LUDIC PROOF

This book represents a new departure in science studies: an analysis of a scientific style of writing, situating it within the context of the contemporary style of literature. Its philosophical significance is that it provides a novel way of making sense of the notion of a scientific style. For the first time, the Hellenistic mathematical corpus – one of the most substantial extant for the period – is placed center-stage in the discussion of Hellenistic culture as a whole. Professor Netz argues that Hellenistic mathematical writings adopt a narrative strategy based on surprise, a compositional form based on a mosaic of apparently unrelated elements, and a carnivalesque profusion of detail. He further investigates how such stylistic preferences derive from, and throw light on, the style of Hellenistic poetry. This important book will be welcomed by all scholars of Hellenistic civilization as well as historians of ancient science and Western mathematics.

Reviel Netz is Professor of Classics at Stanford University. He has written many books on mathematics, history, and poetry, including, most recently, *The Transformation of Mathematics in the Early Mediterranean World* (2004) and (with William Noel) *The Archimedes Codex* (2007). *The Shaping of Deduction in Greek Mathematics* (1999) has been variously acclaimed as “a masterpiece” (David Sedley, *Classical Review*), and “The most important work in Science Studies since Leviathan and the Air Pump” (Bruno Latour, *Social Studies of Science*). Together with Nigel Wilson, he is currently editing the Archimedes Palimpsest, and he is also producing a three-volume complete translation of and commentary on the works of Archimedes.
LUDIC PROOF

Greek Mathematics and the Alexandrian Aesthetic

REVIEL NETZ
To Maya, Darya and Tamara
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Preface

This, my third study on Greek mathematics, serves to complete a project. My first study, *The Shaping of Deduction in Greek Mathematics* (1999) analyzed Greek mathematical writing in its most general form, applicable from the fifth century BC down to the sixth century AD and, in truth, going beyond into Arabic and Latin mathematics, as far as the scientific revolution itself. This form – in a nutshell, the combination of the lettered diagram with a formulaic language – is the constant of Greek mathematics, especially (though not only) in geometry. Against this constant, the historical variations could then be played.* The historical variety is formed primarily of the contrast of the Hellenistic period (when Greek mathematics reached its most remarkable achievements) and Late Antiquity (when Greek mathematics came to be re-shaped into the form in which it influenced all of later science). My second study, *The Transformation of Mathematics in the Early Mediterranean* (2004), was largely concerned with the nature of this re-shaping of Greek mathematics in Late Antiquity and the Middle Ages.

This study, finally, is concerned with the nature of Greek mathematics in the Hellenistic period itself. Throughout, my main concern is with the form of writing: taken in a more general, abstract sense, in the first study, and in a more culturally sensitive sense, in the following two.

The three studies were not planned together, but the differences between them have to do not so much with changed opinions as with changed subject matter.

I have changed my views primarily in the following two ways. First, I now believe my reconstruction of the historical background to Greek mathematics is incomplete.

* Some reviewers have made the fair criticism that my evidence, in that book, is largely drawn from the works of the three main Hellenistic geometers, Euclid, Archimedes, and Apollonius (with other authors sampled haphazardly). I regret, in retrospect, that I did not make my survey more obviously representative. Still, even if my documentation of the fact was unfortunately incomplete, it is probably safe to say that the broad features of lettered diagram and formulaic language are indeed a constant of Greek mathematics as well as of its heirs in the pre-modern Mediterranean.
mathematics, as formulated in Netz 1999, did suffer from emphasizing the underlying cultural continuity. The stability of the broad features of Greek mathematical writing should be seen as against the radically changing historical context, and should be understood primarily (I now believe) in the terms of self-regulating conventions discussed in that book (and since, in Netz 2003a). I also would qualify now my picture of Hellenistic mathematics, as presented in Netz 2004. There, I characterized this mathematics as marked by the “aura” of individual treatises – with which I still stand. However, as will be made obvious in the course of this book, I now ground this aura not in the generalized polemical characteristics of Greek culture, but rather in a much more precise interface between the aesthetics of poetry and of mathematics, operative in Alexandrian civilization.

Each of the studies is characterized by a different methodology, because the three different locks called for three different keys. Primarily, this is an effect of zooming in, with sharper detail coming into focus. In the first study, dealing with cross-cultural constants, I took the approach I call “cognitive history.” In the second study, dealing with an extended period (covering both Late Antiquity and the Middle Ages), I concentrated on the study of intellectual practices (where I do detect a significant cultural continuity between the various cultures of scriptural religion and the codex). This study, finally, focused as it is on a more clearly defined period, concentrates on the very culturally specific history of style. Taken together, I hope my three studies form a coherent whole. Greek mathematics – always based on the mechanism of the lettered diagram and a formulaic language – reached its most remarkable achievements in the Hellenistic period, where it was characterized by a certain “ludic” style comparable to that of contemporary literature. In Late Antiquity, this style was drastically adjusted to conform to the intellectual practices of deuteronomistic texts based on the commentary, giving rise to the form of “Euclidean” science with which we are most familiar.

My theoretical assumption in this book is very modest: people do the things they enjoy doing. In order to find out why Hellenistic mathematicians enjoyed writing their mathematics (and assumed that readers would be found to share their enjoyment), let us look for the kinds of things people enjoyed around them. And since mathematics is primarily a verbal, indeed textual activity, let us look for the kind of verbal art favored in the Hellenistic world. Then let us see whether Greek mathematics conforms to the poetics of this verbal art. This is the underlying logic of the book. Its explicit structure moves in the other direction: the introduction and the first three chapters serve to present the aesthetic characteristics of
Hellenistic mathematics, while the fourth chapter serves (more rapidly) to put this mathematics within its literary context.

The first chapter, “The carnival of calculation,” describes the fascination, displayed by many works of Hellenistic mathematics, with creating a rich texture of obscure and seemingly pointless numerical calculation. The treatises occasionally lapse, as it were, so as to wallow in numbers – giving up in this way the purity of abstract geometry.

The second chapter, “The telling of mathematics,” follows the narrative technique favored in many Hellenistic mathematical treatises, based on suspense and surprise, on the raising of expectations so as to quash them. I look in particular on the modulation of the authorial voice: how the author is introduced into a seemingly impersonal science.

The third chapter, “Hybrids and mosaics,” discusses a compositional feature operative in much of Hellenistic mathematics, at both small and large dimensions. Locally, the treatises often create a texture of variety by producing a mosaic of propositions of different kinds. Globally, there is a fascination with such themes that go beyond the boundaries of geometry, either connecting it to other scientific genres or indeed connecting it to non-scientific genres such as poetry.

This breaking of boundary-genres, in itself, already suggests the interplay of science and poetry in Hellenistic civilization. The fourth chapter, “The literary interface,” starts from the role of science in the wider Hellenistic genre-system. I also move on to describe, in a brief, largely derivative manner, the aesthetics of Hellenistic poetry itself.

In my conclusion, I make some tentative suggestions, qualifying the ways in which the broadly descriptive outline of the book can be used to sustain wider historical interpretations.

The book is thematically structured: two chronological questions are briefly addressed where demanded by their thematic context. A final section of chapter 2, building on the notion of the personal voice in mathematics, discusses the later depersonalization of voice in Late Antiquity giving rise to the impersonal image of mathematics we are so familiar with. A discussion of the basic chronological parameters of Hellenistic mathematics is reserved for even later in the book – the conclusion, where such a chronological discussion is demanded by the question of the historical setting giving rise to the style as described.

The focus of the book is description of style – primarily, mathematical style. I intend to write much more on the mathematics than on its literary context, but the reasons for this are simple: the poetics of Hellenistic literature are generally more familiar than those of Hellenistic mathematics.
while I, myself, know Hellenistic mathematics better than I do Hellenistic literature. Further, there are gaps in our historical evidence so that more can be said at the descriptive level than at the explanatory one. Most important, however, is that my main theme in this work is sustained at the level of style, of poetics or – even more grandly put – of semiotics. The precise historical underpinning of the semiotic practices described here is of less concern for my purposes.

This brings me to the following general observation. A few generations back, scholars of Hellenistic literature identified in it a civilization in decline, one where the poet, detached from his polity, no longer served its communal needs but instead pursued art for art’s sake. More recently, scholars have come to focus on the complex cultural realities of Hellenistic civilization and on the complex ways by which Hellenistic poetry spoke for a communal voice. This debate is framed in terms of the historical setting of the poetry. Any attempt, such as mine, to concentrate on the style, and to bracket its historical setting, could therefore be read – erroneously – as an effort to revive the picture of Hellenistic poets as pursuing art for art’s sake. But this is not at all my point: my own choice to study Hellenistic style should not be read as a claim that style was what mattered most to the Hellenistic authors. I think they cared most for gods and kings, for cities and their traditions – just as Greek geometers cared most for figures and proportions, for circles and their measurements. Style came only after that. So why do I study the styles, the semiotic practices, after all? Should I not admit, then, that this study is dedicated to a mere ornament, to details of presentation of marginal importance? To the contrary, I argue that my research project addresses the most urgent question of the humanities today: where do cultural artifacts come from? Are they the product of the universally “human,” or of specific cultural practices? My research focuses on mathematics, the human cultural pursuit whose universality is most apparent. I try to show how it is indeed fully universal – in its objective achievement – and at the same time how it is fully historical – in the terms of its semiotic practices, which vary sharply according to historical and cultural settings. Seen from this research perspective, it becomes important indeed to look at the semiotic practices typical of the third century BC.

I hope this serves to contextualize this project for my readers, whether they come from science studies or from Hellenistic literature and history. A few more qualifications and clarifications will be made in the conclusion – where once again I address the difficulties involved in trying to account for
the semiotic practices in terms of their historical setting. A few preliminary clarifications must be made right now. The title of my book is a useful slogan but it may also mislead if taken literally. I therefore add a glossary, so to speak, to the title.

First, the title mentions an “Alexandrian” aesthetic. The city of Alexandria no doubt played a major role in the cultural history of the period, but I use the word mostly for liking the sound of “Alexandrian aesthetic” better than that of “Hellenistic aesthetic.” (For an attempt to quantify the well-known central position of Alexandria in post-classical science, see Netz 1997. In general on the cultural role of Alexandria the best reference remains Fraser 1972. “Hellenistic” would have been the more precise term, but it too would not be quite precise; the period of most interest to us lies from the mid-third to the mid-second centuries BC, i.e. not the “Hellenistic” period as a whole. The death of Alexander, as well as the ascendance of Augustus, both had little to do, directly, with the history of mathematics.

Second, the term “the aesthetics of X” might be taken to mean “the aesthetics that X has consciously espoused,” so that a study of, say, the aesthetics of Hellenistic poetry could be understood to mean an analysis of ars-poetic comments in Hellenistic poems, or a study of ancient treatises in aesthetics such as Philodemus’ On Poems. This is of course an important field of study, but it is not what I refer to in my title. I use the term “the Hellenistic aesthetic” as an observer concept, to mean “the aesthetics identifiable (by us) in Hellenistic texts,” referring to the stylistic properties of those texts, regardless of whether or not such stylistic properties were articulated by the Hellenistic actors themselves.

Third, the “Greek mathematics” in my title sometimes means “Greek geometry” (this terminological looseness is inevitable with the Greek mathematical tradition), and nearly always refers to elite, literate mathematical texts. This does not deny the existence of other, more demotic practices of calculation, measurement and numeracy, which obviously fall outside the scope of this book, as belonging to very different stylistic domains. (For the less-literate traditions, see Cuomo 2001, a study rare for its bringing the literate and the demotic together.)

Fourth, the word “Ludic” in the title typically encodes a certain playful spirit and, in one central case, it encodes the mathematics of a certain game – the Stomachion. But most often in this book “ludic” should be read as no more than an abbreviated reference to “works sharing certain stylistic features” (which, to anticipate, includes in general narrative surprise, mosaic structure and generic experiment, and, in an important set
of works, a certain “carnivalesque” atmosphere). I do not suggest that Hellenistic mathematics – or, for that matter, Hellenistic poetry – were not “serious.” Even while serious, however, they were definitely sly, subtle, and sophisticated – a combination which the term “ludic” is meant to suggest.

To sum up, then, this book is about the study of a certain sly, subtle, and sophisticated style identifiable by us in elite Greek mathematical (especially geometrical) works of about 250 to 150 BC, as seen in the context of the elite poetry of the same (and somewhat earlier) period.

The book serves at three levels. The first, as already suggested, is descriptive. It offers a new description of Hellenistic mathematics, one focused on a neglected yet major aspect, namely its style of writing. The second is explanatory: by situating mathematics within its wider cultural context, it aims to explain – however tentatively – both its form, as well as its very flourishing at that period. The third is methodological. I am not familiar with extended studies in the history of mathematics – or indeed of science in general – focused on the aesthetics of its writing. This is an obvious lacuna and, I believe, a major one. There are of course references to aesthetics as a phenomenon in science. Since Hutcheson in the eighteenth century – indeed, since Plato himself – it has been something of a commonplace to discuss the “beauty” of certain scientific objects (possessing symmetry, balance, simplicity, etc.). Scientists and mathematicians not infrequently refer to the aesthetic impulse driving their work (see e.g. Chandrasekhar 1987 for a physicist, or Aigner and Ziegler 1998 for a mathematical example). There is a minor research tradition in the philosophy of science, looking for “beauty” as a principle accounting for the scientific choice between theories; McAllister 1996 forms an example. With rare and marginal exceptions, all of this touches on the aesthetics of the scientific object of study and not on the aesthetics of the scientific artifact itself.

The brief argument above – that people do what they enjoy doing – should suffice to point our attention to the importance of such studies. I realize, of course, that more argument is required to make the claim for the need for studies in the historical aesthetics of science. This book, then, makes the argument by providing one such study.

My gratitude extends widely. Audiences at Stanford, Brown, and Groningen helped me think through my argument. Serafina Cuomo, Marco Fantuzzi, Paula Findlen, and Sir Geoffrey Lloyd all read through my entire text and returned with useful comments. Susan Stephens’ comments on an early version were especially valuable in helping me rethink my interpretation of the interface of science and poetry in the Hellenistic world. Errors and
Preface

omissions, I know, remain, and remain mine. The first draft of this book was composed through the year of a fellowship at Stanford’s Center for Advanced Studies in Behavioral Sciences. The draft was made into a book at Stanford’s Department of Classics, and Cambridge University Press has seen it into publication. I am grateful to have resided in such places that welcome all – including the playful.