1 Reflection on personal mathematics experiences and abilities

LEARNING OBJECTIVES

After reading this chapter, you should be able to:

- describe possible factors that have contributed to your mathematical knowledge and attitude to mathematics
- recall the factors that may contribute to mathematics anxiety
- understand the reasons why we need positive mathematics dispositions
- explain how number sense and a growth mindset can benefit mathematics learners.

INTRODUCTION

In this chapter we focus on the importance of competence and confidence in mathematics. We will provide opportunities for you to reflect on your own strengths and weaknesses in various areas of mathematics and explore ways to develop skills in areas where you feel you might need to improve. Not only is this important in terms of developing your skills and knowledge around mathematics, but the ability and willingness to engage with learning experiences to strengthen your understanding are key to being a lifelong learner. Rather than provide mathematical learning activities, in this chapter we provide reflection activities to assist you to think about aspects of mathematics that you find challenging.

For the last 15 years we have worked with primary and secondary pre-service teachers in postgraduate and undergraduate education programs as well as primary and secondary in-service teachers. This has given us a privileged perspective on many educators' mathematics learning experiences. It is more common than one might expect to find that many of these people have stories to tell about personal difficulties or issues with mathematics. These challenges take many forms and their origins are often quite complex. Some people report concerns with specific mathematics content areas, some report a general lack of confidence in their mathematics ability, and others have feelings of anxiety aroused by engaging with mathematics. (For example, many people report feeling that working with fractions causes them some problems. This has been noted even by some students who achieve well in mathematics.) Of course, some people have a mixture of these experiences and circumstances. For many people with whom we have worked, their frustration has increased over time as they struggle to effect positive change in their relationship with mathematics. So the pressing question is, how can challenges in one's own interactions with mathematics be addressed and improved?

At this point, we would like to remind readers that this book is not for learning how to teach mathematics to children. It targets adults who feel they need some extra support to become more confident in mathematics. We know that the educators with whom we work have attempted to address their personal mathematics issues (often many times and over a very long time). They sometimes report progress and sometimes frustrations. So we are keenly aware that to improve one's mathematical circumstances is a very personal journey. Our hope is that by using this text to analyse these circumstances, readers may gain some insights, hints, motivations and new ideas to assist them to continue their journey of mathematical self-improvement.

An analogy: What if mathematics were a jigsaw puzzle? People might fall into one of the following categories; they might:

- love jigsaw puzzles and do them easily and often
- enjoy jigsaw puzzles for the challenge they present
- have all the pieces but sometimes struggle to put them together
- always seem to be missing some pieces and become frustrated
- have had a bad experience and prefer not to do jigsaw puzzles.

To improve as a jigsaw 'doer' it would be important to self-assess before beginning to make changes for improvement. Similarly, for mathematics, it is important to determine your personal circumstance and the reasons underlying it before proceeding to take steps to improve. From a mathematics perspective where do you feel you are?

In this chapter we ask that you consider your personal mathematical world. We ask that you be honest with yourself and attempt to clearly identify issues that you may have with mathematical knowledge and **dispositions**. We will discuss dispositions in more detail in Chapter 2, but for now let's say that our dispositions cause us to respond in a particular way. For example, some people might have a number of positive dispositions to learning particular subjects, such as persistence, creativity or curiosity, whereas others might have more negative dispositions, such as fear or disinterest.

Disposition refers to the way in which someone responds to a situation (Claxton, 2014) – for example, with resilience, determination, empathy, confidence.

WHAT IS MY MATHEMATICAL CIRCUMSTANCE?

As we have discussed in the previous section, we all have our own experiences and personal circumstances related to mathematics learning. In this section we will examine these in more detail. This personal emphasis may seem odd – even paradoxical – in a book designed to help people improve their mathematical skills and knowledge; still, we are going to use a popular mathematical approach to problem solving devised by Polya (1957) to get started on the personal analysis of your mathematical situation. Polya is well known for articulating a set of steps to follow in an attempt to solve a mathematical problem: (1) Understand the problem; (2) Devise a plan; (3) Carry out the plan; (4) Look back at what you've done. We feel that these steps are equally pertinent to solving many problems in life (not just in mathematics, although we will look in depth at this in Chapter 9). While the first step, which involves both identifying and understanding the problem, seems obvious and simple, we humans often tend to do everything but. Some of the classic 'non-identifying' strategies that we are all guilty of from time to time include:

- denying we have a problem (I haven't put on too much weight over the festive season)
- admitting we have a problem but blaming someone else (I've put on a few kilos but if my partner stopped cooking amazing desserts it wouldn't happen)
- hiding the problem (I'll wear a large, sloppy, kilo-hiding sweater for a while).

This is why some (many?) problems in our lives persist; we simply don't honestly identify them so they can be addressed. Life problems, when not identified and addressed, can continue to grow, and cause us great distress (think finances, relationships, fitness and health). The same can happen if we have concerns about mathematics.

Understanding the problem: a personal perspective

In this section we will begin to consider the possible underlying concerns that you may have in relation to mathematics. These may be related to knowledge, attitudes, or past experiences – or even a combination of these.

Reflection activity 1.1

Write down your own thoughts and feelings about mathematics. You might write about positive or negative aspects of your own learning experiences, your understanding of mathematical concepts, your confidence in your mathematical ability, or how you feel about mathematics (either as a learner or user of maths). Which of these are negative? How might you begin to address these challenges? Which are positive? Is there a way to build on the positive ideas?

It is likely that at some stage you experienced either difficulties with mathematical concepts or you developed the perception that you are not mathematical or that mathematics is too difficult for you. It may be that the knowledge gaps developed first and this led to negative attitudes about mathematics or about your mathematical self-perceptions. It might be that you have a belief that even when you try, you will have difficulties and this impacts on your engagement with mathematics (see Wilkie & Sullivan [2018] for a discussion about motivation and mathematics learning). We know that there are strong links between motivation and achievement (Middleton, 2013) and this is something we hope to assist you with in this book. In the next sections we invite you to think about some scenarios and reflect on your own experiences.

Knowledge gaps

It is amazing that so often people can articulate exactly when they feel their difficulties with mathematics began. They often describe in great detail the situation, the year, the topic, even the teacher they had at the time.

Scenario 1.1

THE FEELINGS OF OTHERS

Here are a few experiences shared with us over the years. See if you can relate to any of them or perhaps you have your own story (feel free to contact the authors if you want to share).

- I had a period of absence at school and when I returned, I struggled to catch up.
- I changed schools a few times and one time I found myself behind in maths and never caught up.
- When we started learning fractions, the teacher moved forward really quickly and I 'didn't get it'.
- I found it hard to remember all the formulae and rules.
- I felt confident until mathematics became more abstract and letters were introduced.
- One year the teacher and I didn't get on, I just got further and further behind.
- Things were really bad at home and I couldn't concentrate in school.

Reflection activity 1.2

Do any of these statements resonate with you? Is it possible to identify what you consider to be your main knowledge gaps? Try to write a list that might allow you to identify concepts that you would like to focus on in future chapters of this book.

The experiences described in Scenario 1.1 all resulted in people missing a 'chunk' of mathematics in their school years and we will begin to look at some of the key elements of fundamental mathematical knowledge later in this chapter. These missing chunks can have quite a powerful knock-on effect as the mathematics curriculum builds on itself over the years. The mathematics curriculum is often referred to as a spiral curriculum. The content from one year is used as a foundation for the content of the next as it expands. Therefore, missing a chunk of knowledge from one year can have multiple ramifications for the ensuing years. As these gaps in knowledge grow, another situation often develops: The negative feelings towards mathematics begin to magnify the impact of the initial missing knowledge.

Dispositional responses

Sometimes past experiences in school or in our personal lives have a profound effect on our attitudes to mathematics or our self-perceptions as mathematics learners. This doesn't happen to all people in this situation but from our experience it happens to many and these negative feelings can be powerful inhibitors to moving forward in mathematics. The group of stories in Scenario 1.2 exemplify how a particular attitude or self-perception may develop.

Scenario 1.2

THE SOURCE OF NEGATIVE DISPOSITIONS

In these experiences people described how gaps in their mathematics knowledge impacted their dispositions toward mathematics.

- Missing out on some maths learning meant I got behind and then failing led me to dislike and even fear maths.
- We used to play a maths game where if you didn't get the correct answer you had to stay standing. I hated it and then I hated maths.
- I would be embarrassed when the teacher asked me questions (even when I didn't have my hand up). I would get them wrong and then I would become afraid of being asked another.
- I learned to fear failure in maths, even when I knew the work, and this would often make me so anxious that I couldn't think straight. Things spiralled down from there.
- I'm a high achiever, I expect to do well in everything I do. I couldn't cope when I didn't do so well at maths. It really knocked my confidence and I started questioning my abilities.

Reflection activity 1.3

Once again, we invite you to reflect on the stories in Scenario 1.2. Are any of them similar to your own experiences? Are your feelings related to identified knowledge gaps or are they more attitudinal or related to your perceptions of yourself as a mathematics learner – or are they more complex?

If you have seen yourself in any of these stories, or you have other experiences that have had a detrimental effect on your mathematics knowledge, ability or confidence then perhaps it can be said that the source of the issue has been identified. Now comes the important time to take some affirming steps. Remember we're not going to deny the

problem, blame others or continue to hide the problem. It is natural and easy to blame Teacher X from Year 4 for the way you feel, but that element of the situation cannot continue to be a blockage for your personal progress in mathematics. Our goal through this book is to support you to develop your mathematical knowledge and skills and, as a result, we hope to help you gain confidence and (maybe) even some enjoyment in learning and doing mathematics.

Been there, done that

Having identified the problem, the second step in Polya's problem-solving strategy is to devise a plan. This may be easier said than done in the case of overcoming problems with mathematics knowledge or anxiety. Again, from our experiences we understand that many people have struggled long and hard to overcome their areas of difficulties with mathematics. It would be insulting just to say, 'Try harder!'.

Using the jigsaw analogy from earlier in this chapter, this text aims, in particular, to help people who have all the mathematics pieces but are not always able to put them together, those who don't have all the pieces and therefore struggle to see the big picture, or those who are sick and tired of trying to find and fit the pieces. This text offers perhaps some different ways of thinking about some core mathematical ideas. We focus on core concepts. We don't cover all possible concepts and we certainly don't cover all concepts in minute detail. As often as possible the mathematics is taken out of the textbook or mathematics classroom and situated in the real world. By doing this, some knowledge gaps can be addressed and the negative mathematics baggage that many readers may carry with them might be assuaged, by engaging in a different but authentic approach. Our intention is to take mathematics into the real world where it becomes numeracy (the focus of the next chapter). By trying to engage with this you can't be accused of doing the same thing over and over and expecting a different outcome (which some might regard as the definition of madness) but you will be able to perhaps view mathematics from a different perspective: an everyday, real-world, practical help to your daily life. Before we focus on the initial steps in devising a plan, let's continue our focus on Polya's first step of understanding the problem.

UNDERSTANDING THE PROBLEM: A RESEARCH PERSPECTIVE

This section describes the nature of mathematics anxiety, the factors that contribute to it, and its impact on mathematics learners. We have a twofold purpose for this focus. First, it is important to understand that you are not alone and that there are many factors that may

have led you to having negative dispositions toward mathematics. Second, and equally importantly, if you are an educator it is essential that you understand how your own students may be feeling and that you do all you can to build their positive dispositions and not impact negatively on your students as a result of your own mathematics experiences. It should also be said that not all people who experience challenges with mathematics have mathematics anxiety and we also acknowledge that you may be using this book simply as a means to improve your mathematical skills and knowledge.

What is mathematics anxiety?

Mathematics anxiety is a feeling of fear or apprehension that can interfere with our ability to use or learn mathematics.

Mathematics anxiety is a feeling of fear or apprehension that arises when we are asked to learn mathematics and that can interfere with our ability to use or learn mathematics (Brady & Winn, 2017; Olson & Stoehr, 2019). The relationships between mathematics anxiety and the factors that impact negatively on learners of mathematics are often cyclic in nature; the situation is almost analogous with the chicken and egg scenario: mathematics anxiety can inhibit our ability to learn or understand mathematics just as our lack of understanding of mathematics or past negative experiences can lead to mathematics anxiety. In short, mathematics anxiety can be caused by a number of factors and it in turn can be a significant factor that influences our engagement and persistence in learning or using mathematics.

The key factors that drive mathematics anxiety are related to the relationship between our experiences in mathematics learning and the resultant feelings that we develop. Our past experiences can strongly influence our self-perceptions (Adelson & McCoach, 2011; Bandura, 2001). Negative experiences in mathematics learning during school (especially when young) or poor past performance in mathematics can lead to worry and anxiety about mathematics learning (Eccles & Roesner, 2011). Sometimes we may not be influenced by negative personal experiences but by our own or other people's views that only certain (smart) people are good at mathematics (Brady & Winn, 2017). As mentioned already, mathematics anxiety can also challenge or inhibit our capacity to learn mathematics and thus impact negatively on our chances of achieving in mathematics (see Olson & Stoehr, 2019).

What is the impact of mathematics anxiety?

When we experience the negative emotions described in the previous section, it is logical that we would not find enjoyment in learning or using mathematics. Indeed, research has shown that when people who are anxious about mathematics are faced with a mathematical situation, their brain responds in the same way as it would if they had detected a threat or

were experiencing pain (Lyons & Beilock, 2012). This being the case, it is important that we address the causes of these feelings so that we can start to understand the challenges we may be facing. Mathematics anxiety or the factors that lead to it can result in learners feeling so worried or fearful that their ability to learn and understand is reduced because their working memory is inhibited (Haylock, 2019). A fear of not being able to achieve at mathematics can also develop into learners losing confidence and they may begin avoiding situations involving mathematics or they may disengage from mathematics learning altogether, including in subjects where mathematics is essential for success, such as science (Quinnell, Thompson & LeBard, 2013).

It is fair to say that regardless of whether you experience mathematics anxiety or whether some of the factors that can lead to it are at the core of your concerns about using or learning mathematics, it is important to try to identify some strategies that may assist you to feel more willing to engage with mathematics learning activities and to feel more confident and comfortable with your ability to use and learn mathematics. This is essential for two main reasons. First, it is important for us as individuals to rise to the challenges posed by mathematics. Second, as educators we have to accept our role as numeracy teachers and to recognise that our students need us to be the most numerate we can be.

Always remember that a negative disposition can be a strong barrier for some people, and it takes time and effort to change our feelings. The following anecdote illustrates how the mere thought of mathematics is enough to cause a negative reaction. One of our past teaching colleagues was a brilliant English teacher who was very anxious about mathematics. While supervising a senior mathematics exam during block testing, the teacher was standing next to a student who raised his hand. The teacher rushed across the assembly hall and said, 'you go and help that student, I'm terrible at maths'. It turned out that the student simply wanted to ask for more blank paper!

WHY IS IT IMPORTANT THAT TEACHERS ADDRESS THEIR MATHEMATICS KNOWLEDGE AND DISPOSITIONS?

Sadly, during their primary school years many children develop beliefs and attitudes about their capability in mathematics as well as whether they enjoy mathematics, and these attitudes and beliefs influence both engagement and achievement in mathematics (Dowker, Bennett & Smith, 2012; Olson & Stoehr, 2019). Those students who have achieved success in mathematics early in school are more likely to continue to engage with mathematics because they have positive attitudes, enjoy mathematics and have high mathematical self-perceptions (Adelson & McCoach, 2011; Goos, Dole & Geiger,

2011). In addition to improved engagement, positive attitudes have a positive influence on students' achievement in mathematics, which makes it imperative for teachers to find ways to enhance their students' attitudes and engagement (Barkatsas, Kasimatis & Gialamas, 2009; Hilton, 2018).

Research has shown that as educators, if we have negative mathematics dispositions or mathematics anxiety we can risk passing these on to our students (Beilock & Willingham, 2014; Burnett & Wichman, 1997). It has been found that mathematics anxious teachers are more likely to use inflexible teaching approaches and emphasise algorithmic thinking over conceptual understanding (e.g. by focusing on procedures rather than the concepts underpinning the mathematics) and memorisation over sense making. According to Olson and Stoehr (2019), teachers who are anxious about mathematics sometimes resort to these ways of teaching because they are concerned about teaching students 'correctly' and fear that they might confuse their students. Such teachers also spend less time helping students with their questions compared to teachers who are not anxious about mathematics (see Ramirez et al., 2018). It is also important to understand that our students are quite aware of what constitutes good mathematics teaching and learning and that they are strongly influenced by effective teaching strategies (Attard, 2011; Reys et al., 2017).

Taken together, these are compelling reasons to ensure that we focus on ways to develop our own mathematics knowledge and our dispositions to mathematics teaching and learning.

ADDRESSING MATHEMATICS DIFFICULTIES

Earlier in this chapter we described the steps in Polya's problem-solving process. Step 2 involves formulating a plan. In this section we will introduce some ideas that will be central to the approaches used in this book and that we hope will assist readers to begin devising the plan to respond to their own mathematics challenges. Some of these ideas relate to mathematical knowledge and skills while others relate more to your attitudes and beliefs around learning mathematics.

Number sense

As could be imagined, there are myriad elements to mathematics, many of which incorporate a huge range of detail and levels of complexity. The first decision made when writing this text was which elements of mathematics would be most valuable for people to consider as they try to improve. Broadly, we have called our focus *improving number sense*. **Number sense** is a term that means many things to many people (Berch, 2005) and this can also

Number sense is the ability to see, understand and use numbers and to understand how numbers relate and work together.