VISCOELASTIC WAVES IN LAYERED MEDIA

This book is a rigorous, self-contained exposition of the mathematical theory for wave propagation in layered media with arbitrary amounts of intrinsic absorption. The theory, previously not published in a book, provides solutions for fundamental wave-propagation problems in the general context of any media with a linear response (elastic or anelastic). It reveals physical characteristics for two- and three-dimensional anelastic body and surface waves, not predicted by commonly used models based on elasticity or one-dimensional anelasticity. It explains observed wave characteristics not explained by previous theories.

This book may be used as a textbook for graduate-level courses and as a research reference in a variety of fields such as solid mechanics, seismology, civil and mechanical engineering, exploration geophysics, and acoustics. The theory and numerical results allow the classic subject of fundamental elastic wave propagation to be taught in the broader context of waves in any media with a linear response, without undue complications in the mathematics. They provide the basis to improve a variety of anelastic wave-propagation models, including those for the Earth's interior, metal impurities, petroleum reserves, polymers, soils, and ocean acoustics. The numerical examples and problems facilitate understanding by emphasizing important aspects of the theory for each chapter.

Roger D. Borcherdt is a Research Scientist at the U.S. Geological Survey and Consulting Professor, Department of Civil and Environmental Engineering at Stanford University, where he also served as visiting Shimizu Professor. Dr. Borcherdt is the author of more than 180 scientific publications including several on the theoretical and empirical aspects of seismic wave propagation pertaining to problems in seismology, geophysics, and earthquake engineering. He is the recipient of the Presidential Meritorious Service and Distinguished Service awards of the Department of Interior for Scientific Leadership in Engineering Seismology, and the 1994 and 2002 Outstanding Paper Awards of Earthquake Spectra. He is an honorary member of the Earthquake Engineering Research Institute, a past journal and volume editor, and an active member of several professional societies.

VISCOELASTIC WAVES IN LAYERED MEDIA

ROGER D. BORCHERDT, PH.D.
United States Geological Survey
Stanford University, USA

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521898539

First published 2009

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

Library of Congress Cataloging in Publication data
Borcherdt, Roger D.
Viscoelastic waves in layered media / Roger D. Borcherdt.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-521-89853-9
1. Waves – Mathematics. 2. Viscoelasticity. 3. Viscoelastic materials. I. Title.
QA935.B647 2008
532′.0533–dc22
2008037113

ISBN 978-0-521-89853-9 hardback

Cambridge University Press
978-0-521-89853-9 - Viscoelastic Waves in Layered Media
Roger D. Borcherdt
Frontmatter
More information

In Memory of My Mother and Father
Dedicated to My Family

Contents

Preface

This book provides a self-contained mathematical exposition of the theory of monochromatic wave propagation in layered viscoelastic media. It provides analytic solutions and numerical results for fundamental wave-propagation problems in arbitrary linear viscoelastic media not published previously in a book. As a text book with numerical examples and problem sets, it provides the opportunity to teach the theory of monochromatic wave propagation as usually taught for elastic media in the broader context of wave propagation in any media with a linear response without undue complications in the mathematics. Formulations of the expressions for the waves and the constitutive relation for the media afford considerable generality and simplification in the mathematics required to derive analytic solutions valid for any viscoelastic solid including an elastic medium. The book is intended for the beginning student of wave propagation with prerequisites being knowledge of differential equations and complex variables.

As a reference text, this book provides the theory of monochromatic wave propagation in more than one dimension developed in the last three to four decades. As such, it provides a compendium of recent advances that show that physical characteristics of two- and three-dimensional anelastic body and surface waves are not predictable from the theory for one-dimensional waves. It provides the basis for the derivation of results beyond the scope of the present text book. The theory is of interest in the broad field of solid mechanics and of special interest in seismology, engineering, exploration geophysics, and acoustics for consideration of wave propagation in layered media with arbitrary amounts of intrinsic absorption, ranging from low-loss models of the deep Earth to moderate-loss models for soils and weathered rock.

The phenomenological constitutive theory of linear viscoelasticity dates to the nineteenth century with the application of the superposition principle in 1874 by Boltzmann. He proposed a general mathematical formulation that characterizes linear anelastic as well as elastic material behavior. Subsequent developments in the theory did not occur until interest increased in the response of polymers and other synthetic materials during the middle of the twentieth century. During this time considerable simplification of the mathematical aspects of the theory occurred with developments in the theory of linear functionals (Volterra 1860–1940) and the introduction of integral transform techniques. Definitive accounts and contributions to the mathematical theory of viscoelasticity include the works of Volterra (2005), Gross (1953), Bland (1960), Fung (1965), and Flugge (1967), and the rigorous account by Gurtin and Sternberg (1962). Gross (1953) suggested that "The theory of viscoelasticity is approaching completion. Further progress is likely to be made in applications rather than on fundamental principles." Hunter (1960) indicated that the application of the general theory of linear viscoelasticity to other than one-dimensional wave-propagation problems was incomplete. Subsequent to Hunter's review, significant advances occurred in the 1970s and 1980s concerning application of viscoelasticity to wave propagation in layered two- and three-dimensional media. This exposition provides a self-contained mathematical account of these developments as well as recent solutions derived herein. It provides closed-form analytic solutions and numerical results for fundamental monochromatic wave-propagation problems in arbitrary layered viscoelastic media.

Recent theoretical developments show that the physical characteristics of the predominant types of body waves that propagate in layered anelastic media are distinctly different from the predominant type in elastic media or one-dimensional anelastic media. For example, two types of shear waves propagate, each with different amounts of attenuation. Physical characteristics of anelastic waves refracted across anelastic boundaries, such as particle motion, phase and energy velocities, energy loss, and direction of energy flux, vary with angle of incidence. Hence, the physical characteristics of the waves propagating through a stack of layers are no longer unique in each layer as they are in elastic media, but instead depend on the angle at which the wave entered the stack. These fundamental differences explain laboratory observations not explained by elasticity or one-dimensional viscoelastic wave-propagation theory. They have important implications for some forward-modeling and inverse problems. These differences lend justification to the need for a book with a rigorous mathematical treatment of the fundamental aspects of viscoelastic wave propagation in layered media.

Viscoelastic material behavior is characterized herein using Boltzmann's formulation of the constitutive law. Analytic solutions derived for various problems are valid for both elastic and linear anelastic media as might be modeled by an infinite number of possible configurations of elastic springs and viscous dashpots. The general analytic solutions are valid for viscoelastic media with an arbitrary amount of intrinsic absorption. The theory, based on the assumption of linear strain, is valid for media with a nonlinear response to the extent that linear approximations are valid for sufficiently small increments in time. Wave-propagation results in this book are based on those of the author as previously published and explicitly derived herein including recent results for multilayered media and Love-Type surface waves.

The book is intended for use in a graduate or upper-division course and as a reference text for those interested in wave propagation in layered media. The book provides a self-contained treatment of energy propagation and dissipation and other physical characteristics of general P, Type-I S, and Type-II S waves in viscoelastic media with arbitrary amounts of absorption. It provides analytic and numerical results for fundamental reflection–refraction and surface-wave problems. The solutions and resultant expressions are derived from first principles. The book offers students the opportunity to understand classic elastic results in the broader context of wave propagation in any material with a linear response.

Chapter 1 provides an introduction for new students to basic concepts of a linear stress–strain law, energy dissipation, and wave propagation for one-dimensional linear viscoelastic media. It provides examples of specific models derivable from various configurations of springs and dashpots as special cases of the general formulation.

Chapter 2 extends the basic concepts for viscoelastic media to three dimensions. It provides the general linear stress–strain law, notation for components of stress and strain, the equation of motion, and a rigorous account of energy balance as the basis needed for a self-contained treatment of viscoelastic wave propagation in more than one dimension.

Chapter 3 provides a thorough account of the physical characteristics of harmonic waves in three-dimensional viscoelastic media. It provides closed-form expressions and corresponding quantitative estimates for characteristics of general (homogeneous or inhomogeneous) P, Type-I S, and Type-II S waves in viscoelastic media with both arbitrary and small amounts of intrinsic absorption. It includes expressions for the physical displacement and volumetric strain associated with various wave types. This chapter is a prerequisite for analytic solutions derived for reflection–refraction and surface-wave problems in subsequent chapters.

Chapter 4 specifies the expressions for monochromatic P, Type-I S, and Type-II S wave solutions needed in subsequent chapters to solve reflection–refraction and surface-wave problems in layered viscoelastic media. The solutions are expressed in terms of the propagation and attenuation vectors and in terms of the components of the complex wave numbers.

Chapter 5 provides analytic closed-form solutions for the problems of general (homogeneous or inhomogeneous) P, Type-I S, and Type-II S waves incident on a plane boundary between viscoelastic media. Conditions for homogeneity and inhomogeneity of the reflected and refracted waves and the characteristics of energy flow at the boundary are derived. Careful study of results for the problems of an incident Type-I S and Type-II S wave is useful for understanding reflection–refraction phenomena and energy flow due to an inhomogeneous wave incident on a viscoelastic boundary.

Chapter 6 provides numerical results for various single-boundary reflection–refraction problems using the analytic solutions derived in the previous chapter. Examples are chosen to provide quantitative estimates of the physical characteristics for materials with moderate and small amounts of intrinsic material absorption as well as a comparison with laboratory measurements in support of theoretical predictions. Study of these examples, especially the first three, is recommended for developing an improved understanding of the physical characteristics of waves reflected and refracted at viscoelastic boundaries.

Chapter 7 provides theoretical solutions and quantitative results for problems of a general Type-I S, P, or Type-II S wave incident on the free surface of a viscoelastic half space. Results are included to facilitate understanding and interpretation of measurements as might be detected on seismometers and volumetric strain meters at or near the free surface of a viscoelastic half space.

Chapter 8 presents the analytic solution and corresponding numerical results for a Rayleigh-Type surface wave on a viscoelastic half space. Analytic and numerical results illustrate fundamental differences in the physical characteristics of viscoelastic surface waves versus those for elastic surface waves as originally derived by Lord Rayleigh.

Chapter 9 provides the analytic solution for the response of multilayered viscoelastic media to an incident general Type-II S wave. It provides numerical results for elastic, low-loss, and moderate-loss viscoelastic media useful in understanding variations in response of a single layer due to inhomogeneity and angle of incidence of the incident wave.

Chapter 10 provides the analytic solution and corresponding numerical results for a Love-Type surface wave on multilayered viscoelastic media. It derives roots of the resultant complex period equation for a single layer needed to provide curves showing the dependencies of wave speed, absorption coefficient, and amplitude distribution on frequency, intrinsic absorption, and other material parameters for the fundamental and first higher mode.

New students desiring a basic understanding of Rayleigh-Type surface waves, the response of a stack of viscoelastic layers to incident inhomogeneous waves, and Love-Type surface waves will benefit from a thorough reading of Chapters 8, 9, and 10.

Chapter 11 provides appendices that augment material presented in preceding chapters. They include various integral and vector identities and lengthy derivations relegated to the appendices to facilitate readability of the main text.

A special note of respect is due those who have developed the elegant constitutive theory of linear viscoelasticity, such as Boltzmann, Volterra, Gurtin, and Sternberg. Their important contributions make applications such as that presented here straightforward. I would like to express my appreciation to Professor Jerome L. Sackman, whose guidance and expertise in viscoelasticity initiated a theoretical journey I could not have imagined. Review comments received on advance copies from colleagues are appreciated, especially those of Professors J. Sackman, W. H. K. Lee, J. Bielak, and C. Langston. Discussions with colleagues, contributions of former coauthors, and assistance with formatting some figures in Chapters 6 and 7 by G. Glassmoyer are appreciated.

Advice and encouragement are appreciated, as received during preparation from friends and colleagues, including Professors H. Shah, A. Kiremidjian, G. Deierlein, H. Krawinkler, and K. Law, of Stanford University, Professors J. Sackman, A. Chopra, J. Moehle, J. Penzien, A. Der Kiureghian, the late B. A. Bolt, T. V. McEvilly, and P.W. Rodgers, of the University of California, Berkeley, and Dr. F. Naeim of John A. Martin and Associates.

A special note of appreciation is due my family (Judy, Darren, Ryan, and Debbie) for their patience and support for the many late hours needed to finish the book. Without their understanding the opportunity to experience the personal excitement associated with discovering characteristics of waves basic to seismology and engineering through the elegance and rigors of mathematics would not have been possible.