1 Introduction

The structures discussed in this book are assemblages of elements (e.g. beams, columns, struts, ties) that form a construction of some practical use. For example, a light steel gantry may be needed to support a cable to power electric trains; or simple portal frames, steel or concrete, may house a factory; or the elements may be combined into a framework for a multi-storey building. A *theory of structures* is necessary to ensure that the design of any particular construction will be satisfactory when built.

The designer decides on the general form of a structure – for example, using girders working in bending for a small-span bridge, rather than a lattice truss with members working in tension or compression (alternative forms may be examined simultaneously to achieve a best design). Design requirements (e.g. specified imposed loads, permitted maximum deflexions) are stated, and the designer's task is to satisfy those requirements. The design process falls logically into two stages: dimensions are assigned to the members of the chosen form, and the theory of structures is then used to ensure that the members are comfortable, and that the overall behaviour of the structure meets the criteria. This process, in general, is circular; the structural analysis cannot be made until the sizes of the members have been chosen, but those sizes depend on the results of the structural analysis. In some cases it may be possible to achieve a direct design without this circular process of trial and error (and, certainly, computer programs may be written to achieve rapidly convergent designs). This book is concerned with the *analysis* of structural forms to ensure that design criteria are met.

The three major structural criteria are strength, stiffness and stability. Successive chapters are devoted to these topics. Individual members must certainly be strong enough to carry the loads they are designed to bear, but the overall strength of a complex structure may well be determined by the interaction of those members. The strength of structures is examined in Chapter 2.

Similarly, to be serviceable a structure must have displacements with acceptable limits – it must be stiff enough under the prescribed loading so that deflexions are not developed which might interfere with its design function (e.g. overhead rails in a factory building must remain sufficiently rigid to ensure that a gantry crane can operate without difficulty; an electric cable must be reachable by the pantogram of a train). Such deflexions are almost always elastic, and their calculation is explained in Chapter 3, and continued in Chapter 4.

Finally, the structure must remain stable. A familiar form of instability may be observed in the buckling of columns, but other forms are possible, and they include the instability of

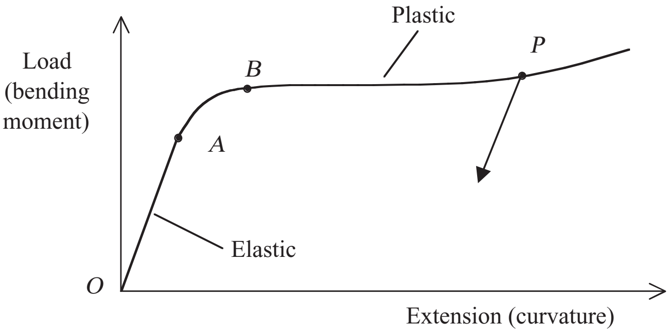

Figure 1.1. Load/extension curve for a ductile material.

the structure overall. Buckling of an individual steel or concrete member may be sudden, and could prove disastrous for the structure as a whole, although in certain types of construction (for example, plates and thin shells, which are outside the scope of this book), stable buckling can occur. Stability is studied in Chapter 5.

1.1 Structural assumptions

A first requirement of a material that is structurally useful is that it should be ductile. That is, steel, reinforced concrete (preferably under-reinforced), aluminium alloys, and perhaps wrought iron are acceptable, but cast iron and glass are not; they will shatter if incorporated as load-bearing members in a practical structure. Figure 1.1 shows schematically the results of a tensile test on a prismatic mild steel bar of a grade typically used in structural work. As the tensile load on the specimen is increased the resulting extension is at first elastic and proportional to the load (Hooke's Law), and is recoverable.

However, when the yield stress of the steel is reached the specimen extends at a more or less constant load, and behaves in a *plastic* way. If the test is interrupted at this stage, and the load is removed, the extension is not fully recoverable, and the unloading from a point such as P is elastic. The transition from elastic to plastic behaviour in such a test may be sharp, and points A and B may almost coincide. However, the important property of the schematic sketch of fig. 1.1 is that possible plastic extensions, for mild steel, are many times the extension at first yield (more than a factor of 10 before indeed the load starts to increase with the onset of strain hardening).

Such a mild steel bar is used in the example of a truss in Chapter 2, but the bar could equally be made from aluminium alloy. In that case the load/extension characteristic differs from that shown in fig. 1.1 in that portion BP of the curve would rise gently instead of being virtually horizontal. However, a design based on the load at point B of the curve would be safe for the alloy construction, and in both cases, steel and aluminium alloy, the plastic region is sufficiently large that extensions may be assumed to be unlimited, and to take place at constant load (provided there is no danger of instability; see below for the third structural assumption). The load/extension characteristic is in fact idealized as shown in fig. 1.2.

If the mild steel member is used not in tension, but in bending in the form of a beam in a structural frame, then fig. 1.1 represents – again schematically, but with some accuracy – the moment/curvature characteristic of the member. As before, the initial response is linear and elastic, but at yield large

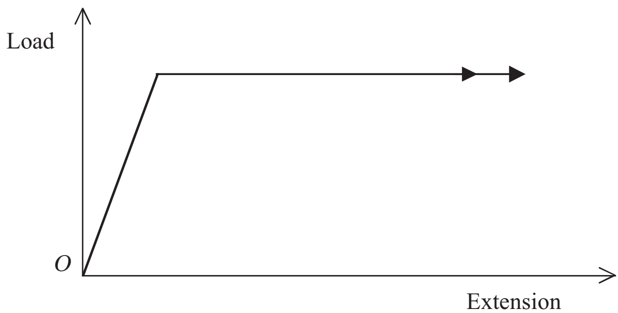

Figure 1.2. Idealised load/extension curve.

increases in curvature can occur in the beam. The yield zones are localized at *plastic hinges*; large rotations of the hinges can occur at a constant value of bending moment, defined as the *full plastic moment* of the beam. This value corresponds to the plateau *BP* in fig. 1.1. As will be seen, the formation of a single (or indeed more than one) plastic hinge does not necessarily imply that the structure has attained a limiting strength; that limit is reached when a sufficient number of hinges form so that unacceptably large deformations can occur.

A second structural assumption is concerned with the magnitude of the deformations. It is possible to construct analyses which allow for finite displacements, but the straightforward theory of structures assumes that working deformations (that is, displacements before the limiting strength is attained) are small compared with the overall dimensions of the structure. By small it is implied that changes in the overall geometry of the structure under load are negligible; thus the angles between the bars of a truss framework stay virtually unchanged, so that equilibrium equations involving these angles, and written for the undeformed structure, remain valid for the deformed state.

Finally, a third major assumption concerns the stability of structural members. This question is explored later but, essentially, care must be taken if a member is used whose load/deflexion characteristic does not exhibit the ductile plateau of the schematic fig. 1.1, but instead involves a decrease of load with increasing deformation.

1.2 Structural equations

The theory of structures is a branch of solid mechanics which deals with slightly deformable bodies, and there are only three types of basic equation which may be written to perform a structural analysis. The first set of equations expresses the static equilibrium of a structure – that is, internal structural resultants (e.g. bar forces in a truss, bending moments in a beam or frame, and so on) must be in equilibrium with the external loads acting on the structure. The familiar equations of statics – resolving forces, taking moments – are used to ensure this equilibrium. As will be seen in the next chapter, these basic equations may be used to determine the strength of a structure constructed from materials whose limiting strength (e.g. a yield stress or value of full plastic moment) is known.

The other two structural criteria – stiffness and stability – require the use of the other two sets of master structural equations. Straightforwardly, if elastic deflexions are to be calculated, then the elastic properties of the material must enter the analysis. For the trusses and beams considered in this book only the value of Young's modulus is needed. Once the value is specified for a given structural member, that member's elastic deformation can be calculated in terms of the applied internal forces (i.e. tension, compression, bending moment).

Problems involving shear deformation (which are not considered here) require the value of a second (independent) material constant, the shear modulus; this modulus is needed, for example, if the effects of twisting of a member (e.g. a steel hollow-box section) are to be investigated. (There are, in theory, 21 elastic constants for materials which possess no isotropy or other elastic symmetry. Wood, for example, has three mutually perpendicular planes of symmetry, two along the grain and one at right angles. In this case, the number of elastic constants required in theory to specify elastic behaviour is reduced to 9. However, for a reasonably homogeneous and isotropic material like steel or aluminium alloy the two constants suffice.)

Finally, the elastic deformations must be such that the members still fit together when the structure is loaded, and the structure as a whole must obey whatever boundary conditions may be specified (e.g. a beam rests on a given number of supports, a frame has its footings rigidly attached to foundations, and so on). Considerations such as these are expressed in the third set of master equations, the so-called compatibility conditions.

2 Strength

2.1 Trussed frameworks

The three equal bars shown in fig. 2.1 are supposed to be rigid and infinitely strong; they are pinned together at B and C with frictionless joints, and similar pins at A and D connect the assemblage to a rigid foundation. Evidently the figure does not represent a (two-dimensional) structure – it is a mechanism (a four-bar chain, counting the ground AD as one of the bars) incapable of carrying load. The addition of a diagonal member AC enables load to be applied – for example, the horizontal force W at joint C, fig. 2.2. The statical analysis of the truss is shown in fig. 2.3, in which the bar forces shown have been obtained by resolving horizontally and vertically at the frictionless joints. At B the two members meeting at right angles must each carry zero load, while the resolution of forces at joint C shows that the added member AC carries a tension $W\sqrt{2}$, while the (rigid) member CD is subject to a compression W. (In accordance with the assumption of small

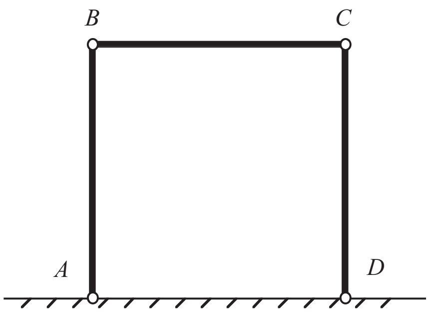

Figure 2.1. Mechanism of pinned bars.

deformations, the 90- and 45-degree angles in fig. 2.3 remain unchanged for the purpose of the resolution of forces.)

In contrast to the original three rigid members, the diagonal AC is a structural element which elongates slightly under the action of its tensile load. If the load/extension characteristic of member AC is known (that is, it has a known cross-sectional area and elastic modulus), then its extension can be calculated in terms of the force W, and the deflexion of point C may be determined.

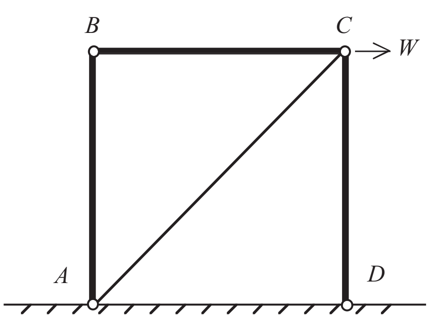

Figure 2.2. Structure capable of carrying load.

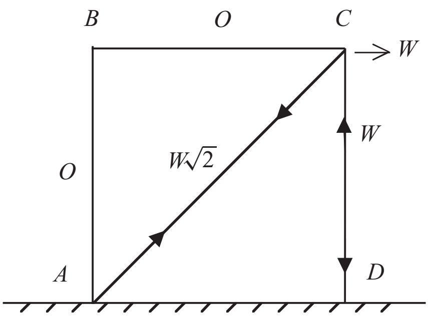

Figure 2.3. Bar forces due to load W.

The problem of the stiffness of the truss – that, is the determination of the deflexion of joint C – is discussed in Chapter 3. The present objective is to calculate the strength of the simple structure shown in fig. 2.2. If the member AC can sustain a maximum load of value T, then clearly the greatest value of W is $T/\sqrt{2}$. At this load, indefinite ductile extension of bar AC occurs in accordance with the idealized characteristic of fig. 1.2, and deflexions of the structure occur which are no longer small – a mechanism of collapse (the four-bar chain) has developed.

This analysis can hardly be dignified by the label Theory of Structures. The structural problem proper is illustrated in fig. 2.4, in which a second structural member BD has been added to the truss; as before, all joints are supposed to be freely pinned, and the two diagonals have no connexion where they cross. Under the action of the applied load W tensions P_1 and P_2 are developed in the two diagonal members, as shown. Resolution of forces at joint B leads to the marked values of tension in bars BC and BA. Tensile forces are denoted positive, so that the tension $-P_2\sqrt{2}$ marked for bar