Modern Electrodynamics

An engaging writing style and a strong focus on the physics make this comprehensive, graduate-level textbook unique among existing classical electromagnetism textbooks.

Charged particles in vacuum and the electrodynamics of continuous media are given equal attention in discussions of electrostatics, magnetostatics, quasistatics, conservation laws, wave propagation, radiation, scattering, special relativity, and field theory. Extensive use of qualitative arguments similar to those used by working physicists makes *Modern Electrodynamics* a must-have for every student of this subject.

In 24 chapters, the textbook covers many more topics than can be presented in a typical two-semester course, making it easy for instructors to tailor courses to their specific needs. Close to 120 worked examples and 80 applications boxes help the reader build physical intuition and develop technical skill. Nearly 600 end-of-chapter homework problems encourage students to engage actively with the material. A solutions manual is available for instructors at www.cambridge.org/Zangwill.

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There are more things in heaven & earth connected with electromagnetism than are yet dream't of in philosophy.

Joseph Henry, letter to Lewis C. Beck (1827)

Reprinted from *The Papers of Joseph Henry*, Volume I, edited by Nathan Reingold (Washington, DC: Smithsonian Institution Press). Used by permission of the Smithsonian Institution. Copyright 1972.

The search for reason ends at the shore of the known; on the immense expanse beyond it only the ineffable can glide.

Abraham Joshua Heschel, *Man is Not Alone* (1951)

Why repeat all this? Because there are new generations born every day. Because there are great ideas developed in the history of man, and these ideas do not last unless they are passed purposely and clearly from generation to generation.

Richard Feynman, *The Meaning of It All* (1963)

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Preface

A textbook, as opposed to a treatise, should include everything a student must know, not everything the author does know.
Kenneth Johnson, quoted by Francis Low (1997)

In his *Lectures on Physics*, Richard Feynman asserts that "ten thousand years from now, there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics". Whether this prediction is borne out or not, it is impossible to deny the significance of Maxwell's achievement to the history, practice, and future of physics. That is why electrodynamics has a permanent place in the physics curriculum, along with classical mechanics, quantum mechanics, and statistical mechanics. Of these four, students often find electrodynamics the most challenging. One reason is surely the mathematical demands of vector calculus and partial differential equations. Another stumbling block is the non-algorithmic nature of electromagnetic problem-solving. There are many entry points to a typical electromagnetism problem, but it is rarely obvious which lead to a quick solution and which lead to frustrating complications. Finally, Freeman Dyson points to the "two-level" structure of the theory.[1] A first layer of linear equations relates the electric and magnetic fields to their sources and to each other. A second layer of equations for force, energy, and stress are quadratic in the fields. Our senses and measurements probe the second-layer quantities, which are determined only indirectly by the fundamental first-layer quantities.

Modern Electrodynamics is a resource for graduate-level readers interested to deepen their understanding of electromagnetism without minimizing the role of the mathematics. The book's size was dictated by two considerations: first, my aim that it serve both as a classroom text and as a reference volume; second, my struggle to apply the epigraph at the top of the page.[2] Physicists are a prickly and opinionated bunch, so it is not surprising that there is very little agreement about "everything a student must know" about electromagnetism at the graduate level. Beyond a very basic core (the main content of undergraduate texts), the topics which appear in graduate-level textbooks and (electronically) published lecture notes depend strongly on the research background of the writer and whether he or she is a theorist or an experimenter. Some instructors view the subject as a convenient setting to illustrate the methods of mathematical physics and/or computational physics. Others see it as an opportunity to introduce topics (optics, plasma physics, astrophysics, biophysics, etc.) into a curriculum which might otherwise not include them. Still others teach electromagnetism for the main purpose of introducing the methods of relativistic field theory to their students.

Given the many uses of this foundational course, *Modern Electrodynamics* purposely contains much more material than can be comfortably covered in a two-semester course. Presentations with quite

[1] F.J. Dyson, "Why is Maxwell's theory so hard to understand?", in *James Clerk Maxwell Commemorative Booklet* (The James Clerk Maxwell Foundation, Edinburgh, 1999). Available at www.clerkmaxwellfoundation.org/DysonFreemanArticle.pdf.

[2] From the preface to F.E. Low, *Classical Field Theory* (Wiley, New York, 1997), p. xi.

different emphases can be constructed by making different choices from among the many topics offered for discussion. All instructors will omit various sections and probably entire chapters. Consistent with this point of view, I do not offer a single, idiosyncratic "vision" of electromagnetism. Rather, I have aimed to present what seemed (to me) to be the pedagogically soundest approach for students coming to this material after a first serious exposure at the junior/senior undergraduate level. In many cases, the same issue is examined from more than one point of view. The mathematics of the subject is given its proper due, but the qualitative and physical arguments I provide may ultimately remain with the reader longer.

The organization of this book reflects my personal experience as an instructor. After experimenting with relativity-first, Lagrangian-first, and radiation-first approaches, I concluded that the majority of students grasped the subject matter best when I used a traditional arrangement of topics. The text is purposely repetitive. This is done both to reinforce key ideas and to help readers who do not read the text in chapter order. My background as a condensed matter physicist appears in various places, including an emphasis on the practical (rather than the formal) aspects of microscopic averaging, a discussion of the limitations of the Lorentz model of dielectric and magnetic matter, and the presence of an entire chapter devoted to the experimentally important subject of quasistatics.

Every chapter of *Modern Electrodynamics* contains worked examples chosen either to develop problem-solving skills or to reveal subtleties of the subject which do not appear when one's exposure is limited to a few standard examples. Every chapter also contains several "applications" drawn from all the major subfields of physics. By and large, these are topics I was unwilling to relegate to the end-of-chapter homework for fear many readers would never see them. About half the chapters include a boxed excursion into an issue (often historical) where words serve better than equations, and every chapter ends with an annotated list of *Sources, References, & Additional Reading* to acknowledge my debt to others and to stimulate inquisitive readers. Finally, every chapter contains a large number of homework problems. These range from undergraduate-type drill problems to more challenging problems drawn directly from the research literature. Like most textbook authors, I emphasize that active engagement with the homework problems is an important part of the learning process. This is particularly important for electromagnetism where the struggle with difficult problems has somehow (wrongly) been elevated to a rite of passage. My desired outcome is a reader who, after completion of a course based on this book, can comfortably read and understand (if not necessarily reproduce in detail) a non-trivial electromagnetic argument or calculation which appears in the course of his or her research or reading.

The modernity of the text indicated by its title is *not* associated with the use of particularly "modern" mathematical methods. Rather, it derives from the inclusion of topics which have attracted new or renewed attention in recent decades. Examples include the electrostatics of ion channels, the modern theory of electric polarization, magnetic resonance imaging, the quantum Hall effect, optical tweezers, negative refraction, the time-domain approach to radiation, the polarization anisotropy of the cosmic microwave background, near-field optics, and relativistic heavy ion collisions. To keep the text finite, some familiar special topics from other texts have been omitted or barely touched upon. Examples include collisions and energy exchange between charged particles, the method of virtual quanta, transition radiation, energy loss in matter, and classical models of the electron. On the other hand, *Modern Electrodynamics* includes an overview of Dirac's Hamiltonian approach to electrodynamics and an update on certain "perpetual" problems like the correctness of the Lorentz-Dirac equation of motion for classical point particles and the Abraham-Minkowski controversy over the electromagnetic energy-momentum tensor in matter. All of these are illustrative of the self-refreshing nature of a subject which is re-invented by every new generation to meet its needs.

Finally, two choices I have made may give pause to some readers. One is my use of SI units throughout. The other is my use of the imaginary number i to impose the metric in special relativity. The technical rationale for using SI units is given in Section 2.6. An equally good reason is simply that

this system has become the worldwide standard and nearly all undergraduate textbooks use it without apology. However, because the physics literature is replete with books and research papers which use Gaussian units, Appendix B discusses this system and provides an algorithm to painlessly convert from SI to Gaussian and vice versa.

My reason for using the "old-fashioned" Minkowski metric is purely pedagogical and cannot be stated more clearly than Nobel prize winner Gerard 't Hooft did in the preface to his *Introduction to General Relativity* (2001), namely: "In special relativity, the i has considerable practical advantage: Lorentz transformations are orthogonal, and all inner products only come with + signs. No confusion over signs remains". Although he switches to a metric tensor to discuss general relativity (as he must), 't Hooft further champions his use of i in special relativity with the remark, "I see no reason to shield students against the phenomenon of changes in convention and notation. Such transitions are necessary whenever one switches from one research field to another. They better get used to it". That being said, Appendix D outlines the use of the metric tensor $g_{\mu\nu}$ in special relativity.

It would have been impossible for me to write this book without help. At the top of the list, I gratefully acknowledge the contributions of Wayne Saslow (Texas A&M University) and Glenn Smith (Georgia Tech). These colleagues read and commented on many chapters and were always willing to talk electromagnetism with me. My colleague Brian Kennedy (Georgia Tech) repeatedly put aside his research work and helped on the many occasions when I managed to confuse myself. Special thanks go to Michael Cohen (University of Pennsylvania), who tried to teach me this subject when I was a graduate student and who generously shared many of his insights and thoughtful homework problems. I regret that Mike never found the time to write his own book on this subject.

I am happy to acknowledge David Vanderbilt (Rutgers University) and Stephen Barnett (University of Strathclyde) who provided essential help for my discussions of dielectric polarization and radiation pressure, respectively. Olivier Darrigol (CNRS, Laboratoire SPHERE) answered my questions about the history of Lagrangians in electrodynamics and Andrew Scherbakov (Georgia Tech) gave assistance with the homework problems. I also thank the brave souls who provided feedback after testing some of this material in their classrooms: Michael Pustilnik, Roman Grigoriev, and Pablo Laguna (Georgia Tech), Peter McIntyre (Texas A&M University), Brian Tonner (University of Central Florida), Jiang Xiao (Fudan University), and Kapil Krishan (Jawaharlal Nehru University). Finally, I am delighted to thank my editor Simon Capelin and his first-rate staff at Cambridge University Press. Simon never failed to show enthusiasm for this project, now over more years than either of us would care to remember.

I dedicate this work to my wife Sonia and daughter Hannah. They suffered, but not in silence.