Fluid Film Lubrication

Second Edition

Fluid film bearings are machine elements that should be studied within the broader context of tribology. The three subfields of tribology – friction, lubrication, and wear – are strongly interrelated. The last decade has witnessed significant advances in the area of fluid film lubrication and its applications, and this second edition offers a look at some of these advances. This edition adds to the fundamentals of fluid film lubrication a discourse on surface effects and the inclusion of treatment of flow with significant inertia within the section on turbulence. Basic ideas of the multigrid method are conveyed along with multilevel multi-integration in the treatment of elastohydrodynamic lubrication. The chapter on lubrication with non-Newtonian fluids discusses the impact of the so-named qualitative EHL. This chapter also contains a thorough discussion of blood as a lubricant, with a view of the application of lubrication theory to LVADs. New chapters have been included on ultra-thin films, both liquid and gaseous, and lubrication of articulating joints and their replacement. Some of the most recent literature is discussed.

Andras Z. Szeri is the Robert Lyle Spencer Professor of Mechanical Engineering at the University of Delaware. He is a member of ASME and AAM and is on the editorial board of Microsystem Technologies and Applied Mechanics and Engineering.
To my wife Mary for her continued encouragement
# Contents

*Preface to the Second Edition*  
*Preface to the First Edition*

1 Introduction  
1.1 Historical Background  
1.2 Tribological Surfaces  
1.3 Friction  
   - Laws of Friction  
   - Asperity Contact  
   - Adhesion Theory of Friction  
   - Junction Growth  
   - Ploughing  
   - Friction of Metals  
   - Friction of Polymers  
   - Friction of Ceramics  
   - Thermal Effects of Friction  
1.4 Wear  
   - Sliding Wear  
   - Abrasive Wear  
1.5 Effect of Lubrication  
   - Thick-film Lubrication  
   - Mixed Lubrication  
   - Boundary Lubrication  
   - Solid Lubrication  
1.6 Fluid Film Bearings  
   - Hydrostatic Bearings  
   - Hydrodynamic Bearings  
   - Elastohydrodynamic Lubrication  
1.7 Bearing Selection  
   - Rubbing Bearings  
   - Rolling-Element Bearings  
   - Fluid Film Bearings  
1.8 Nomenclature  
1.9 References

2 Basic Equations  
2.1 Fluid Mechanics  
Kinematics
3 Thick-Film Lubrication

3.1 Externally Pressurized Bearings
- Pad Characteristics
- Optimization
- Operation with Flow Restrictors

3.2 Journal Bearings
- Short-Bearing Theory
- Boundary Conditions
- Long-Bearing Theory
- Sommerfeld Condition
- Gumbel Condition
- Swift-Stieber Conditions
- Finite Journal Bearings
- Cavitation Algorithm

3.3 Thrust Bearings
- Plane Slider
- Sector Thrust Bearing

3.4 Effects of Surface Topography
- Surface Roughness
- Statistical Methods
- Homogenization
- Surface Texturing

3.5 Nomenclature
3.6 References

4 Dynamic Properties of Lubricant Films

4.1 Fixed Pad
- Linearized Force Coefficients
- Analytical Solutions
- Coordinate Transformations

4.2 Stability of a Flexible Rotor
## Contents

4.3 Pivoted-Pad Journal Bearings 159  
Pad Assembly Method 162  
Pad Perturbation Method 171  
4.4 Pivoted-Pad Thrust Bearing 173  
4.5 Nomenclature 181  
4.6 References 183  

5 Effects of Fluid Inertia 184  
5.1 Temporal Inertia Limit, $R_\varepsilon \to 0, \Omega^* \geq 1$ 185  
5.2 Convective Inertia Limit, $\Omega^* \to 0, R_\varepsilon \geq 1$ 185  
   Journal Bearings 186  
   Hydrostatic Bearings 199  
5.3 Total Inertia Limit, $\Omega^*/R_\varepsilon \to 1, Re \geq 1$ 205  
   The Method of Small Perturbations 205  
   Squeeze Flow Between Parallel Plates 208  
   The Method of Averaged Inertia 213  
5.4 Nomenclature 218  
5.5 References 219  

6 Flow Stability and Transition 222  
6.1 Stability 223  
   Stability Criteria 223  
   Stability Analysis 225  
   Energy Stability 226  
   Linear Stability 227  
   Bifurcation Analysis 228  
6.2 Flow between Concentric Cylinders 229  
6.3 Flow between Eccentric Cylinders 232  
   Critical Reynolds Number 235  
   Local Iteration 238  
6.4 Rotating Disk Flows 243  
   Linear Stability Analysis 244  
6.5 Nomenclature 248  
6.6 References 249  

7 Turbulence 254  
7.1 Equations of Turbulent Motion 254  
7.2 Turbulence Models 259  
7.3 Constantinescu’s Model 264  
7.4 Ng-Pan-Elrod Model 269  
7.5 Bulk Flow Model of Hirs 274  
7.6 Turbulence with Inertia Retained 279  
   Method of Averaged Inertia 279  
7.7 Nomenclature 281  
7.8 References 283  

8 Elastohydrodynamic Lubrication 285  
8.1 Rigid Cylinder Rolling on a Plane 285
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.2</td>
<td>Elastohydrodynamic Theory</td>
<td>288</td>
</tr>
<tr>
<td>8.3</td>
<td>Contact Mechanics</td>
<td>295</td>
</tr>
<tr>
<td>8.4</td>
<td>Nondimensional Groups</td>
<td>299</td>
</tr>
<tr>
<td></td>
<td>Lubrication Regimes</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>Film-Thickness Design Formulas</td>
<td>303</td>
</tr>
<tr>
<td>8.5</td>
<td>Analysis of the Line Contact Problem</td>
<td>304</td>
</tr>
<tr>
<td></td>
<td>Elastic Deformation</td>
<td>304</td>
</tr>
<tr>
<td></td>
<td>Problem Formulation</td>
<td>307</td>
</tr>
<tr>
<td></td>
<td>Numerical Considerations</td>
<td>309</td>
</tr>
<tr>
<td>8.6</td>
<td>Analysis of the Point Contact Problem</td>
<td>313</td>
</tr>
<tr>
<td></td>
<td>Relaxation</td>
<td>314</td>
</tr>
<tr>
<td></td>
<td>The Multigrid (MG) Method</td>
<td>317</td>
</tr>
<tr>
<td></td>
<td>Application to Linear Operators</td>
<td>317</td>
</tr>
<tr>
<td></td>
<td>The Intergrid Operators</td>
<td>320</td>
</tr>
<tr>
<td></td>
<td>Application to Nonlinear Operators</td>
<td>321</td>
</tr>
<tr>
<td></td>
<td>Problem Formulation</td>
<td>323</td>
</tr>
<tr>
<td></td>
<td>Multilevel Multi-Integration</td>
<td>329</td>
</tr>
<tr>
<td>8.7</td>
<td>Rolling-Contact Bearings</td>
<td>332</td>
</tr>
<tr>
<td></td>
<td>Bearing Types</td>
<td>333</td>
</tr>
<tr>
<td></td>
<td>Rolling Friction</td>
<td>337</td>
</tr>
<tr>
<td></td>
<td>Frictional Losses in Rolling Contact Bearings</td>
<td>338</td>
</tr>
<tr>
<td></td>
<td>Specific Dynamic Capacity and Life</td>
<td>339</td>
</tr>
<tr>
<td></td>
<td>Specific Static Capacity</td>
<td>342</td>
</tr>
<tr>
<td></td>
<td>Fatigue Wear Out</td>
<td>342</td>
</tr>
<tr>
<td>8.8</td>
<td>Minimum Film Thickness Calculations</td>
<td>342</td>
</tr>
<tr>
<td></td>
<td>Nominal Line Contact</td>
<td>342</td>
</tr>
<tr>
<td></td>
<td>Nominal Point Contact</td>
<td>344</td>
</tr>
<tr>
<td>8.9</td>
<td>Nomenclature</td>
<td>346</td>
</tr>
<tr>
<td>8.10</td>
<td>References</td>
<td>347</td>
</tr>
<tr>
<td>9</td>
<td>Thermal Effects</td>
<td>351</td>
</tr>
<tr>
<td>9.1</td>
<td>Effective Viscosity</td>
<td>351</td>
</tr>
<tr>
<td>9.2</td>
<td>Thermohydrodynamic Theory</td>
<td>357</td>
</tr>
<tr>
<td></td>
<td>The Energy Equation</td>
<td>357</td>
</tr>
<tr>
<td></td>
<td>The Pressure Equation</td>
<td>365</td>
</tr>
<tr>
<td>9.3</td>
<td>Journal Bearings</td>
<td>367</td>
</tr>
<tr>
<td></td>
<td>Bearing Temperature</td>
<td>367</td>
</tr>
<tr>
<td></td>
<td>The Role of Nondimensional Parameters</td>
<td>369</td>
</tr>
<tr>
<td></td>
<td>Friction Factor</td>
<td>371</td>
</tr>
<tr>
<td></td>
<td>Journal Locus and Dynamic Coefficients</td>
<td>372</td>
</tr>
<tr>
<td></td>
<td>Thermal Deformation</td>
<td>373</td>
</tr>
<tr>
<td>9.4</td>
<td>Thrust Bearings</td>
<td>376</td>
</tr>
<tr>
<td></td>
<td>The Pressure Equation</td>
<td>377</td>
</tr>
<tr>
<td></td>
<td>The Energy Equation</td>
<td>379</td>
</tr>
<tr>
<td></td>
<td>The Heat Conduction Equation</td>
<td>381</td>
</tr>
<tr>
<td></td>
<td>Pad Deformation</td>
<td>381</td>
</tr>
</tbody>
</table>
## Contents

9.5 Nomenclature 385
9.6 References 386

10 Lubrication with Non-Newtonian Fluids 389

10.1 Hydrodynamic Lubrication 390
  Summary of Previous Work 390
  Lubrication with Power Law Fluid 391
  Fluids of the Differential Type 393
  Lubrication with a Third Grade Fluid 396

10.2 Elastohydrodynamic Lubrication 402
  Constitutive Models 403
  A Generalized non-Newtonian Reynolds Equation for EHL 406

10.3 Quantitative Elastohydrodynamic Lubrication (EHL) 411

10.4 The Piezoviscous Fluid 416

10.5 Lubrication with Emulsions 421
  Fundamentals of Mixture Theory 422
  Constitutive Model 424
  Lubrication Approximation 426
  Applications 427

10.6 Blood as Lubricant 431
  The Rheology of Blood 433
  Rheological Models 435
  Blood Trauma Models 437

10.7 Nomenclature 440
10.8 References 442

11 Gas Lubrication 451

11.1 Reynolds Equation for Gas Lubricant 453

11.2 Self Acting Gas Bearings 455
  Journal Bearings 457
  Infinitely Long Step Slider 460

11.3 Nomenclature 464
11.4 References 464

12 Molecularly Thin Films 466

12.1 Gas Flow 467
  Velocity Slip at the Boundary 468
  Molecular Gas Lubrication 476
  Direct Simulation Monte Carlo 480

12.2 Liquid Flow 483
  Molecular Dynamics Simulation 483
  Velocity Slip at Solid Boundary 484
  Density Oscillation Near Solid Boundary 491
  Interactive Force Between Closely Spaced Solid Surfaces 492
  Van der Waals Forces 493
  Double-Layer Forces 494
  Solvation Forces 494
<table>
<thead>
<tr>
<th>Contents</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Response to Shear</td>
<td>501</td>
</tr>
<tr>
<td>Ultrathin Film Lubrication</td>
<td>504</td>
</tr>
<tr>
<td>12.3 Nomenclature</td>
<td>505</td>
</tr>
<tr>
<td>12.4 References</td>
<td>506</td>
</tr>
<tr>
<td>13 Biotribology</td>
<td>511</td>
</tr>
<tr>
<td>lubrication of Articular Joints</td>
<td>511</td>
</tr>
<tr>
<td>13.1 Natural Joints</td>
<td>512</td>
</tr>
<tr>
<td>Properties of the Cartilage</td>
<td>512</td>
</tr>
<tr>
<td>Elastohydrodynamic Models</td>
<td>513</td>
</tr>
<tr>
<td>Boosted Lubrication</td>
<td>515</td>
</tr>
<tr>
<td>Weeping Lubrication</td>
<td>516</td>
</tr>
<tr>
<td>Biphasic Models</td>
<td>517</td>
</tr>
<tr>
<td>Boundary Lubrication</td>
<td>520</td>
</tr>
<tr>
<td>13.2 Artificial Joints</td>
<td>525</td>
</tr>
<tr>
<td>Types of Total Hip Replacement (THR)</td>
<td>525</td>
</tr>
<tr>
<td>Mathematical Modeling</td>
<td>527</td>
</tr>
<tr>
<td>Hard-on-Soft THR</td>
<td>530</td>
</tr>
<tr>
<td>Hard-on-Hard THR</td>
<td>533</td>
</tr>
<tr>
<td>13.3 Nomenclature</td>
<td>537</td>
</tr>
<tr>
<td>13.4 References</td>
<td>537</td>
</tr>
<tr>
<td>Index</td>
<td>543</td>
</tr>
</tbody>
</table>
Preface to the Second Edition

For the successful operation of mechanical devices, from spinning computer disks to automobiles to large electric generators of nuclear power stations, it is essential that the components that are destined to move relative to one another do so with low friction and rate of wear. This is made possible through appropriate design and utilization of fluid film bearings. Traditionally, fluid film lubrication was a purely mechanical subject, but with the emergence of bioengineering, the technology also finds application in lubricating artificial joints, contact lenses, and mechanical heart pumps, to name a few. In this second edition, I have tried to give a flavor of some of these advances. The contents of the first edition remain valid by and large, as they deal with the fundamentals that have changed little. Thus, this edition represents addition, rather than revision, of material. Chapter 2 is rewritten, however, to align it with a more complete discussion of constitutive theory. Chapter 3, the chapter covering thick-film lubrication, features a section on surface texturing; another section treats surface roughness in a more thorough manner. The chapter on turbulence includes the handling of flow with significant inertia. In the treatment of elastohydrodynamic lubrication, covered in Chapter 8, I tried to convey basic ideas of the multigrid method and touched on multilevel multi integration. The chapter on lubrication with non-Newtonian fluids discusses the “qualitative” EHL, and contains a discourse on lubrication with piezoviscous fluids, relative to the Reynolds equation. This chapter also comprises a thorough discussion of blood as lubricant, with a view to the application of lubrication theory to artificial organs. Chapters 12 and 13 are new. In the first of these, I concentrate on ultra-thin films, both liquid and gaseous. The chapter discusses and classifies recent research results and, particularly for gas films but also for liquid films, outlines design principles. The chapter closes with the listing of 92 up-to-date references. The last chapter, Biotribology, is devoted to lubrication of the hip joint. Its two principal sections discuss lubrication of natural joints and artificial joints. The first of these presents the various theories of natural joint lubrication including microelastohydrodynamic lubrication, biphasic models, and boundary lubrication. The second section lists the various existing constructs of total hip replacement and their relative performance. This last chapter surveys 60 references in all.

Compared to the first edition, the second edition contains more than 70 new pictorial representations of recent research results. I trust that the reader will find these additions worthwhile.
Preface to the First Edition

Fluid film bearings are machine elements which should be studied within the broader context of tribology, “the science and technology of interactive surfaces in relative motion and of the practices related thereto.” The three subfields of tribology – friction, lubrication, and wear – are strongly interrelated. Fluid film bearings provide but one aspect of lubrication. If a bearing is not well designed, or is operated under other than the design conditions, other modes of lubrication, such as boundary lubrication, might result, and frictional heating and wear would also have to be considered.

Chapter 1 defines fluid film bearings within the context of the general field of tribology, and is intended as an introduction; numerous references are included, however, should a more detailed background be required. Chapters 2, 3, and 4 outline classical lubrication theory, which is based on isothermal, laminar operation between rigid bearing surfaces. These chapters can be used for an advanced undergraduate or first-year graduate course. They should, however, be augmented with selections from Chapter 8, to introduce the students to the all-important rolling bearings, and from Chapter 9, to make the student realize that no bearing operation is truly isothermal. Otherwise, the book will be useful to the industrial practitioner and the researcher alike. Sections in small print may be omitted on first reading – they are intended for further amplification of topics. In writing this book, my intent was to put essential information into a rational framework for easier understanding. So the objective was to teach, rather than to compile all available information into a handbook. I have also included thought-provoking topics; for example, lubrication with emulsions, the treatment of which has not yet reached maturity. I expect significant advances in this area as it impacts on the environment.

The various chapters were read by Dr. M. L. Adams, Case Western Reserve University; Dr. M. Fillon, University of Poitiers, France; Dr. S. Jahanmir, National Institute for Standards and Technology; Dr. F. E. Kennedy, Dartmouth College; Mr. O. Pinkus, Sigma Inc.; Dr. K. R. Rajagopal, Texas A & M University; Dr. A. J. Szeri, University of California at Berkeley; and Dr. J. A. Tichy, Rensselaer Polytechnic Institute. However, in spite of the considerable assistance I received from various colleagues, any mistakes are mine alone.

The typing was expertly done by my daughter Maria Szeri-Leon and son-in-law Jorge Leon. I am grateful to them for their diligence and perseverance; not even their wedding interrupted the smooth flow of the project. I would also like to thank Ms. Florence Padgett, Editor at Cambridge University Press, for suggesting the project and for having confidence in me. My thanks are also due to Ms. Ellen Tirpak, Senior Project Manager at TechBooks, for providing expert editing of the manuscript.