Index of notation

Sets and sequences

We denote by $\mathcal{P}(A)$ the set of subsets of a set A (its powerset), and by $\mathcal{P}_f(A)$ its set of finite subsets. The difference of two sets A and B is the set $A - B := \{a \in A \mid a \notin B\}$. The empty set is $\emptyset$.

We denote by $Seq(A)$ the set of finite sequences of elements of a set A, and by $s[i]$ the i-th element of $s \in Seq(A)$. The empty sequence is $()$. If A is an alphabet, we denote $Seq(A)$ also by A^*. Its elements are then called words, which are sequences of letters (or symbols). The empty word is also denoted by ε.

We denote by $|A|$ the cardinality of a set A, and also by $|s|$ the length of a sequence s (in particular, of a word). The cardinality of a set A is also denoted by $Card(A)$ in certain cases, for better readability of formulas. We denote by $|s|_a$ the number of occurrences of $a \in A$ in a sequence $s \in Seq(A)$.

Integers

We denote by $\mathcal{Z}$ the set of integers, by $\mathcal{N}$ the set of nonnegative ones and by $\mathcal{N}_+$ the set of positive ones. For $n, m \in \mathcal{Z}$, we let $[n,m] := \{i \in \mathcal{Z} \mid n \leq i \leq m\}$ and $[m] := [1,m]$. We have $[m] = \emptyset$ if $m \leq 0$ and $[n,m] = \emptyset$ if $m < n$.

If $p, q \in \mathcal{N}$, $q \geq 2$, we let $\mathrm{mod}_q(p)$ be the unique integer r in $[0, q-1]$ such that $p \equiv r \pmod{q}$. If $n \in \mathcal{N}$, then $\exp(n)$ denotes 2^n. All logarithms are in base 2. The function $\exp : \mathcal{N}^2 \to \mathcal{N}$ is defined by $\exp(0,n) = n$ and $\exp(d+1,n) = 2^{\exp(d,n)}$; thus, $\exp(1,n) = \exp(n)$.

Binary relations and functions

If $R \subseteq A \times A$ is a binary relation on a set A, then R^* denotes its reflexive and transitive closure, R^+ its transitive closure and R^{-1} its inverse $\{(x,y) \mid (y,x) \in R\}$. The identity relation $\{(x,x) \mid x \in A\}$ is denoted by Id_A, or just by Id when A is clear from the context.

If $R \subseteq A \times B$ and $S \subseteq B \times C$ are two binary relations, then $R \cdot S$ denotes their composition, i.e., the relation $\{(x,z) \in A \times C \mid (x,y) \in R$ and $(y,z) \in S$ for some $y \in B\}$. If R and S are functional, i.e., if they define partial functions $f : A \to B$ and $g : B \to C$ respectively, then $R \cdot S$ defines the partial function $g \circ f : A \to C$; in that case we denote $R \cdot S$ also by $S \circ R$. We denote by $\bigcirc_{i \in I} f_i$ the composition in any order of functions f_i that commute pairwise (i.e., such that $f_i \circ f_j = f_j \circ f_i$ for all $i,j \in I$).

The domain of a binary relation $R \subseteq A \times B$ is denoted by $Dom(R) \subseteq A$, and its image is $R(A) \subseteq B$. Thus, $Dom(R) = \{a \in A \mid (a,b) \in R$ for some $b \in B\} = R^{-1}(B)$. For a subset C of A, $R(C) = \{b \in B \mid (a,b) \in R$ for some $a \in C\}$.

The restriction of a mapping $f : A \to B$ to a subset C of A is denoted by $f \restriction C$. Two mappings $f : A \to B$ and $g : A' \to B'$ *agree* if $f(a) = g(a)$ for every $a \in A \cap A'$, i.e., if $f \restriction (A \cap A') = g \restriction (A \cap A')$. We denote by $f \cup g$ their common extension into a mapping $: A \cup A' \to B \cup B'$.

We denote by $[A \to A]_f$ the set of mappings $: A \to A$ that are the identity outside of a finite subset of A and by $Perm_f(A)$ the subset of those that are permutations, i.e., bijections $: A \to A$.

Other symbols

Notation that is self-explanatory (e.g., $Loops(G)$ for the set of loops of a graph G), or that is used in a single section, is not listed. The order below is conceptual: concepts that are close mathematically are put together as closely as possible. General concepts are given before the more technical ones. Symbols are followed by short explanations and the page numbers where they are defined.

Terms and rooted trees (Sections 2.1 and 2.6.1, Definitions 2.13 and 2.14)

Graph algebras: graphs with sources (Section 2.3)

Graph algebras: graphs with ports (Section 2.5)

Equational and recognizable sets; automata (Chapters 3, 4 and 6)

Cambridge University Press
978-0-521-89833-1 - Graph Structure and Monadic Second-Order Logic: A Language-Theoretic Approach
Bruno Courcelle and Joost Engelfriet
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More information

Relational structures (Chapters 5, 7 and 9)

Logic (Chapters 5 and 6)

Standard logical notation is not reviewed.

Monadic second-order transductions (Chapters 7 and 9)

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[1] This index refers to sections, definitions, theorems etc. by using the following abbreviations: a = application, c = corollary, d = definition, e = example, ℓ = lemma, p = proposition, r = remark, s = section and t = theorem.