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Jacques Heyman
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BASIC STRUCTURAL THEORY

This book introduces the basic equations of the theory of structures. Conventional presentations of these equations follow the ideas of elastic analysis, introduced nearly two hundred years ago. The present text is written against the background of advances made in structural theory during the last fifty years, notably by the introduction of the so-called plastic theory. Tests on real structures in the twentieth century revealed that structural states predicted by elastic analysis cannot in fact be observed in practice, whereas plastic ideas can be used to give accurate estimates of strength. Strength is discussed in the first part of this book without reference to equations of elastic deformation. However, the designer is concerned also with stiffness, for which elastic analysis is needed, and the standard equations (suitable, for example, for computer programming) are presented. Finally, stability is analyzed, which again is essentially an elastic phenomenon, and it is shown that a higher factor of safety is required to guard against buckling than is required to guarantee straightforward strength. The emphasis throughout this book is on the derivation and application of the structural equations, rather than on details of their solution (nowadays best done by computer), and the numerical examples are deliberately kept simple.

Jacques Heyman is former Head of the Department of Engineering at the University of Cambridge and the author of fourteen books, including *The stone skeleton, Elements of the theory of structures, Structural analysis: a historical approach, Elements of stress analysis* and the two-volume set, *Plastic design of frames: Volume 1, Fundamentals* and *Volume 2, Applications*, with Lord Baker. He is a Fellow of the Society of Antiquaries, the Institution of Civil Engineers and the Royal Academy of Engineering, and an Honorary Fellow of the Royal Institute of British architects. He was a consulting engineer for a number of English cathedrals and a member of the Architectural Advisory Panel for Westminster Abbey and the Cathedrals Fabric Commission for England. He also has served on many British Standards committees. Professor Heyman's *The stone skeleton* won the Choice Outstanding Academic Books Award in 1996.

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Preface

University courses in structural theory (as in any other branch of engineering) aim to teach the principles of the subject. It is in fact difficult, if not impossible, to discuss a principle in the abstract, and students are usually engaged in carrying out an assortment of algebraic or numerical calculations for particular examples of structures, in the hope that fundamental truths will be revealed.

Structural equations are straightforward and are, with some exceptions, linear. The equations may be written easily, but unfortunately they are very numerous. Although their solution presents no conceptual difficulty, the work involved is so heavy that, before the advent of the electronic computer, it was virtually impossible to obtain exact numerical results for any but the simplest structure. Advances in structural theory in the nineteenth century, and in the twentieth, were directed on the one hand to the establishment of basic theorems to guide the engineer towards easier formulation of the equations, and on the other hand to the development of

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computational techniques which could lead to approximate solutions of any required degree of accuracy.

Many of these advances were made by scientists and engineers of experience and insight, and they show great creative genius. Thus the student's understanding will be enlightened by, for example, the elastic reciprocal theorems, the concepts of strain and potential energy, and by the theory underlying the testing of models. Side by side with these fundamental elastic properties, however, the student may well be presented with a host of techniques such as deflexion coefficients, slope/deflexion equations, and moment/area methods, which might seem to be basic to the theory of structures whereas, despite their intellectual power, they are really no more than aids to the solution of the structural equations. The student may well feel aggrieved to have spent time mastering methods of calculation, when the modern computer is furnished with programs which can produce numerical solutions for any complex structure.

Some of these topics are presented in Chapter 4, but the discussion is brief. It is the intention of this book to present the basic ideas of structural theory, rather than to review the many techniques of calculation for elastic structures. These basic ideas will enable the engineer to appreciate the way in which a computer program delivers its solutions, without necessarily investigating every detail of the computation. All of these analyses, the theory and the calculations, fall within the gigantic intellectual framework of the classical theory of structures, enunciated formally by Navier in 1826, and developed

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over the next century to the point where it forms the basis of most design codes throughout the world.

There is a second and powerful reason for not concentrating on the conventional syllabus of nineteenth and twentieth century courses on structural engineering. The methods of elastic analysis, whether executed by hand or by computer, purport to describe the actual state of a given structure under a given loading system. The Navier theory appeared to be so self-evidently correct that it was almost a century before tests on real structures revealed that the results of elastic analysis cannot be observed in practice. There is now overwhelming evidence that the state of an actual structure may be very different from that calculated confidently by the elastic designer.

A seemingly artificial example, the four-legged table, reveals the problem. The tripod is an ideal structure – the forces in the three legs which result from a given loading can be found easily and unequivocally from simple equations. However, those same equations are insufficient to furnish the leg forces for the conventional table with four legs, and the full apparatus of elastic structural theory leads to those difficult calculations to which the computer can now give a precise answer.

This computer output is the Navier elastic solution for this theoretical structure. The real table, placed on a hard floor, will rock, and if a leg is clear of the floor by a mere fraction of a millimetre, it is certain that the force in that leg is actually zero, whereas the computer has supplied a definite value for

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the force. Moreover, a cork wedge may be used to make the table comfortable for its users, which exposes the task facing the structural engineer: how are the leg forces to be evaluated, so that the legs may be designed, when any one of the four may be in contact with a (supposedly) rigid floor, or clear of the floor, or supported by an elastic wedge of unknown properties?

A real structure is, in fact, supported externally in a way which is unknown (and unknowable) to the engineer, who nevertheless is required to make a design. In modelling the structure for analysis, the conventional elastic designer is forced to make some assumptions (as is the computer program) – for the table, for example, that all four legs are in contact initially with a (rigid) floor. These assumptions, seemingly innocuous and actually of small consequence, can lead to structural solutions widely different from those observed in practice. Very small differences in *boundary conditions* can lead to wholly disproportionate differences in internal structural forces, in real multi-storey buildings as well as in the simple model of the table. The foundations of a steel or concrete frame can settle by small but – for the user – acceptable amounts; a bolted joint, assumed to be inflexible, may slip on first loading; frame members may be manufactured with slight dimensional errors. Such defects seem trivial, and they do not, in fact, affect the basic strength of a structure, but it is these defects which reveal that elastic calculations give a poor indication of how a structure carries its loads.

The anomaly was fully revealed in the first half of the twentieth century by tests on buildings under construction,

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and the results led to the development of the so-called plastic theory as an alternative to elastic analysis. Plastic theory in its simple form makes no use of (unknowable) boundary conditions in the assessment of strength – indeed, no attempt is made to calculate the actual state of a structure. Instead, new and powerful theorems – above all, the safe theorem – give the designer confidence in plastic methods. Paradoxically, it is the safe theorem which shows that conventional elastic methods, the Navier schema – while predicting a state which is not experienced by the real structure, and which will lead to a design which is usually wasteful of material – nevertheless gives a safe estimate of strength.

However, simple plastic theory is concerned only with the prediction of strength, and traditional elastic computations cannot be rejected. Even if, as is the case, the working state of a structure is essentially unknowable, in the sense that the internal forces are critically dependent on seemingly trivial unknown imperfections, the designer may be required to make estimates of stiffness to ensure that deflexions lie below specified limits. A structure may well yield on first loading, but stiffness is basically an elastic structural property, and must be estimated somehow by the engineer. Similarly, although buckling may also involve some yielding, the onset of instability can be determined (with some empirical imprecision) by the use of classical elastic differential equations.

This book starts, then, with a discussion of the strength of structures, and it will be seen that statements can be made with some confidence. Subsequent chapters discuss stiffness and stability, and some of the traditional methods of elastic

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analysis are presented in order to predict deflexions and the onset of buckling. The examples are deliberately kept simple, and the necessary mathematical foundations of the subject are outlined in three short appendices.

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