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978-0-521-89678-8 - Guided Explorations of the Mechanics of Solids and Structures: Strategies for Solving Unfamiliar Problems
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Excerpt
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Introduction

> Thus, for a complete investigation of dynamical systems, we require not only a computer and the direct integration methods. These provide no more than an ideal computer laboratory in which an arbitrary number of experiments can be performed, yielding an immense data flow. We require, in addition, certain principles according to which the data may be evaluated and displayed, thus giving an insight into the astonishing variety of responses of dynamical systems.
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> J. Argyris and H.-P. Mlejnek [3]

Suppose the existence of a very powerful computer, so powerful that it can execute any given command such as *build* a cantilever beam, *excite* the beam with this force history, *record* the velocity histories, and the like. Suppose, further, that it cannot answer questions such as *what* is elasticity? *why* is resonance relevant to vibrations? *how* are vibration and stiffness related? Here then is the interesting question: With the aid of this powerful computer, how long would it take a novice to discover the law governing the vibration of structures? The answer, it would seem, is never, unless the novice is a modern Galileo.

Now suppose we add a feature to the computer; namely, access to a vast bibliographic database (in the spirit of Google or Wikipedia) that can respond to such library search commands as *find* every reference to vibrations, *sort* the find according to the type of structure, *report* only those citations that combine experiment with analysis, and so on. With this database the process of discovery will be much more rapid.

How can an engineer with a powerful finite element (FE) program and modicum of background solve an unfamiliar problem? More specifically, how can this engineer construct a sequence of commands to the program that will eventually lead to a solution? The current generation of FE programs is like the preceding powerful computer but without the reference library – they can solve (almost) any problem posed to them, but they cannot pose the question itself. That is the role, the very difficult role, of the engineer. The role of the teacher is akin to the reference library, the teacher gives the engineer access to the ideas, insights, and methods of previous generations of engineers. Thus the ideal situation for a novice engineer would be to have a powerful computer, a huge library on-line, and a personal professor always ready to answer questions and suggest solutions. Of course, this is not realistic, which brings us to the purpose of this book. This book introduces a set of strategies and tools whereby a novice can leverage a given knowledge base to solve an unfamiliar problem. These strategies are elucidated within a computer simulation program called QED, which is also introduced as part of this book.

Computer Simulation and Accelerated Experiences

Consider experimental studies, for example, as a means to understanding an unfamiliar problem. An experiment by its nature is a single realization – a single geometry, material, or load case; multiple test cases and examples are just not economically feasible. The same can be said of classroom examples if the time devoted to them is to remain reasonable. But engineers, being introduced to something new, something unfamiliar, need to see other examples as well as variations on the given examples. For instance, in the stress analysis of a symmetrically notched specimen, these are some logical questions to ask:

- What if the notches are bigger or smaller?
- What if there is one instead of two notches?
- What if the notches are closer or farther apart?
- What if the material is changed?
- What if instead of a notch there is a crack?
- What if the clamped boundary has some elasticity?

These questions are too cumbersome and expensive to answer experimentally but are very appropriate for a computer simulation program.

As computer modeling became easier to use and faster to run, this opened up the possibility of understanding problems through parameter variation and visualizing the results. Whereas computer programs once sufficed to provide numbers – discrete solutions of "stress at a point" and the like – now complete simulations, which present global behavior, trends, and patterns, can be presented and sensitivity studies analyzing the relative importance of geometric and material parameters evaluated. Being able to observe phenomena and zoom in on significant parameters, making judgments concerning those that are significant and those that are not, greatly enhances the depth of understanding.

This understanding can come only through experiences, and here the simulation program can help considerably to accelerate the process of accumulating experiences.

Strategies for Solving Unfamiliar Problems

We take as given that the FE program can solve the problem in the sense that once it is properly posed the program can generate the requisite numbers. Our meaning of solution is not the generation of these numbers, but rather, understanding the formulation of the problem and understanding the significance of the generated numbers. But problems are not solved (in our sense) in a knowledge vacuum; the engineer must bring some background knowledge, understanding, and experience to bear on the problem. The objective here is to systematically leverage or migrate this background to the problem at hand. Furthermore, the FE program through the simulation program is to be part of this learning process.

Assume that a preliminary search on the Web has not yielded any useful hits; then six possible strategies for solving unfamiliar problems have been identified and are listed as follows.

I. Establish a basic knowledge base by duplicating a textbook example for the theory.
II. Perform a parameter sensitivity study.
III. Start with a known solution to a different (but somewhat related) problem and slowly morph the problem to the one of interest.
IV. Start with the given complex problem and remove features so as to identify those features that do and do not affect essential behaviors.
V. Identify separate features of the complex problem, make a model for each, and understand their behavior. Then construct a compound model that combines multiple features so as to understand their interaction.
VI. Construct a simple analytical model that contains the essential features of the problem.

The first strategy is the most basic in that there must be some knowledge base before the unfamiliar problem can be tackled. The power of duplicating a textbook example should not be underestimated because the very process of constructing the FE model often highlights aspects not apparent in the theoretical analysis. The second strategy is always available because it treats the problem as a black box; the objective is to identify the significant parameters from which a simpler reduced model can be constructed. Strategies III and IV are the converse of each other and share the idea of *morphing*, that is, slowly changing a given problem into another. Strategy V explores complexity in a problem. In a sense, the ability to construct a simple analytical model that has the essential features of a complex problem is testament to an understanding of the complex problem, and thus Strategy VI is often the capstone of an analysis.

A further word about *simple analytical models*. There was a time when simple models were the only available analytical tool to study complex problems. With the advent of computers, and especially the development of FE codes, almost any complex problem can now be tackled with as much sophistication as needed. However, when trying to understand a complex problem by using the preceding strategies, it becomes necessary to make many computer runs, and voluminous data can easily be generated that are quite difficult to assimilate. Ironically, this power and versatility of current FE programs have renewed the interest in constructing simple models – not as solutions per se but as organizational principles for seeing through the numbers produced by the codes. Another facet often overlooked is that simple models provide the language with which to describe things and therefore help the thinking process itself. That is, their solutions are more generic or archetypal and therefore more generally applicable.

Outline of the Book

QED is the simulation program written specifically to help implement the preceding strategies for solving problems in the mechanics of solids and structures. Chapter 1 gives an overview of QED; this is not a manual for its use, rather it tries to give an insight into the design and organization of the simulation environment and its connection to the mechanics formulation of problems. Chapter 2 considers some basic problems in the analysis of structures under static loads; the primary considerations are those of stiffness, stress, and equilibrium. Chapter 3 considers dynamic effects in the form of structural vibrations; this is complemented in Chapter 4 with the study of transient dynamic responses, in particular, wave-propagation responses. Chapter 5 introduces some aspects of the nonlinear behavior of structures; the examples discussed include large deflections, large strains, and nonlinear material behaviors. Chapter 6 refines the concept of equilibrium (both static and dynamic) while exploring stability of the equilibrium. The book concludes with Chapter 7, which discusses the construction of simple analytical models; in the process, this chapter also delves deeper into the basic concepts of mechanics introduced in Chapter 1.

The explorations are designed under the assumption that the explorer has little specific analysis background. Thus the first exploration in each chapter is introductory in nature and covers the basic concepts of the problem. As much as possible, these are made visual without overburdening analyses. The subsequent explorations in each chapter then delve deeper into the topics. Each exploration is given as much detail as possible to accomplish the data collection tasks without necessarily explaining why. Means of data analysis are given at the end of each exploration, often with a simple model for comparison. Partial results from the data analysis are presented for guidance – providing complete results would overly prejudice the sense of exploration and discovery.

The ostensible goal of the explorations is to introduce various topics in the mechanics of solids and structures; the deeper goal, however, is to develop the strategies for solving unfamiliar problems, and for this reason the range of topics is both broad and deep. That is, a narrow focus on topics (vibrations only, say) would not engender the sense of unfamiliarity that is sought; similarly, the inclusion of the concept of wave dispersion requires the use of important strategies and tools that would otherwise be overlooked.

The explorations collect data, observe trends, foster experiences, and so on, but they do not explain why. Chapter 7, with the development of simple analytical models, in part, tries to explain why, but obviously must be limited in scope. Each exploration is therefore accompanied by some pertinent further readings that can be used to extend the knowledge base and motivate variations on the explorations. Purposely, the QED explorations are like physical experiments, and so many of these readings refer to physical experimental studies.

1 QED the Computer Laboratory

Structures are to be found in various shapes and sizes with various purposes and uses. These range from the human-made structures of bridges carrying traffic, buildings housing offices, airplanes carrying passengers, all the way down to the biologically made structures of cells and proteins carrying genetic information. Figure 1.1 shows some examples of human-made structures. Structural mechanics is concerned with the behavior of structures under the action of applied loads – their deformations and internal loads.

The primary function of any structure is to support and transfer externally applied loads. It is the task of structural analysis to determine two main quantities arising as the structure performs its role: internal loads (called *stresses*) and changes of shape (called *deformations*). It is necessary to determine the first in order to know whether the structure is capable of withstanding the applied loads because all materials can withstand only a finite level of stress. The second must be determined to ensure that excessive displacements do not occur – a building, blowing in the wind like a tree, would be very uncomfortable indeed even if it supported the loads and did not collapse.

Modern structural analysis is highly computer oriented. This book takes advantage of that to present QED, which is a learning environment that is simple to use but rich in depth. The QED program is a visual simulation tool for analysis. Its intent is to provide an interactive simulation environment for understanding a variety of problems in solid and structural mechanics. This chapter gives an overview of QED and the underlying mechanics and programs. The subsequent chapters give detailed instructions on using QED to solve mechanics problems; here we concentrate on installing QED and providing a big picture overview of the program and its environment. To put all this in perspective, the first section begins with an overview of structural mechanics.

The design of QED is such that it isolates the user from having to cope with the full flexibility of the underlying enabling programs. Although it is not necessary to understand these programs in order to run QED, a working knowledge of them is helpful in explaining some of the choices available in QED, and therefore a section is devoted to summarizing the capabilities of these programs.

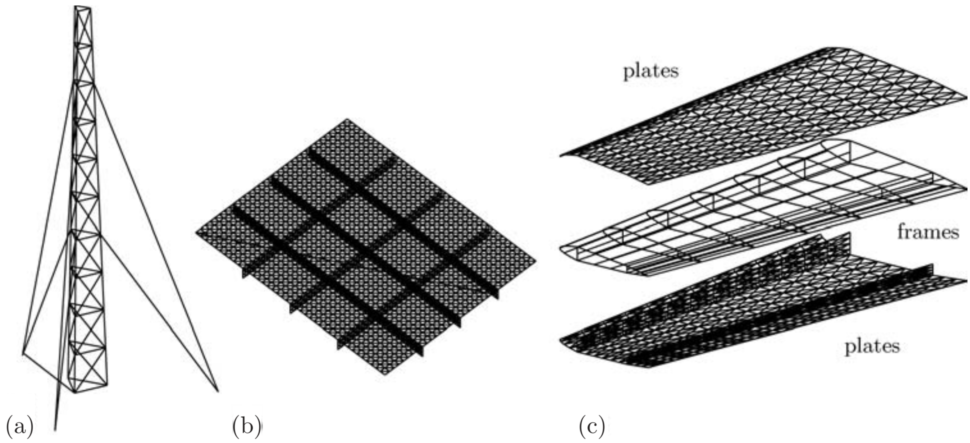

Figure 1.1. Some types of structures: (a) skeletal space truss with cable stays, (b) reinforced shell panel, (c) exploded view of an aircraft wing section.

The explorations in the following chapters utilize a variety of geometries, a variety of analyses, embedded in a variety of learning strategies. This chapter ends with two charts presenting all the explorations arranged according to these aspects.

1.1 Brief Overview of the Mechanics of Structures

This section briefly introduces the primary concepts in the mechanics of solids and structures. The intention is to identify and highlight the main components necessary in the solution of a general structures problem.

Types of Structures

Structures that can be satisfactorily idealized as line members are called *frame* or *skeletal* structures. An example is shown in Figure 1.1(a). Usually the members are assumed to be connected either by frictionless pins or by rigid joints. The members of a frame can support bending (in any direction) as well as axial and torsional loads.

Folded-plate and *shell* structures look very different from their frame counterparts because they are made of two-dimensional (2D) distributed material. For example, a *plate* is an extended continuous body in which one dimension is substantially smaller than the other two and is designed to carry both moments and transverse loads. Figure 1.1(b) shows an example of a plated structure. Shells also support in-plane loads, an action referred to as the *membrane* action.

Real structures are usually constructed as combinations of frame and shell members. Figure 1.1(c) shows an example of an aircraft wing comprising shell and frame members.

When a structural component does not fit into any of the previous special categories, we refer to it as a *solid*. All structural models are approximations of the

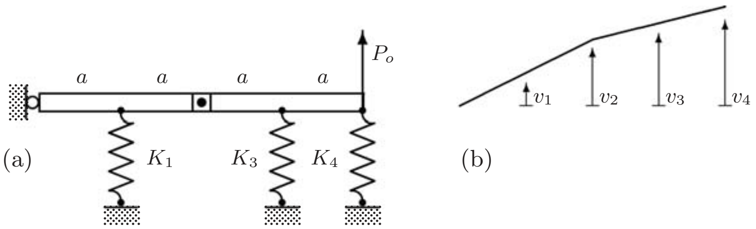

Figure 1.2. Simple structural model with external applied load P_o: (a) two linked rigid bars attached to springs, (b) compatibility of displacements.

three-dimensional (3D) solid; one of the preoccupations in this book is the assessment of the validity of the simpler structural models.

Structural Analyses

There are three fundamental concepts in the mechanics of solids: *deformation and compatibility*, *stress and equilibrium*, and *material behavior*. Each of these is elaborated on here and then made explicit through analyzing the relatively simple structure of Figure 1.2. This structure consists of horizontal rigid bars pinned at the left end and the center and attached to vertical springs.

Even this relatively simple structure has many unknowns; for example, there are the indicated displacements and the forces in the springs and attachment points. Additional steps are therefore required for effecting a solution, and these too are demonstrated.

I. Deformation and Compatibility

Loaded structures undergo deformations (or small changes in shape) and, as a consequence, material points within the structure are displaced to new positions. The principle of *compatibility* is assumed to apply to all of the structures encountered in this book. That is, it is assumed that, if a joint of a structure moves, then the ends of the members connected to that joint move by the same amount, consistent with the nature of the connection. For the pin-jointed members of Figure 1.2, compatibility means that the ends of the members meeting at a joint undergo equal translation. For the rigid bar, it means that all points along the bar have the same rotation.

There are four indicated displacements in Figure 1.2(b); these are referred to as *degrees of freedom* or DoFs. We distinguish between *unconstrained* (i.e., unknown) and *constrained* (i.e., known in some way) DoFs. In the present case, the bars are rigid and therefore impose the constraint that the allowable displacements along each bar must form a straight line. Because the rigid bars are articulated, then only two independent displacements are required for describing the deflections. That is, the four indicated displacements are related to each other by

$$v_1 = \frac{a}{2a} v_2 = \tfrac{1}{2} v_2 \,, \qquad v_3 = v_2 + \frac{a}{2a}[v_4 - v_2] = \tfrac{1}{2} v_2 + \tfrac{1}{2} v_4. \tag{1.1}$$

Figure 1.3. Free bodies of the simple model: (a) left bar, (b) right bar.

We take v_2 and v_4 as the two (unconstrained) DoFs and write the other two as functions of these.

Perhaps needless to say, an analysis of the deformation alone does not provide a solution to a mechanics problem when loads are applied.

II. Stress and Equilibrium

Loads can be regarded as being either *internal* or *external*. External loads consist of applied forces and moments and the corresponding reactions. The applied loads usually have preset values, whereas the reactions assume values that will maintain the equilibrium of the structure. Internal forces are the forces generated within the structure in response to the applied loads. These internal forces give rise to internal stresses.

If the spring K_3 in Figure 1.2 is detached from the bar, then its effect is replaced with that of a force resisting the deflection. If the second bar is removed from the springs and the joint, then the effect of these too is replaced with forces, as shown in Figure 1.3(b). This is called a *free body* or a *free-body diagram*.

The free-body diagrams of the separated bars are shown in Figure 1.3. For each free body, the (vector) sum of the applied loads (both forces $\hat{F}$ and moments $\hat{M}$) must be zero. That is,

$$\sum \hat{F} = 0\,, \qquad \sum \hat{M} + \sum \hat{r} \times \hat{F} = 0.$$

Note that the assumed initial direction of the reactions is not important; however, at separated joints they must be equal and opposite, as shown in the figure for reaction R_2. The forces F_i are due to the deformation of the springs, and their sense is made consistent with the displacements v_i. Summing the forces vertically and the moments about the left end gives, respectively,

$$\begin{aligned} R_0 - F_1 + R_2 &= 0, \qquad -F_1 a - R_2 2a = 0, \\ R_2 - F_3 - F_4 + P_o &= 0, \qquad -F_3 a - F_4 2a + P_o 2a = 0. \end{aligned} \tag{1.2}$$

This problem is *statically indeterminate* because equilibrium alone is insufficient for a solution (four equations, five unknowns: R_0, F_1, R_2, F_3, F_4). Generally, it should be expected that structural problems are statically indeterminate, and compatibility and material relations must be incorporated to effect a solution.

III. Material Behavior

For illustrative purposes, we restrict ourselves to materials that are *linearly elastic*. By elastic we mean that the stress–strain (or force–deflection) curve is the same for both loading and unloading. The restriction to linear behavior allows us to use the very important concept of superposition.

For our simple structure of Figure 1.2, the only components exhibiting elasticity are the springs. Linear springs have a force–deflection relation described by

$$P = Ku,$$

where P is the applied load, u is the relative deflection of the ends, and K is the spring stiffness. The force in each spring is given by

$$F_1 = K_1 v_1 = \tfrac{1}{2} K_1 v_2\,, \qquad F_3 = K_3 v_3 = \tfrac{1}{2} K_3 v_2 + \tfrac{1}{2} K_3 v_4\,, \qquad F_4 = K_4 v_4. \quad (1.3)$$

These are additional equations but in terms of the already identified unknowns. Hence we are now in a position to solve the problem.

IV. Solution Procedure

A total of nine unknowns were previously identified: four displacements, three spring forces, and two reactions. It is possible to set up nine simultaneous equations for the nine unknowns; however, it is more judicious to first identify the minimum set of unknowns, and this is generally done in terms of the unconstrained DoFs of the structure. As previously discussed, these have been identified as the displacements v_2 and v_4. Thus it should be possible to establish a minimum set of $[2 \times 2]$ simultaneous equations.

Get from the second equilibrium equation that $R_2 = -\frac{1}{2} F_1$ and combine with the first equilibrium equation to get $R_0 = +\frac{1}{2} F_1$. Substitute these into the remaining equilibrium equations and replace F_i in terms of the displacements and spring stiffnesses to get

$$\begin{bmatrix} \frac{1}{4} K_1 + \frac{1}{2} K_3 & \frac{1}{2} K_3 + K_4 \\ \frac{1}{2} K_3 a & \frac{1}{2} K_3 a + 2K_4 a \end{bmatrix} \begin{Bmatrix} v_2 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} P_o \\ 2a P_o \end{Bmatrix}. \quad (1.4)$$

Observe that this is a stiffness relation analogous to that of a simple spring, but this generalized spring has two DoFs. Furthermore, observe that this spring stiffness is a combination of the individual spring stiffnesses plus the geometry.

This equation can now be solved. For example, suppose $K_1 = K_3 = K_4 = K$; then

$$K \begin{bmatrix} \frac{3}{4} & \frac{3}{2} \\ \frac{1}{2} a & \frac{5}{2} a \end{bmatrix} \begin{Bmatrix} v_2 \\ v_4 \end{Bmatrix} = \begin{Bmatrix} P_o \\ 2a P_o \end{Bmatrix} \qquad \Rightarrow \qquad v_2 = -\frac{4P_o}{9K}\,, \qquad v_4 = +\frac{8P_o}{9K}.$$

This solution indicates that the second bar rotates clockwise, forcing the first bar to deflect downward.

V. Postprocessing

The solution procedure solved for just the minimum set of unknowns, but of course other quantities such as the spring forces and reactions may also be of interest. In the

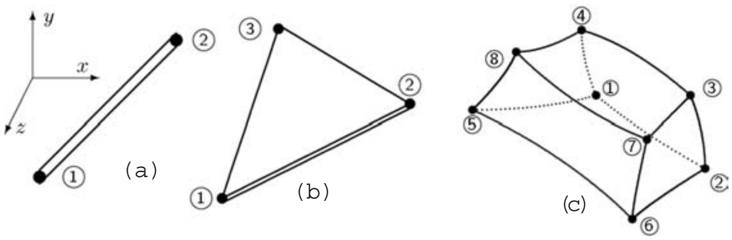

Figure 1.4. Some finite elements: (a) frame, (b) shell, (c) solid. The numbers in circles are the node numbers.

general structures problem, the stresses, strains, and so on may also be of interest. All of these are obtained as part of the postprocessing of the displacements.

It is clear, for example, how the other displacements are obtained from Equations (1.1) and the spring forces from Equations (1.3).

General Structural Analyses

Although the previous example is relatively simple, it nonetheless identified the major components of a structural analysis. The key stage is that of Equations (1.4), which is the minimum set of simultaneous equations to determine the unknowns of the problem.

We write this equation as

$$[K]\{u\} = \{P\},$$

where $[K]$ is the $[N \times N]$ *structural stiffness matrix*, $\{u\}$ is the $[N \times 1]$ *vector of degrees of freedom* used to describe the deformation of the structure, $\{P\}$ is the $[N \times 1]$ *vector of applied loads*, and N is the minimum number of DoFs. All (linear static) structural problems can be represented by this equation; the complexity of the problem in terms of geometry, materials, loads, and so on is reflected in the size of N.

A complex structure is conceived as a collection of components. In the finite element (FE) method, the smallest component is the element – Figure 1.4 shows three such elements. The process of assemblage to form $[K]$ is essentially the summing of the stiffnesses of the individual elements. Almost any general structure can be modeled conveniently and efficiently by a combination of the indicated elements.

Before we discuss these elements in a little more depth, a word about how the deformation of a general structure, modeled with FEs, is described in terms of its DoFs. The number of possible displacement components at each node is known as the *nodal DoF*; the nodal DoFs for different structural types are shown in Table 1.1. From this table, it is clear how the frame and shell structures share common types of DoFs. This is what allows them to be combined to form complex structures such as thin-walled shell structures reinforced with frame members.