Economic Growth

How can society improve its living standards? What are the conditions necessary for prosperity? These are the questions that define the essence of growth theory. In this user-friendly book, Olivier de La Grandville provides a fascinating introduction to the theory of economic growth and shows how many results from this field are of paramount importance for society. The classical mechanics of the growth process are carefully explained, with two chapters devoted to the fundamental issue of the substitution of labour for capital in the growth process (co-written with Robert M. Solow, winner of the Nobel Prize in Economics). The book also addresses the fundamental question of the optimal investment rate of an economy. In addition, La Grandville shows us that by unifying the descriptive and normative aspects of growth theory we can generate many fresh insights, including a proof of Adam Smith's "Invisible Hand" conjecture.

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OLIVIER DE LA GRANDVILLE

Economic Growth

A Unified Approach

With two special contributions by Robert M. Solow

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This book is lovingly dedicated
to my wife Ann,
to our children Diane, Isabelle and Henri,
and to their own children

CONTENTS

INTRODUCTION

Why should you read a book on economic growth? Because the subject is important: it is about the well-being of our societies today and in the future; and because it is beautiful. It carries wonderful ideas, some exposed more than 2000 years ago, spanning all civilizations. You will certainly marvel at Ibn Khaldun's prescience, at Mo Tzu's wisdom, at Solow's depiction of transition phases, at Dorfman's incredible intuition in solving variational problems.

This book is not quite the same as other books. Economic growth has attracted, particularly in the last hundred years, countless, excellent writers who have developed the field into an immense array of topics, from theoretical to empirical. Rather than trying to cover all developments – of which you can have an idea through the bibliography – I have wanted to tell you what I found fascinating in the subject. But my hope is also that you will find here a useful introduction to this wide area of research, because a lot of the book is not only on ideas but on methodology as well.

A further reason for me to write this text was to submit personal views and present new results. For too many years I have expounded growth theory by dividing the subject, as many did, into two main strands of thought: positive, or descriptive theory on the one hand, normative on the other. I am now convinced that those two strands should be unified – hence the title of the book. For clarity's sake, I think however that both approaches to the theory should be first presented separately (parts I and II), and then unified (part III). Such unification proceeds not from any personal whim, but from logical reasons: the results of both strands of thought mutually imply each other, as will be shown.

Let me now underline the new results you will find in this book:

• A proof of one of the most important, daring conjectures ever made in economics or social sciences. We owe this conjecture, known as "the invisible hand", to Adam Smith who wrote, in his *Inquiry into the Nature and the Causes of the Wealth of Nations* (1776):

> Every individual is continually exerting himself to find out the most advantageous employment for whatever capital he can command. It is his own advantage, indeed, and not that of the society, which he has in view. But

> the study of his own advantage naturally, or rather necessarily, leads him to prefer that employment which is most advantageous to the society . . .
>
> He generally, indeed, neither intends to promote the public interest, nor knows how much he is promoting it. . . . He intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention.[1]

Note how far reaching Adam Smith's conjecture is. Not only does the author hold that the search by individuals for the most advantageous employment of their capital stock is advantageous to society; he takes one step further by stating that this advantage is maximized.

What is then the exact benefit that Smith could have referred to in his conjecture? How is it to be measured? The answer is quite surprising. It will be shown that it is not only one, but *two* magnitudes that are simultaneously maximized for society:

(1) the sum of the discounted consumption flows society can acquire from now to infinity

(2) the benefits of society's activity at any point of time t – including today; those benefits are the sum of the consumption flows received at time t and the rate of increase in the value of the capital stock at that time.

The proof of this theorem, which you will find in the last chapter, uses the methodology and results expounded throughout this text, and draws both on positive and normative theory. If only for this very proof, there would be ample reason to justify the unified approach I am advocating.

• A thorough analysis of the importance of the elasticity of substitution in the growth process. Too often do we see growth models carrying the convenient, beloved hypothesis of an elasticity of substitution equal to 1, equivalent to the Cobb–Douglas function. We now have evidence, however, that in any economy we might consider the elasticity of substitution is significantly different from 1, with a tendency of growing – which is good news, as the reader will discover. A surprise is in store: an increase in the elasticity of substitution will be shown to have far more importance for society's future welfare than a similar increase in the rate of technical progress.

• A detailed examination of the consequences of using utility functions in optimal growth theory. Traditional treatment of the theory usually leads to a system of differential equations which does not possess analytical solutions. In my opinion, this issue, involving solving numerically such a system, has been taken too lightly, and short shrift has been given to results taking the form of exceedingly high "optimal" savings rates. It can even be read in the literature that some family of

[1] A. Smith, *op. cit.*, Dent and Sons, London, 1975, pp. 398–400.

utility functions can be used, although it turns out that no equilibrium point exists. Great care will be taken in analysing the optimal time-paths of the economy, both in terms of their associated initial values and their ultimate evolution.

• A formula for the optimal savings rate of an economy. Until now optimal savings rates could be calculated numerically only – no closed form was available and, as mentioned, those values often made little sense. The formula I submit is expressed in terms of the fundamental characteristics of the economy and society's rate of preference for the present, and yields reasonable, very reachable values.

• Applications and extensions of Dorfman's modified Hamiltonian. With remarkable insight, Robert Dorfman had introduced a new Hamiltonian to tackle the variational problems encountered in optimal growth theory. To honour Professor Dorfman's memory, I propose to call his concept a Dorfmanian. It will play a fundamental role in the proof of Smith's conjecture. The reader will also find here extensions of the Dorfmanian which can yield all high-order equations of the calculus of variations – including the Euler–Poisson and the Ostrogradski equations.

• The final reason why you should read this book is that Robert Solow and myself need your help: you will be invited to exercise your sagacity and try to prove a conjecture we are offering at the end of chapter 4. The conjecture, of a mathematical nature, is as formidable a challenge to prove as it is easy to express: the general mean of two numbers, considered as a function of its order, has one and only one inflection point. Why is it important? Because as a result, income per person behaves exactly like a function of production whose dependent variable is the elasticity of substitution, with a first phase of increasing returns, followed by decreasing returns, and our economies seem to be in the very neighbourhood of this point of inflection.

It is my pleasure to thank a number of persons whose role has been essential in the realization of this book. First and foremost, I would like to express my deepest gratitude to Robert Solow, who not only co-authored a large chapter (chapter 5) and the appendix of chapter 4, but also gave me invaluable advice on many other important parts of the book. Needless to say, I alone remain responsible for any remaining shortcomings, and the personal views expressed here are not necessarily condoned by him.

My colleague Ernst Hairer, of the Department of Mathematics at the University of Geneva, has used his program DOPRI for solving numerically the differential equations of chapters 9 and 10; the stunning phase diagrams – in particular the one on the cover of this book – are his work. Ernst Hairer's generosity led him also to write the program yielding the initial values leading to equilibrium

for any system of parameters characterizing the economy. I am in great debt to him.

Claudio Sfreddo let us use for our regressions in chapter 6 the data he had developed on a common basis for 16 OECD countries in his PhD thesis; we are very grateful to him.

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