Ancient Mathematics

Foreword

The twentieth century saw great strides in our understanding of the mathematics of ancient times. This was often achieved through the combined work of archaeologists, philologists, and historians of mathematics.

We especially see how this understanding has grown in the study of the mathematics of Mesopotamia. Although the clay tablets on which this mathematics was written were excavated beginning in the nineteenth century, it was not until early in the twentieth century that a careful study of the mathematics on some of these tablets was undertaken. In particular, the tablet known as Plimpton 322 was first published by Neugebauer and Sachs in 1945, who determined that the numbers in each row of the tablet always included two out of the three numbers of a Pythagorean triple. Since that time, there has been a great scholarly debate on how those numbers were found, as well as the general purpose of the tablet. In our opening papers, we present two discussions of this issue, one by R. Creighton Buck and a second by Eleanor Robson. Both of these papers illustrate the necessity of applying ideas from several disciplines to help us make sense of the past.

Greek mathematics has, of course, been studied ever since the demise of Greek civilization. A survey of the history of Greek mathematics, as it was understood in the 1940s, is presented here by Max Dehn, a prominent mathematician in his own right—he solved one of the Hilbert problems. Dehn's article originally appeared in four parts in the Monthly, each part dealing with a different chronological period. The first part considers the work of Pythagoras and his school; the second deals with Euclid; the third considers Apollonius and Archimedes; while the fourth gives us a summary of the mathematics in Greek culture under the domination of the Roman empire.

Two of the mathematicians mentioned by Dehn are dealt with in more detail in the following articles, one on Diophantus and two on Hypatia. J. D. Swift examines several problems posed by Diophantus and explains some of his ingenious solutions. A. W. Richeson discusses the life of Hypatia through a detailed analysis of the sources available to him in 1940. In a more recent article, Michael Deakin considers the latest research on the work of Hypatia. He explains how we know what we do know, especially in regard to her mathematical work, and what remains as speculation.

Frank Swetz presents a detailed survey of what we know about mathematics in ancient China. Not only does he explain in detail certain mathematical techniques of the Chinese, but he also presents a detailed bibliography so that the reader may explore further. Swetz briefly mentions the third-century mathematician Liu Hui, whose work is explored in greater detail by Philip Straffin in the next article. There we learn not only about Liu Hui's commentaries and extensions of the Chinese classic *Nine Chapters on the Mathematical Art,* but also about Liu's use of a limit argument to determine the volume of a pyramid. Although Liu used what is now called Cavalieri's principle to determine certain volumes, he could not figure out how to determine the volume of a sphere. Straffin shows us how a later mathematician, Zu Gengzhi, ultimately determined the correct formula for that volume through a creative use of the same principle.

This section concludes with three discussions of mathematics in the Americas. First, W. C. Eells reports on the data from many years of linguistic study and analyzes the structure of the number systems in numerous groups of North American Indians. Next, A. W. Richeson looks at the number system of Mayas, displaying both the head-variant form of the monuments as well as the more familiar written form in the codices. Marcia Ascher, in an article written to commemorate the 500th anniversary of Columbus's first visit to the western hemisphere, then discusses the mathematics of two of the civilizations living there at the time. She explains the quipus of the Incas of Peru and Ecuador and then deals anew with the Mayans, concentrating in particular on the types of mathematical problems that they could solve in their number system.

Sherlock Holmes in Babylon

R. CREIGHTON BUCK

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Let me begin by clarifying the title "Sherlock Holmes in Babylon." Lest some members of the Baker Street Irregulars be misled, my topic is the archaeology of mathematics, and my objective is to retrace a small portion of the research of two scholars: Otto Neugebauer, who is a recipient of the Distinguished Service Award, given to him by the Mathematical Association of America in 1979, and his colleague and long-time collaborator, Abraham Sachs. It is also a chance for me to repay both of them a personal debt. I went to Brown University in 1947, and as a new Assistant Professor I was welcomed as a regular visitor to the Seminar in the History of Mathematics and Astronomy. There, with a handful of others, I was privileged to watch experts engaged in the intellectual challenge of reconstructing pieces of a culture from random fragments of the past. (See [4], [5].)

This experience left its mark upon me. While I do not regard myself as a historian in any sense, I have always remained a "friend of the history of mathematics"; and it is in this role that I come to you today. Let me begin with a sample of the raw materials. Figure 1 is a copy of a cuneiform tablet measuring perhaps 3 inches by 5. The markings can be made by pressing the end of a cut reed into wet clay. Dating such a tablet is seldom easy. The appearance of this tablet suggests that it may have been made in Akkad in the city of Nippur in the year −1700, about 3700 years ago.

Confronted with an artifact from an ancient culture, one asks several questions:

(i) What is this and what are its properties?

(ii) What was its original purpose?

(iii) What does this tell me about the culture that produced it?

In the History of Science, one expects neither theorems nor rigorous proofs. The subject is replete with conjectures and even speculations; and in place of proof, one often finds mere confirmation: "I believe P implies Q; and because I also believe Q, I therefore also believe P."

In Figure 1, we draw a vertical line to separate the first two columns. In the first column, we recognize what seem to be counting symbols for the numbers from 1 through 9. Paired with these in the second column we see 9, then 1 and 8, then 2 and 7, and then 3 and 6. This suggests that what we have is a "table of 9's", a multiplication table for the factor 9. Checking further, we see 5 and 4 across from the

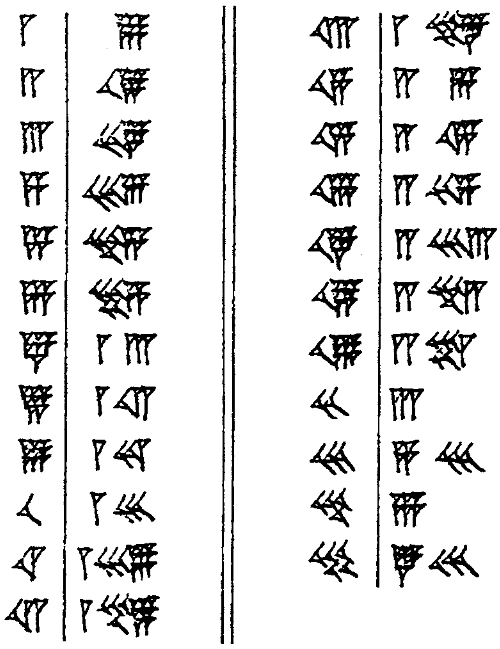

Figure 1.

counting symbol for 6, which confirms the conjecture. However, in the next line we see 7 and then across from it what seems to be a 1 and a 3.

We modify our conjecture; instead of an ordinary decimal system, we are dealing with a hybrid. There is a decimal substratum, using one type of wedge for units and another for tens but the system is base 60 in the large. The 1 and 3 in fact represent $60+3=63$. We then immediately conjecture that the same wedge symbol will be used for 1, for 60, for $(60)^2$, $(60)^3$, and so on, while the digits will be given in a decimal form.

Thus from a single tablet we might have conjectured a complete sexagesimal numeral system. We would then seek confirmation of this by examining other tablets, hoping to see the same patterns there. Indeed, this was done in the last century, and among the thousands of Babylonian tablets many were found that bear multiplication tables of the same general type as that given in Figure 1, generated by various multiplication factors. There are a great many duplicates.

We find the Babylonian numeral system cumbersome to write. In this paper, base 60 numerals will be written by putting the digits (0 through 59) in ordinary Arabic base ten, separating consecutive digits by the symbol "/". The "units place" will be on the right as usual. Thus

$$7/13/28 \text{ represents } 28+13(60)+7(60)^2 = 26,008.$$

Addition is easy:

$$\begin{array}{r} 14/28/31 \\ \underline{3/35/45} \\ 18/4/16 \end{array}$$

If the tablets that bear multiplication tables are catalogued, something strange is seen. Many tables of 9's, 12's, etc., are found; but there are also multiplication tables for unlikely factors, while many tables we would have expected never appear. In Figure 2, we list those that occur frequently.

We are left with three puzzles:

(i) Why are some tables missing? (For example, 7, 11, 13, 14, etc.?)

(ii) Why are there tables with factors such as 3/45, 7/12, 7/30, and 44/26/40?

(iii) Why are there so many tablets with exactly the same multiplication tables on them?

Some clues are found; for example, there are tablets that contain two versions of the same multiplication

2	18	1/15 = 75	7/12 = 432
3	20	1/20 = 80	7/30 = 450
4	24	1/30 = 90	8/20 = 500
5	25	1/40 = 100	12/30 = 750
6	30	2/15 = 135	16/40 = 1000
8	36	2/24 = 144	22/30 = 1350
9	40	2/30 = 150	44/26/40 = 160,000
10	45	3/20 = 200	
12	48	3/45 = 225	and a scattering of others
15	50	4/30 = 270	
16		6/40 = 400	

Figure 2. Factors Used for Multiplication Tables

table, one done neatly and one less neatly and perhaps with an error or two. I am sure that a familiar picture comes immediately to your mind: a cluster of students, all engaged in copying a model table provided by the teacher who will shortly be grading their efforts. Are we not correct to infer that in Nippur there was probably an extensive school for scribes who were in training to become bureaucrats or priests?

To help answer the first two questions, let us examine another tablet, which for convenience I have transcribed into the slash notation. (See Figure 3.) This again fits the pattern of two matched columns, and we look for an explanation. We note at once that in the first few rows the product of the adjacent column numbers is always 60. There seem to be some exceptions, however. With the pair 9 and 6/40, this product is

$$(9) \times (6/40) = (9) \times (400) = 3600$$

2	30	16	3/45	45	1/20
3	20	18	3/20	48	1/15
4	15	20	3	50	1/12
5	12	24	2/30	54	1/6/40
6	10	25	2/24	1/4	56/15
8	7/30	27	2/13/20	1/12	50
9	6/40	30	2	1/15	48
10	6	32	1/52/30	1/20	45
12	5	36	1/40	1/21	44/26/40
15	4	40	1/30		

Figure 3.

and again

$$(16) \times (3/45) = (16) \times (225) = 3600$$

while still further down, we see

$$(27) \times (2/13/20) = (27) \times (8000) = 216{,}000.$$

The solution becomes obvious if we write these products in Babylonian form; since 60 is 1/0, 3600 is 1/0/0, and 216,000 is 1/0/0/0. For confirmation, look at the last entry in the table:

$$\begin{aligned}(1/21) \times (44/26/40) &= (81) \times (160{,}000)\\ &= 12{,}960{,}000\\ &= 1/0/0/0/0.\end{aligned}$$

If we now follow the Babylonian practice of omitting terminal zeros, we see that Figure 3 is merely a table of reciprocals, written in "sexagesimal floating point." If A is an integer in the first column, the integer paired with it in the second column, A^R, is one chosen so that their product would be written as "1," meaning *any* suitable power of 60. The integers that appear in the table will always be factorable into powers of 2, 3, and 5, since these have terminating reciprocals in base 60. The term "floating-point arithmetic" is today a computer concept but is also understandable to anyone who has used a slide rule or worked with logarithms; the concept would also have been familiar to medieval astronomers who multiplied large numbers by the device called "prosthaphaeresis."

Now that Figure 3 is understood, we can answer the two puzzles left hanging on the previous page. Observe that the integers used to generate multiplication tables, as seen in Figure 2, mostly come from the standard reciprocal table. (There are also tablets that contain nonstandard reciprocals, reciprocals of such numbers as 7, 11, etc., of necessity given in terminating approximate form.) In floating point, $B \div A = B \times A^R$. Thus the combination of a set of multiplication tables and a reciprocal table makes it easy to carry out floating-point division, provided that the divisor is one of the "nice" numbers in base 60, of the form $2^\alpha 3^\beta 5^\gamma$. For example, let us divide 417 by 24; in base 60, this will be $6/57 \div 24 = 17/22/30$.

Method:

$$6/57 \div 24 = (6/57) \times (24)^R = (6/57) \times (2/30):$$

$$\begin{aligned}6/57 \times 2 &= 12 + 1/54 = 13/54\\ 6/57 \times 30 &= 3 + 28/30 = \underline{\ 3/28/30\ }\\ \text{answer} &= 17/22/30\end{aligned}$$

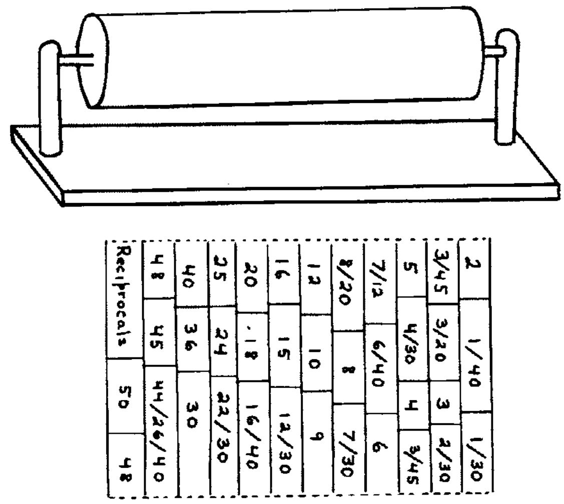

Figure 4.

The last steps in this calculation are easier if one recalls that $30 = 2^R$, so that multiplication by 30 is the same as halving. (Of course the scribe must be sure to keep track of the actual magnitudes and place values.)

That common calculations were made in this fashion becomes even more plausible in the light of one remarkable discovery. This is an inscribed cylinder, carrying on its curved face a copy of the standard reciprocal table and each of the standard multiplication tables. (In Figure 4, we show this restored, with each multiplication table indicated by its generator.) With the help of this cylinder, perhaps mounted on a stand, a scribe could easily keep track of taxes and calculate wages; perhaps we have here the Babylonian version of a slide rule or desk calculator!

With this brief introduction to the arithmetic of the Babylonians, we turn to another tablet whose mathematical nature had been overlooked until the work of Neugebauer and Sachs. It is in the George A. Plimpton Collection, Rare Book and Manuscript Library, at Columbia University, and usually called Plimpton 322. (See Figure 5, which is reproduced here by permission of the Library.) The left side of this tablet has some erosion; traces of modern glue on the left edge suggest that a portion that had originally been attached there has since been lost or stolen. Since it was bought in a marketplace, one may only conjecture about its true origin and date, although the style suggests about −1600 for the latter. As with most such tablets, this had been assumed to be a commercial account or inventory report. We will attempt to show why one can be led to believe otherwise.

Figure 5. Plimpton 322

Column A	Column B	Column C
15	1/59	2/49
58/14/50/6/15	56/7	3/12/1
1/15/33/45	1/16/41	1/50/49
5 29/32/52/16	3/31/49	5/9/1
48/54/ 1/40	1/5	1/37
47/ 6/41/40	5/19	8/1
43/11/56/28/26/40	38/11	59/1
41/33/59/ 3/45	13/19	20/49
38/33/36/36	9/1	12/49
35/10/2/28/27/24/26/40	1/22/41	2/16/1
33/45	45	1/15
29/21/54/ 2/15	27/9	48/49
27/ 3/45	7/12/1	4/49
25/48/51/35/6/40	29/31	53/49
23/13/46/40	56	53

Figure 6. Plimpton 322

First, let us transcribe it into the slash notation, as seen in Figure 6. We have reproduced the three main columns, which we have labeled A, B, and C. We note that there are gaps in column A, due to the erosion. However, it seems apparent that the numbers there are steadily decreasing. We note that some of the numerals there are short and some long, apparently at random. In contrast with this, all the numerals in columns B and C are rather short, and we do not see any evidence of general monotonicity.

Since it is easier for us to work with Arabic numerals, let us translate columns B and C into these numerals and look for patterns. (See Figure 7.) We see at once that B is smaller than C, with only two exceptions. Also, playing with these numbers, we find that column B contains exactly one prime, namely, 541, while column C contains eight numbers that are prime.

In the first 20,000 integers, there are about 2,300 primes, which is about 10 percent; among 15 inte-

B	C	B	C
119	169	541	769
3367	11521	4961	8161
4601	6649	45	75
12709	18541	1679	2929
65	97	25921	289
319	481	1771	3229
2291	3541	56	53
799	1249		

Figure 7.

$C+B$	$C-B$
288	50
14888	8154
11250	2048
31250	5832
162	32
800	162
5832	1250
2048	450
1310	228
13132	3200
120	30
4608	1250
26210	-25632
5000	1458
109	-3

Figure 8.

B	C	(a,b)
119	169	12,5
3367	11521	?
4601	6649	75,32
12709	18541	125,54
65	97	9, 4
319	481	20,9
2291	3541	54,25
799	1249	32,15
541	769	?
4961	8161	81,40
45	75	?
1679	2929	48,25
25921	289	?
1771	3229	50,27
56	53	?

Figure 9.

gers, selected at random from this interval, we might, then, expect to see one or two primes, but certainly not eight! This at once tells us that the tablet is mathematical and not merely arithmetical. (Imagine your feelings if you were to find a Babylonian tablet with a list of the orders of the first few sporadic simple groups.)

Encouraged, one attempts to find further visible patterns, for example, by combining the entries in columns B and C in various ways. One of the earliest tries is immediately successful. In Figure 8, we show the results of calculating $C+B$ and $C-B$. If you are sensitive to arithmetic you will note that, in almost every case, the numbers are each twice a perfect square.

If $C+B = 2a^2$ and $C-B = 2b^2$, then $B = a^2 - b^2$ and $C = a^2 + b^2$. Thus the entries in these columns could have been generated from integer pairs (a,b). In passing, we note that B, being $(a-b)(a+b)$, is not apt to be prime; on the other hand, when a and b are relatively prime, every prime of the form $4N+1$ can be expressed as a^2+b^2.

In Figure 9, we have recopied columns B and C, together with the appropriate pairs (a,b) in the cases where this representation is possible. As a further confirmation that we are on the right track, we note that in every such pair the numbers a and b are both "nice", that is, factorable in terms of 2, 3, and 5. In five cases, the pattern breaks down and no pair exists. It will be a further confirmation if we can explain these discrepancies as errors made by the scribe who produced the tablet. We make a simple hypothesis and assume that B and C were each computed independently from the pair (a,b) and that a few errors were made but each affected only one number in each row. Thus in each vacant place we will assume that either B or C is correct and the other wrong, and attempt to restore the correct entry. Since we do not know the correct pair (a,b) we must find it; because of the evidence in the rest of the table, we insist that an acceptable pair must be composed of "nice" sexagesimals.

We start with line 9; here, $B = 541$, which happens to be the only prime in Column B. We therefore assume B is wrong and C is correct, and thus write $C = 769 = a^2 + b^2$. This has a single solution, the pair $(25, 12)$. (We also note that both happen to be nice sexagesimals.) If this is correct, then B should have been $(25)^2 - (12)^2 = 481$, instead of 541 as given. Is there an obvious explanation for this mistake? Yes, for in slash notation, $541 = 9/1$ and $481 = 8/1$. The anomaly in line 9 seems to be merely a copy error.

Turn now to line 13; here, B is far larger than C, which is contrary to the pattern. Assume that B is in error and C is correct, and again try $C = 289 = a^2 + b^2$. There is a "nice" unique solution, $(15, 8)$, and using these, we are led to conjecture that the correct value of B is $(15)^2 - (8)^2 = 161$. Again, we ask if there is an obvious explanation for arriving at the incorrect value given, 25921. A partial answer is immediate: $(161)^2 = 25921$; so that for some reason the scribe recorded the *square* of the correct value for B.

Continuing, consider line 15. Since $B = 56$ and $C = 53$, we have $B > C$, which does not match the general pattern. However, it is not clear whether B is too large or C too small. Trying the first, we assume C is correct and solve $53 = a^2 + b^2$, obtaining the unique answer $(7, 2)$. We reject this, since 7 is not a nice sexagesimal. Now assume that B is correct, and write $56 = a^2 - b^2 = (a+b)(a-b)$. This has two solutions, $(15, 13)$ and $(9, 5)$. We reject the first and use the second, obtaining $92 + 52 = 106$ as the correct value of C. Seeking an explanation, we note that the value given by the scribe, 53, is exactly half of the correct value.

Turning now to line 2 of Figure 9, we have $B = 3367$ and $C = 11521$, either of which might be correct. Assume that $C = a^2 + b^2$ and find two solutions $(100, 39)$ and $(89, 60)$. While 100 and 60 are nice, 39 and 89 are not, so we reject both pairs and assume that B is correct. Writing $3367 = (a-b)(a+b)$ and factoring 3367 in all ways, we find four pairs: $(1684, 1683), (244, 237), (136, 123), (64, 27)$, of which we can accept only the last. This yields $(64)^2 + (27)^2 = 4825$ as the correct C. Comparing this with the number 11521 that appeared on the tablet, we see no immediate naive explanation for the error. For example, since $4825 = 1/20/25$ and $11521 = 3/12/1$, it does not seem to be a copy error. Without an explanation, we may have a little less confidence in this reconstruction of the entries in line 2.

The last misfit in the table is line 11, where we have $B = 45$ and $C = 75$. This is unusual also because this is the only case where B and C have a common factor. The sums-and-differences-of-squares pattern failed because neither $C + B = 120$ nor $C - B = 30$ is twice a square. However, everything becomes clearer if we go back to base 60 notation and remember that we use floating point; for $120 = 2/0$, which is twice $1/0$ and which we can also write as 1, clearly a perfect square. In the same way, 30 is twice 15, which is also 4^R and which is the square of 2^R. The pattern is preserved and no corrections need be made in the entries: with $a = 1 = 1/0$ and $b = \frac{1}{2} = 2^R = 30 = 0/30$, we have $a^2 = 1/0$ and $b^2 = 0/15$, and

$$C = a^2 + b^2 = 1/0 + 0/15 = 1/15 = 75$$
$$B = a^2 - b^2 = 1/0 - 0/15 = 0/45 = 45.$$

(Another aspect of the line 11 entries will appear later.)

With this, we have completed the work of editing the original tablet. In Figure 10, we give a corrected table for columns B and C, together with the appropriate pairs (a, b) from which they can be calculated.

B	C	(a, b)
119	169	12, 5
3367	4825	64, 27
4601	6649	75, 32
12709	1854	125, 54
65	97	9, 4
319	481	20, 9
2291	3541	54, 25
799	1249	32, 15
481	769	25, 12
4961	8161	81, 40
45	75	$1, \frac{1}{2} = 30$
1679	2929	48, 25
161	289	15, 8
1771	3229	50, 27
56	106	9, 5

Figure 10. Corrected Version

It is now the time to raise the second canonical question: What was the purpose behind this tablet? Speculation in this direction is less restricted, since the road is not as well marked. We can begin by asking if numbers of the form $a^2 - b^2$ and $a^2 + b^2$ have any special properties. In doing so, we run the risk of looking at ancient Babylonia from the twentieth century, rather than trying to adopt an autochthonous viewpoint. Nevertheless, one relation is extremely suggestive, involving both algebra and geometry. For any numbers (integers) a and b,

$$(a^2 - b^2)^2 + (2ab)^2 = (a^2 + b^2)^2. \qquad (1)$$

In addition, if we introduce $D = 2ab$, then B, C, and D can form a right-angled triangle with $B^2 + D^2 = C^2$. And finally, these formulas generate all Pythagorean triplets (triangles) from the integer parameters (a, b). (See Figure 11.)

There is no independent information showing that these facts were known to the Babylonians at the time we conjecture that this tablet was inscribed, although, as will appear later, their algebra had already

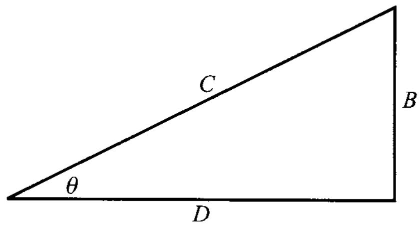

Figure 11. $B = a^2 - b^2, D = 2ab, C = a^2 + b^2$

mastered the solution of quadratic equations. If the tablet indeed is connected with this observation, then the unknown column A numbers ought to be connected in some way with the same triangle. The next step is, then, to proceed as before and try many different combinations of B, C, and D, in hopes that one of these will approximate the entries in column A. Slopes and ratios are an obvious starting point, so one calculates $C \div B, C \div D, B \div D$, etc. After discarding many failures, one arrives at the combination $(B \div D)^2$. In Figure 12, we give the values of this expression, calculated from the corrected values of B and using the hypothetical values of (a, b) to find D. (We remark that it was very helpful to have a programmable pocket calculator that could be trained to work in sexagesimal arithmetic!)

If we now return to Figure 6 and compare the numerals given there in column A with those that appear in Figure 12, we see that there is almost total agreement. For example, in line 10 we have exact duplication of an eight-digit sexagesimal! On probabilistic grounds alone, this is an overwhelming confirmation. Of course, at the top of the tablet where there were gaps due to erosion, Figures 6 and 12 are not the same, but it is evident that the calculated data in Figure 12 can be regarded as filling in the gaps. There are two minor disagreements in the two tables. In line 13, the tablet does not show an internal "0" that is present in Figure 12. This could have been the custom of the scribe in dealing with such

line	value
1	59/0/15
2	56/56/58/14/50/6/15
3	55/7/41/15/33/45
4	53/10/29/32/52/16
5	48/54/1/40
6	47/6/41/40
7	43/11/56/28/26/40
8	41/33/45/14/3/45
9	38/33/36/36
10	35/10/2/28/27/24/26/40
11	33/45
12	29/21/54/2/15
13	27/0/3/45
14	25/48/51/35/6/40
15	23/13/46/40

Figure 12. Calculated Values of $(B \div D)^2$

an event. In line 8, the scribe has written a digit "59" where there should have been a consecutive pair of digits, "45/14". Since $59 = 45 + 14$, it is not difficult to invent several different ways in which an error of this sort could have been made.

It should be remarked that Neugebauer and Sachs did not use $(B \div D)^2$ as a source for column A but rather $(C \div D)^2$. Because of the relationship between B and C, and formula (1), one sees that $(C \div D)^2 = (B \div D)^2 + 1$. Thus, the only effect of the change would be to introduce an initial "1/" before all the sexagesimals that appear in Figure 12, and the reason for their choice was that they believed that this was true for column A on the Plimpton tablet. Others who have examined the tablet do not agree. (I have not seen the tablet, and I do not believe it matters which alternative is used.)

We now know the relationship of columns A, B, and C. Referring to Figure 11, C is the hypotenuse, B the vertical side, and A is the square of the slope of the triangle; thus, in modern notation $A = \tan^2 \theta$. It is interesting to observe that the anomalous case of line 11, with $B = 45$ and $C = 75$, turns out to be the familiar $3, 4, 5$ triangle; in the Babylonian case, this would seem to have been the $\frac{3}{4}, 1, \frac{5}{4}$ triangle, since $45 = 3 \times 4^R$ and $75 = 1/15 = 5 \times 4^R$. Of course the triangle, the side D, and the parameters (a, b) are all constructs of ours and not immediately visible in the original tablet. All that we can assert without controversy is that $A = B^2 \div (C^2 - B^2)$.

Let us reexamine some of our reasoning. In lines 2, 9, 13, and 15, the scribe recorded correct values for A but incorrect values for C, B, B, and C, respectively. This suggests strongly that A was not calculated directly from the values of B and C, but that A, B, and C were all calculated independently from data that do not appear on the tablet; our hypothetical pair (a, b) gains life. (Of course there is the possibility that the tablet before us is merely a copy from another master tablet.) In either case, it seems odd that column A should be error free while columns B and C, involving simpler numbers, should have four errors.

Other questions can be raised. If, as argued by Neugebauer, the purpose of the tablet was to record a collection of integral-sided Pythagorean triangles (triplets), why do we not see the values of D, or at least the useful parameters (a, b)? And why would one want the values in column A which are squares of the slope? And why should the entries be arranged in an order that makes the numbers A decrease monotonically?