition grid	61
The multiplication grid	62
Arrays for learning number facts	64
Putting it all together: the power of number facts	65
Related systems	68
Conclusion	69
Chapter 5 Mental computation	70
Introduction	71
What is mental computation?	71
Why the focus on mental computation?	73
Using number facts to complete mental calculations	75
Properties of numbers and operations	76
Order of operations	76
Distributivity	77
Associativity	78
Estimation and mental computation	80
Choosing strategies to aid mental computation	82
Is the answer to my mental computation reasonable?	84
Conclusion	86
Chapter 6 Additive and multiplicative thinking	87
Introduction	88
Additive and multiplicative thinking	88

Additive thinking	88
Multiplicative thinking	90
Distinguishing between additive thinking and multiplicative thinking	91
Absolute and relative thinking	96
Relative thinking and co-variation	98
Co-variation in nature	99
Confusing relative thinking	100
Making connections	102
Connecting relative thinking to multiplicative thinking	102
Connecting additive to absolute and multiplicative to relative thinking	104
Conclusion	106
Chapter 7 Fractional thinking	108
Introduction	109
Common fractions	109
Common fractions are versatile	111
Why are common fractions useful?	116
Operations with common fractions	117
Equivalent fractions	119
Multiplying fractions	120
Dividing fractions	121
Some possible reasons for difficulty with common fractions	122
Decimals	123
Working with decimals	124
Percentages	126
Working across the three main fraction types	130
Conclusion	132
Chapter 8 Ratio, rate and scale	133
Introduction	134
Ratio	134
Representing ratio	134
Developing ratio understanding	136
Real-world examples of ratio	138
Rate	142
Common rates in everyday life	142
Long-term effects of rates	146
Scale	148
Relative size	149
Reading scales	150
Scale factor	153
One-, two- and three-dimensional scale	154
Not to scale	157
Conclusion	158

Chapter 9	Problem solving	159
	Introduction	160
	Problem solving and life	160
	Problem solving and growth mindset	161
	Contributing skills for problem solving	162
	The language of problem solving	162
	Multiple representations	164
	Thinking mathematically	165
	Polya's Step 1: Understand the problem	166
	Irrelevant facts (red herrings)	167
	Too much information	168
	Missing information	168
	Not understanding all the information	169
	Polya's Step 2: Devise a plan	170
	Using multiple representations	170
	Deciding what operation(s) to use	171
	Working backwards	173
	Multistep problems	174
	Unfamiliar contexts	176
	Polya's Step 3: Carry out the plan	177
	Thinking about possible outcomes	177
	Polya's Step 4: Look back at what you've done	177
	An authentic example of problem solving	178
	Conclusion	179
	Appendix: Answers to learning activities	181
	References	196
	Index	200

PREFACE

Rather than being a ‘how to do’ or ‘how to teach’ mathematics guide, this text specifically aims to assist people who self-identify as needing some additional support in becoming more mathematically confident and competent (although at times we will mention the importance of certain aspects for teachers). The approach taken is to focus on a few foundational concepts that we feel, from our experience, cause common mathematical difficulties for people. The reader is asked to reflect on and self-assess in relation to some core content knowledge and also their personal attitudes to mathematics. From that point, the text is intended for the reader to engage in areas where it is felt assistance is needed. Broadly the focus is to assist the reader to take some steps to improve their number sense. As we progress through the chapters, we will provide learning activities to help clarify ideas and to give readers a chance to apply the ideas. When relevant, the answers to the questions are provided in the Appendix.

ABOUT NUMBER SENSE

Number sense can be many things to many people, so we have no intention of trying to present the myriad possibilities. We have chosen topics (as reflected in the contents pages) that we feel are at the core of mathematical understanding and being numerate but which we find are the most common areas of concern. Because mathematics can be inherently complex and abstract, understanding foundational elements that develop personal number sense is vital. Whenever possible, authentic scenarios are used to help the reader contextualise the mathematics concepts and give insight into how they are used and why they are so important.

ABOUT THE AUTHORS

Annette and Geoff each have over 20 years’ experience teaching mathematics in secondary and primary schools respectively. They have taught mathematics education courses at tertiary level for the last 15 years. During this time, they have been researching with and working with in-service teachers in the field of mathematics. In more recent years, they have also had considerable experience helping pre-service education students prepare for or sit their LANTITE (Literacy and Numeracy Test for Initial Teacher Education), a process during which some common issues have become apparent. These many years of experience across the spectrum of mathematics education have informed the focus of this text.

Cambridge University Press
978-1-009-00992-8 — Making Sense of Number
Annette Hilton , Geoff Hilton
Frontmatter
[More Information](#)
