Part I
General
Prime Numbers and the Search for Extraterrestrial Intelligence

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Apart from tabloid devotees, no one knows for sure if we human beings are alone in the universe as an intelligent species or if there are others, perhaps many others. Given the vast distances involved, it is reasonable to guess that if there is ever contact between humans and intelligent aliens, it will be through the radio spectrum. Fair enough, but how possibly might we actually transmit useful information to each other? English is becoming the “universal” language on our planet at this point, but it is ridiculous to assume that at first contact our alien correspondents would communicate with us in English, or any other Earth language.

The problem of accurate communication even between humans is quite daunting. Our history is marked with episodes of misunderstanding often caused by words or nuances that are misinterpreted. Despite the tragic loss of languages spoken by small and isolated groups of people, one might argue that it is a good thing that we are moving towards just a few principal languages.¹

We also have some experiences communicating with other species, such as our pets. A dog owner can sometimes tell from the tenor of a bark or a whimper what’s up with Fido. And sometimes Fido hears and obeys his owner’s command. We have also tried to teach apes to speak in sign language, with limited success, and we have tried to interpret the whistles and clicks of dolphins and other whales. While this research is exciting, we still have not found a Rosetta stone that allows for free communication across species. It may be that as technological beings, we just don’t have enough in common with other Earth species.

Which brings us again to possible extraterrestrial civilizations. Could there possibly be a message they could send us that we would be able to understand? Could we likewise send a message that an alien recipient could decipher?

In 1960, a German/Dutch mathematician and philosopher, Hans Freudenthal, suggested a language he called Lincos to use for possible extraterrestrial communication. It was sort of a pidgin

¹Some have argued for an adaptation of Esperanto as a more logical language that is easily learned, but it has yet to catch on in a meaningful way.
of Latin and logic with short words using the roman alphabet together with logical symbols. One could well argue that such a language is not transparent enough.

A decade later, the American astronomer Frank Drake came up with a brilliant scheme for extraterrestrial communication involving prime numbers. To get an idea of this, say you were to receive a message of dots and dashes:

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and to distinguish it from noise, after a short pause, it repeats, and then again, and again, and so on. The repetitive nature would certainly be an eye opener. But what possibly could it mean?

You begin analyzing. The “message” has 55 symbols. Hmmm. The number 55 can be factored into primes as $5 \times 11$ or $11 \times 5$. This suggests putting the symbols into a rectangular array. Let’s try $11 \times 5$:

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No, this does not seem to help, it still looks like random noise. Let’s try the other way, $5 \times 11$:

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Wait, this no longer looks so random. Clearly there is some sort of diamond shape at the top. Could the aliens be baseball fans? Probably not, it is likely just a pleasing symbol to get our attention, but what is this strange staircase design at the bottom?

With just 55 pixels it is hard to communicate very much, but an inspired guess might be that the staircase design is counting in binary from 1 to 5. Think of a dot as a 1 and a dash as a 0, and read the bottom of the 5 columns from left to right: 001, 010, 011, 100, 101. Yes, these are the base-two representations of the numbers 1, 2, 3, 4, 5.

It is easy to imagine that using a longer repetitive message, one might easily send more detailed images. Even images of photographic quality. And by then sending different pictures, it is conceivable that one could transmit a movie in this way. Which one would you send? My favorite: Attack of the Killer Tomatoes. Well, maybe not, we don’t want to give the aliens any ideas.

The key thing is that each repetitive string should have length $pq$ where $p$, $q$ are primes, so there are at most two essentially different ways of putting the string into a rectangular array.\footnote{See \url{http://www.geocities.com/monicavdv/informatie-hams/Lincon.html}.}

In the novel Contact by Carl Sagan, this is exactly the method that the aliens choose to communicate with us earthlings. Currently, SETI (Search for ExtraTerrestrial Intelligence) is systematically searching for messages from outer space, enlisting the aid of people all over the world who have spare computer time.\footnote{If $p = q$ there is just one way, so maybe this is a better scheme for sending messages.}
They haven’t found anything too remarkable as yet, but in some sense we have only just begun our search. In the meantime, have we sent any such messages specifically for alien consumption? Yes, we have. When the Arecibo radio telescope was inaugurated in Puerto Rico, one of its first jobs was to send a repetitive message into space. This message had length 1679, which factors as $23 \times 73$. Arranged the long way from top to bottom, the picture contains an amazing wealth of information: the numbers from 1 to 10, the atomic numbers of hydrogen, carbon, nitrogen, oxygen, and phosphorus (the principal elements on which most life on Earth is based), the formulas for sugars and bases in nucleotides of DNA, the number of nucleotides in DNA, a picture of a double helix of DNA, a picture of a person, the human population on Earth, the height of a human, a schematic showing our solar system with Earth in a distinguished position, a schematic of the radiotelescope at Arecibo, and the diameter of the telescope. Measures such as the telescope diameter and the height of a person were calibrated using the wavelength of the transmission of the message, namely 12.6 centimeters. Perhaps a bit crowded to my taste, but it was merely a demonstration of an idea. The Arecibo message was beamed to the globular cluster M13 which is about 25,000 light years away. Maybe in 50,000 years we will get a reply!

It is interesting that prime numbers would have an application that makes communication transparently simple. Especially so, since many cryptographic systems, designed to hide messages from unauthorized recipients, are also based on prime numbers. And it was not so long ago that prime numbers were merely the province of the curious, with no discernible applications at all.

What are the chances that there is an alien species in existence close enough for us to contact? Of course this question is very difficult to answer. Those who think there are other civilizations out there point to the enormous number of stars in the universe. Even if only 1/10 of them have planetary systems, and even if only 1/10 of these have a planet such as Earth that can support life
as we know it, and even if only 1/10 of these actually have spawned life, and even if only 1/10 of these have life that has evolved into intelligent creatures, that still leaves a tremendous number of intelligent aliens out there. On the minus side, we might argue that human civilization has only existed for a very brief time on a cosmic scale, and the technological aspect of our civilization only for a few hundred years at most. Moreover, there are indications that we will not be around very long! We use our technology to foul our own planet and make it less habitable, maybe even uninhabitable. Even though we have clear archaeological and historical records of earlier human civilizations that vanished because of climate changes, we continue to burn fossil fuels as if there were no tomorrow, literally. And when the most advanced and richest country, which produces the most pollutants, is called on the question, its leaders claim that it would hurt their economy to significantly cut back on emissions, and this greenhouse thing really hasn’t been proved anyway. So, if one were to extrapolate to alien civilizations in outer space, one might conceivably guess that if they are at all like us, they won’t be around very long either. So, while there may have been many intelligent civilizations, the chance that one exists at the same brief time as ours, and also within shouting distance, may be very slim indeed.  

Nevertheless it is fun to contemplate such contact. And the exercise may help us to be a bit more introspective and responsible as stewards of our own lovely planet.

For further reading on prime numbers:

For further reading on the likelihood of extraterrestrial civilizations, and how we might detect and communicate with them:
S. Webb, *If the universe is teeming with aliens ... where is everybody? Fifty solutions to the Fermi Paradox and the problem of extraterrestrial life*, Copernicus, New York, 2002.

Finally, a link to the SETI Institute: [http://www.seti-inst.edu/](http://www.seti-inst.edu/).

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3 The thought of multiplying various probabilities, such as we suggested above to assess the likely number of advanced civilizations, and also to assess the longevity of a civilization, is due to Frank Drake. Drake’s equation in fact presents a formula that gives the number of advanced civilizations, once all of these probabilities are estimated. On the longevity question, Drake was led to muse “Is there intelligent life on Earth?”
2

Space Shuttle Geometry

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How can mathematical definitions and theorems help prevent accidents? Read on, and find out. For the exploration in section 4, you will need something with a straightedge (for example, a ruler), a compass, some paper and a pencil.

This talk was inspired by a talk of Robert Bryant (of Duke University) on the same topic.

1 Introduction

The knowledge of mathematical theorems and properties turns out to be useful in many unexpected situations. Perhaps even more important than knowing lots of mathematics, is knowing enough mathematics to realize when you should talk to someone who knows more. In this example, the definition of a circle plays a key role. Do you remember the definition of a circle?1

2 Space Shuttle Background

The U.S. space shuttle program originated in the early 1970’s, as a way to save money while continuing to fly space missions with astronauts on board. Instead of building rockets that could be flown only once, reusable space shuttles were developed. Like airplanes, they would refuel and fly again, but because of the extreme stresses they would undergo, they would need months of repairs and careful testing between flights. The first space shuttle was launched in 1981, and by the year 2000, over one hundred successful flights had taken place.

On January 28, 1986, the space shuttle Challenger exploded 72 seconds after liftoff. All seven crew members were killed, including Ron McNair, the second African-American in space, Judy Resnik, the second American woman in space, and Christa McAuliffe, who was on board as part of the new Teacher in Space Program. The shuttle and the attached boosters and fuel tank were destroyed. The shuttle program was halted for over two years, while the accident was investigated and preventative recommendations were implemented.
Figure 2.1. A space shuttle launch
Figure 2.2. Back row (left to right): Ellison S. Onizuka, Sharon Christa McAuliffe, Gregory B. Jarvis, Judith A. Resnik. Front row: Michael J. Smith, Francis R. (Dick) Scobee, Ronald E. McNair.

The president of the United States appointed a committee to investigate the causes of the accident. Two of the committee members had been astronauts themselves: Neil Armstrong was the first person to set foot on the moon; Sally Ride was a physicist and the first American woman in space. Another physicist appointed to the committee was Richard Feynman, who was a professor at Caltech. Feynman wrote about his experience on the committee in his humorous and candid book "What Do You Care What Other People Think?" Feynman’s book includes many details of the investigation of the accident.

3 The Cause of the Accident

The cause of the accident was determined to be the failure of a seal on one of the boosters attached to the shuttle. This seal failure led to a fuel leak, which led to the explosion. The committee decided there were three main contributing factors to the seal failure: poor booster joint design, slow O-ring response, and out-of-roundness.

3.1 Booster Joint Design

During shuttle lift-off, there are two solid rocket boosters and a fuel tank that are attached to the shuttle at first, and drop off after the fuel inside them is used up. The boosters are each made up of several cylindrical metal pieces that are put together. Figure 2.3 shows how two pieces of a booster fit together at a place called a “joint.” The large forces inside the booster during lift-off cause the joint to bulge outward. (Because a joint is weaker than the other parts of the booster, it will give way first.) After the accident, the joints were redesigned to keep them from bulging outward so much. This helps prevent the joint seal from breaking.
3.2 O-ring Response Time

The o-rings are thin rubber rings that act like washers on a faucet. When they are in place between the metal pieces of the booster, they are compressed, like a sponge. But if a gap is formed between pieces of the booster, then the o-rings are supposed to spring back quickly to their full size, which would prevent any fuel from leaking out of the booster. The morning of the Challenger accident, the temperature was below freezing at the site of the launch. It turns out that o-rings don’t spring back to full size as quickly when they are that cold. Feynman gave an impressive demonstration of this at a public press conference. He used a metal clamp to compress part of an o-ring, and he put the clamp and the o-ring piece in a glass of ice water. After a few minutes, Feynman pulled the o-ring piece out of the water, and removed the clamp. It was easy to see that the piece of o-ring took several seconds to spring back to its normal shape. During lift-off, however, it was estimated that such a lengthy delay in springing back to shape could be disastrous. It was determined that lift-offs should never again occur under such cold conditions.

3.3 Out-of-roundness

After a lift-off from Kennedy Space Center, in Cape Canaveral, Florida, it takes about five minutes for all of the fuel in the shuttle boosters to be used. The empty boosters are then released from the shuttle, and fall into the Atlantic Ocean. They are recovered, taken apart, and the pieces reconditioned. Then each part is refilled with solid fuel, and examined to make sure that it is very round. If the pieces are not round enough, then they have to be reshaped, since the metal pieces
have to fit together precisely to prevent fuel leaks. NASA’s test for roundness at the time of the Challenger lift-off was the following: measure the longest distance (we will call this a “diameter”) across the cross-section at three different places. If the three diameters were all the same, then the booster was declared round.

What do you think of their test? How many diameters would need to be the same in order to guarantee that the booster is round? On a piece of paper, try to draw a curve that has that number of diameters all the same, but isn’t a circle. If you have trouble, get someone else to try it with you. You can make a game: if you give them a number, say, \( n \), they can try to draw a curve (we’ll allow corners and straight pieces) that has \( n \) diameters all the same, but isn’t a circle. Who do you think will win the game? See Figure 2.4 for some attempts to play this game.

![Figure 2.4](image)

What if you could measure all of the diameters, and they were all the same? Would the curve have to be a circle? Try measuring the diameters of the figure shown below (Figure 2.5). It has all of the diameters the same, but it is clearly not a circle! We will call it a “fake circle” or a curve of constant width. It turns out that there are infinitely many fake circles that can be drawn. In the next section, you will learn how to make them yourself.

![Figure 2.5](image)

4 Watch Out! That’s a Fake Circle

How do you construct a fake circle? Start by drawing three straight line segments that form a triangle. It doesn’t matter what kind of triangle is formed. It could be equilateral (all sides having the same length), isosceles (exactly two of the sides having the same length), or scalene (none of the sides having the same length). The line segments should actually be extended beyond the corners, or vertices, of the triangle. (See Figure 2.6.)

To draw the first arc of the fake circle, take a compass, put the sharp point on a corner, or vertex, of the triangle, and put the pencil point on one of the nearest extended segments, reasonably far outside of the triangle. You should make it far enough outside the triangle that you’ll be able to continue the figure without ever crashing into a side of the triangle, but not so far outside the