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978-0-521-89573-6 - Separation of Molecules, Macromolecules and Particles: Principles, Phenomena and Processes

Kamalesh K. Sirkar

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## Separation of Molecules, Macromolecules and Particles

Providing chemical engineering undergraduate and graduate students with a basic understanding of how the separation of a mixture of molecules, macromolecules or particles is achieved, this textbook is a comprehensive introduction to the engineering science of separation.

- Students learn how to apply their knowledge to determine the separation achieved in a given device or process.
- Real-world examples are taken from biotechnology, chemical, food, petrochemical, pharmaceutical and pollution control industries.
- Worked examples, elementary separator designs and chapter-end problems give students a practical understanding of separation.

The textbook systematically develops different separation processes by considering the forces causing the separation, and how this separation is influenced by the patterns of bulk flow in the separation device. Readers will be able to take this knowledge and apply it to their own future studies and research in separation and purification.

**Kamalesh K. Sirkar** is a Distinguished Professor of Chemical Engineering and the Foundation Professor of Membrane Separations at New Jersey Institute of Technology (NJIT). His research areas are membranes and novel membrane based processes.

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“The first comprehensive book that takes the fundamentals of separation on a molecular level as the starting point! The benefit of this approach is that it gives you a thorough insight in the mechanisms of separation, regardless of which separation is considered. This makes it remarkably easy to understand any separation process, and not only the classical ones. This textbook finally brings the walls down that divide separation processes in classical and non-classical.”

*Bart Van der Bruggen, University of Leuven, Belgium*

“This strong text organizes separation processes as batch vs continuous and as staged vs differential. It sensibly includes coupled separation and chemical reaction. Supported by strong examples and problems, this non-conventional organization reinforces the more conventional picture of unit operations.”

*Ed Cussler, University of Minnesota*

“This book fills the need by providing a very comprehensive approach to separation phenomena for both traditional and emerging fields. It is effectively organized and presents separations in a unique manner. This book presents the principles of a wide spectrum of separations from classical distillation to modern field-induced methods in a unifying way. This is an excellent book for academic use and as a professional resource.”

*C. Stewart Slater, Rowan University*

“This book is an excellent resource for the topic of chemical separations. The text starts by using examples to clarify concepts. Then throughout the text, examples from many different technology areas and separation approaches are given. The book is framed around various fundamental approaches to chemical separations. This allows one to use this knowledge for both current and future needs.”

*Richard D. Noble, University of Colorado*

“This book provides a unique and in depth coverage of separation processes. It is an essential reference for the practicing engineer. Unlike more conventional textbooks that focus on rate and equilibrium based separations, Prof Sirkar focuses on how a given separation takes place and how this is used in practical separation devices. Thus the book is not limited by application e.g. chemical or petrochemical separations.

“As chemical engineering becomes increasingly multi-disciplinary, where the basic principles of separations are applied to new frontier areas, the book will become an essential guide for practitioners as well as students.

“The unique layout of the text book allows the instructor to tailor the content covered to a particular course. Undergraduate courses will benefit from the comprehensive and systematic coverage of the basics of separation processes. Whether the focus of a graduate course is traditional chemical separations, bioseparations, or separation processes for production of renewable resources the book is an essential text.”

*Ranil Wickramasinghe, University of Arkansas*

“This advanced textbook provides students and professionals with a unique and thought-provoking approach to learning separation principles and processes. Prof. Sirkar has leveraged his years of experience as a separation scientist and membrane separation specialist, to provide the reader with a clearly written textbook full of multiple examples pulled from all applications of separations, including contemporary bioseparations. Compared to other separations textbooks, Prof. Sirkar’s textbook is holistically different in its approach to teaching separations, yet provides the reader with a rich learning experience. Chemical engineering students and practicing professionals will find much to learn by reading this textbook.”

*Daniel Lepek, The Cooper Union*

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# Separation of Molecules, Macromolecules and Particles

Principles, Phenomena and Processes

Kamlesh K. Sirkar

New Jersey Institute of Technology



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# Contents

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<i>Preface</i>	<i>page xi</i>
<i>List of notation</i>	<i>xiii</i>
<b>Introduction to the book</b>	<b>1</b>
Introduction to chapters	2
Linked footprints of a separation process/technique	3
Classification of separation processes	6
Additional comments on using the book	7
Textbooks, handbooks and major references on separation processes	7
<b>1 Description of separation in a closed system</b>	<b>19</b>
1.1 Binary separation between two regions in a closed vessel	19
1.2 Multicomponent separation between two regions in a closed vessel	22
1.3 Definitions of composition for a binary system in a closed vessel	24
1.4 Indices of separation for binary systems	25
1.5 Comparison of indices of separation for a closed system	29
1.6 Indices for separation of multicomponent systems between two regions	33
1.7 Some specialized nomenclature	35
Problems	35
<b>2 Description of separation in open separators</b>	<b>39</b>
2.1 Preliminary quantitative considerations	39
2.2 Binary separation in a single-entry separator with or without recycle	42
2.2.1 Examples of separation in single-entry separators	45
2.2.2 Single-entry separator with a product recycle	48
2.2.3 Separative power and value function	49
2.3 Binary separation in a double-entry separator	50
2.3.1 Examples of separation in double-entry separators	51
2.3.2 Separation in a double-entry separator with recycle/reflux	53
2.4 Multicomponent systems	53
2.4.1 Size-distributed particle population	54
2.4.2 Continuous chemical mixtures	62
2.4.3 Multicomponent chemical mixtures	64
2.5 Separation in an output stream with time-varying concentration	66
Problems	70
<b>3 Physicochemical basis for separation</b>	<b>76</b>
3.1 Displacements, driving forces, velocities and fluxes	76
3.1.1 Nature of displacements	76
3.1.2 Forces on particles and molecules	77
3.1.3 Particle velocity, molecular migration velocity and chemical species flux	88
3.1.4 Integrated flux expressions for molecular diffusion and convection: single-phase systems	102

3.1.5	Flux expressions in multicomponent systems	112
3.1.6	Additional topics	117
3.2	Separation development and multicomponent separation capability	118
3.2.1	Separation development in a closed system	119
3.2.2	Multicomponent separation capability	124
3.2.3	Particulate systems	127
3.3	Criteria for equilibrium separation in a closed separator	128
3.3.1	Phase equilibrium with equal pressure in all phases	129
3.3.2	Phase equilibrium where different phases have different pressures	130
3.3.3	Single-phase equilibrium in an external force field	132
3.3.4	Equilibrium between phases with electrical charges	132
3.3.5	Equilibrium between bulk and interfacial phases	133
3.3.6	Curved interfaces	136
3.3.7	Solute distribution between phases at equilibrium: some examples	137
3.3.8	Particle distribution between two immiscible phases	159
3.4	Interphase transport: flux expressions	159
3.4.1	Interphase transport in two-phase systems	160
3.4.2	Interphase transport: membranes	170
3.4.3	Interphase transport in two-phase systems with phase barrier membranes	189
	Problems	192
	Appendix: Diffusion coefficients in different systems	202
<b>4</b>	<b>Separation in a closed vessel</b>	<b>205</b>
4.1	Equilibrium separation between two phases or two regions in a closed vessel	205
4.1.1	Gas-liquid systems	206
4.1.2	Vapor-liquid systems	208
4.1.3	Liquid-liquid systems	217
4.1.4	Liquid-solid systems	222
4.1.5	Interfacial adsorption systems	223
4.1.6	Liquid-ion exchanger systems	228
4.1.7	Supercritical fluid-bulk solid/liquid phase	231
4.1.8	Bulk fluid phase - mesophase systems	231
4.1.9	Partitioning between a bulk fluid phase and an individual molecule/macromolecule or a collection of molecules for noncovalent solute binding	234
4.1.10	Gas-solid particle-liquid system in mineral flotation	245
4.2	Equilibrium separation in a single phase in an external force field	245
4.2.1	Centrifugal force field	246
4.2.2	Electrical force field	253
4.2.3	Gravitational force field	256
4.2.4	Particle separation with acoustic forces	260
4.2.5	Externally imposed temperature gradient: thermal diffusion	261
4.3	Equilibrium separation between two regions in a closed vessel separated by a membrane	262
4.3.1	Separation by dialysis using neutral membranes	262
4.3.2	Separation between two counterions in two solutions separated by an ion exchange membrane: Donnan dialysis	264
4.3.3	Separation of a gas mixture by gas permeation	266
4.3.4	Separation of a pressurized liquid solution through a membrane	272
	Problems	273
<b>5</b>	<b>Effect of chemical reactions on separation</b>	<b>280</b>
5.1	Extent of separation in a closed vessel with a chemical reaction	280
5.2	Change in separation equilibria due to chemical reactions	281
5.2.1	Gas-liquid and vapor-liquid equilibria	281
5.2.2	Liquid-liquid equilibrium	289

<b>Contents</b>	<b>vii</b>
5.2.3 Stationary-mobile phase equilibria	299
5.2.4 Crystallization and precipitation equilibrium	303
5.2.5 Surface adsorption equilibrium	306
5.2.6 Complexation in Donnan dialysis	308
5.2.7 Enzymatic separation of isomers	308
5.3 Rate-controlled equilibrium separation processes: role of chemical reactions	309
5.3.1 Absorption of a gas in a reactive liquid	309
5.3.2 Solvent extraction of a species with chemical reaction	315
5.4 Rate-governed membrane processes: role of chemical reactions	318
5.4.1 Reverse osmosis: solute ionization	318
5.4.2 Ultrafiltration: complexation	319
5.4.3 Dialysis: reaction in dialysate	323
5.4.4 Chemical reactions in liquid membrane permeation-separation	324
5.4.5 Separation through solid nonporous membrane	337
Problems	338
<b>6 Open separators: bulk flow parallel to force and continuous stirred tank separators</b>	<b>346</b>
6.1 Sources and nature of bulk flow	348
6.1.1 Hydrostatic pressure induced bulk flow	348
6.1.2 Gravity induced bulk flow	350
6.1.3 Free convection	350
6.1.4 Bulk motion due to capillarity	352
6.1.5 Electroosmotic flow	353
6.1.6 Centrifugal force driven flow	354
6.1.7 Surface tension gradient based flow	354
6.1.8 Drag flow	355
6.1.9 Feed introduction mode vs. bulk flow	356
6.2 Equations of change	358
6.2.1 Equations of change for species concentration in a mixture	358
6.2.2 Equation of motion of a particle in a fluid: trajectory equation	368
6.2.3 General equation of change for a particle population	368
6.3 Bulk flow parallel to force direction	372
6.3.1 External forces	373
6.3.2 Chemical potential gradient driven phase-equilibrium systems	390
6.3.3 Filtration and membrane separation processes	412
6.4 Continuous stirred tank separators	445
6.4.1 Well-mixed separators - CSTS and batch separators	445
6.4.2 Well-mixed separators - membrane based devices	469
Problems	475
<b>7 Separation in bulk flow of feed-containing phase perpendicular to the direction of the force</b>	<b>485</b>
7.1 Chemical potential gradient based force in phase equilibrium: fixed-bed processes	487
7.1.1 Fixed-bed adsorption/desorption processes	487
7.1.2 Pressure-swing adsorption process for gas separation	511
7.1.3 Potential-swing adsorption	519
7.1.4 Parametric pumping	520
7.1.5 Chromatographic processes	527
7.1.6 Expanded bed adsorption (EBA) from a broth/lysate aided by gravitational force	550
7.1.7 Counteracting chromatographic electrophoresis and electrochromatography	551
7.2 Crossflow membrane separations, crossflow filtration and granular filtration	555
7.2.1 Crossflow membrane separations, crossflow filtration	555
7.2.2 Granular filtration of hydrosols (and aerosols)	586
7.3 External force field based separation: bulk flow perpendicular to force	596
7.3.1 Electrical force field	596

viii	Contents
7.3.2 Centrifugal force field	618
7.3.3 Gravitational force field	634
7.3.4 Field-flow fractionation for colloids, macromolecules and particles	640
7.3.5 Magnetic force field	648
7.3.6 Radiation pressure – optical force	654
Problems	655
<b>8 Bulk flow of two phases/regions perpendicular to the direction(s) of the force(s)</b>	<b>670</b>
8.1 Countercurrent bulk flow of two phases or regions perpendicular to the direction(s) of the force(s) driving species	670
8.1.1 Development of separation in countercurrent flow systems	670
8.1.2 Gas (vapor) absorption/stripping	683
8.1.3 Distillation	709
8.1.4 Countercurrent solvent extraction	736
8.1.5 Countercurrent melt crystallization in a column	751
8.1.6 Countercurrent adsorption and simulated moving bed system	754
8.1.7 Membrane processes of dialysis and electro dialysis	761
8.1.8 Countercurrent liquid membrane separation	767
8.1.9 Countercurrent gas permeation	771
8.1.10 Countercurrent gas centrifuge	775
8.1.11 Thermal diffusion and mass diffusion	781
8.2 Cocurrent bulk motion of two phases or regions perpendicular to the direction(s) of the force(s) driving species/particles	782
8.2.1 Cocurrent two-phase flow devices – general considerations	783
8.2.2 Chromatographic separations in cocurrent two-phase flow devices	785
8.2.3 Particle separation in cocurrent gas–liquid flow–Venturi scrubber	788
8.2.4 Cocurrent membrane separators	789
8.3 Crossflow of two bulk phases moving perpendicular to the direction(s) of the driving force(s)	794
8.3.1 Continuous chromatographic separation	794
8.3.2 Crossflow plate in a distillation column	799
Problems	804
<b>9 Cascades</b>	<b>812</b>
9.1 Types of cascades	812
9.1.1 Countercurrent cascades, ideal cascade	812
9.1.2 Other cascade configurations	818
9.2 Cascades for multicomponent mixture separation via distillation	822
9.3 Cascades for multicomponent mixture separation involving other separation processes	824
Problems	824
<b>10 Energy required for separation</b>	<b>827</b>
10.1 Minimum energy required for separation	827
10.1.1 Evaporation of water from sea water	828
10.1.2 Recovery of water by reverse osmosis	828
10.1.3 Net work consumption	830
10.1.4 Minimum energy required for membrane gas permeation, distillation, extraction, and other separation processes	831
10.2 Reducing energy required for separation	836
10.2.1 Evaporation of water for desalination	836
10.2.2 Distillation	839
10.2.3 Free energy of mixing	843
10.2.4 Dilute solutions	844
Problems	845

Cambridge University Press

978-0-521-89573-6 - Separation of Molecules, Macromolecules and Particles: Principles, Phenomena and Processes

Kamalesh K. Sirkar

Frontmatter

[More information](#)

<b>Contents</b>	<b>ix</b>
<b>11 Common separation sequences</b>	<b>847</b>
11.1 Bioseparations	847
11.2 Separation sequences for water treatment	851
11.2.1 Sea-water and brackish-water desalination	851
11.2.2 Ultrapure water production	852
11.2.3 Pharmaceutical grade water	853
11.3 Chemical and petrochemical industries	853
11.4 Hydrometallurgical processes	855
<i>Postface</i>	856
<i>Appendix A Units, various constants and equivalent values of various quantities in different units</i>	857
<i>Appendix B Constants</i>	858
<i>Appendix C Various quantities expressed in different units</i>	859
<i>References</i>	861
<i>Index</i>	877

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[More information](#)

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## Preface

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This is an introductory textbook for studying separation. Primarily, this book covers the separation of mixtures of molecules; in addition, it provides a significant treatment of particle separation methods. Separation of macromolecules has also received some attention. The treatment and coverage of topics are suitable for chemical engineering students at undergraduate and graduate levels. There is enough material here to cover a variety of introductory courses on separation processes at different levels.

This book is focused on developing a basic understanding of how separation takes place, and of how the resulting separation phenomenon is utilized in a separation device. The role of various forces driving molecules or particles from a feed mixture into separate phases/fractions/regions is basic to such an approach to studying separation. The separation achieved is then amplified in an open separator via different patterns of bulk-phase velocities vis-à-vis the direction(s) of the force(s). The forces are generated by chemical potential gradient, electrical field, rotational motion, gravity, magnetic field, etc. The resulting separation is studied under three broad categories of separation processes.

Separation processes driven by a negative chemical potential gradient are generally multiphase systems and are treated under the broad category of phase equilibrium driven processes. External force driven processes populate the second category, and include those operating under an electrical field, rotational motion, magnetic field or gravity; thermal diffusion processes are also briefly included here. The third category of membrane based processes studied is driven generally by a negative chemical potential gradient; however, electrical force is also relevant for some processes. The treatment of any external force driven processes will cover both separation of molecules and particle separations.

These physical separation methods are often reinforced by chemical reactions, which are usually reversible. An elementary treatment of the role of chemical reactions in enhancing separation across a broad spectrum of phase equilibrium driven processes and membrane based processes has been included. The level of treatment in this book assumes familiarity with elementary principles of chemical engineering thermodynamics and traditional

undergraduate levels of knowledge of ordinary differential equations and elementary partial differential equations.

Specific aspects of a given separation process are studied in the chapter devoted to those aspects for all separation processes. To study a particular separation process in great detail, one therefore has to go to different chapters. The footprints of a given separation process are provided at the beginning of the book (Tables 1–7); there are quite a few tables to cover a variety of separation processes. The list of processes is large; however, it is far from being all inclusive. The introductory chapter, which provides additional details about various chapters, as well as about the book, is preceded by a notation section. All references appear at the end of the book.

The description of the extent of separation achieved in a closed vessel for a mixture of molecules is treated in Chapter 1. Chapter 2 illustrates how to describe the separation of molecules in open separators under steady and unsteady state operation; a description of separation for a size-distributed system of particles is also included. Chapter 3 introduces various forces developing species-specific velocities, fluxes and mass-transfer coefficients, and illustrates how the spatial variation of the potential of the force field can develop multicomponent separation ability. The criteria for chemical equilibrium are then specified for different types of multiphase separation systems, followed by an illustration of integrated flux expressions for two-phase and membrane based systems.

Chapter 4 develops the extent of separation achieved in a closed vessel to a variety of individual processes under each of the three broad categories of separation processes. Chapter 5 demonstrates how separation can be considerably enhanced by chemical reactions in phase equilibrium based and membrane based processes under both equilibrium- and rate-controlled conditions. For open separators having bulk flow in and out, including continuous stirred tank separators (CSTs), Chapter 6 provides first the equations of change for molecular species concentration in single-phase and two-phase systems, the trajectory equation for a particle in a fluid and the general equation of change for a particle population. Chapter 6 then treats individual separation processes under each of the three

broad categories of separation processes when the bulk flow is parallel to the direction of the force and in CSTS mode.

Chapter 7 follows this latter approach of treating individual separation processes under each of the three broad categories of separation processes when the bulk flow of feed-containing phase is perpendicular to the direction of the force. Chapter 8 follows the same approach when the bulk flows of two phases/regions in the separator are perpendicular to the direction(s) of the force(s). Chapter 9 briefly elaborates on cascades, which were already introduced in the countercurrent multistaged flow systems of Chapter 8. Chapter 10 introduces the energy required for a number of separation processes. Chapter 11 illustrates a few common separation sequences in a number of common industries involved in bioseparations, water treatment, chemical and petrochemical separations and hydro-metallurgy. Conversion factors between various systems of units are provided in an Appendix.

Virtually all separation processes taught to chemical engineering students in a variety of courses have been covered via the approach illustrated in Chapters 3, 4, 6, 7 and 8; in addition, many particle separation methods have been treated. The structural similarity in the separation method between apparently unrelated separation processes becomes quite clear. A few basic principles equip the students with the capability to understand a wide variety of separation processes and techniques, including emerging ones. To aid the student, there are 118 worked examples, 300 problems, 340 figures, 100 tables and 1011

references. A website will provide guidance for computer simulations for a few selected problems.

The introductory chapter provides references to articles and books which influenced the development of various aspects of this book. I have benefitted considerably from the comments on selected chapters of the book by reviewers, anonymous or otherwise. Comments by Professors C. Stewart Slater, of Rowan University, Steven Cramer, of Rensselaer Polytechnic Institute, and Ranil Wickramasinghe, of Colorado State University (now at University of Arkansas), were particularly useful.

Many doctoral students and postdoctoral fellows were of invaluable help during the long gestation period of this book, either in formulating solutions of the problems or in developing illustrative drawings. I want to mention in particular Amit Sengupta, Theoharris Papadopoulos, Xiao-Ping Dai, Meredith Feins, Dimitrios Zarkadas, Quixi Fan, Praveen Kosaraju, Fei He, Atsawin Thongsukmak, Sagar Roy, Dhananjay Singh, John Tang and John Chau. The first two students helped me when we were at Stevens Institute of Technology. Sarah Matthews of Cambridge University Press patiently provided manuscript preparation guidelines and encouraging comments during an ever-shifting timetable. Irene Pizzie did an extraordinary job as the copy editor. Brenda Arthur of New Jersey Institute of Technology tirelessly typed the draft of the whole manuscript over a considerable length of time, while carrying out many other duties.

I must also mention at the end my wife, Keka, without whose patience, help and understanding this book would never have been finished.

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# Notation

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Equation numbers identify where the symbols have been introduced or defined.

The following styles have been adopted.		$a_{sp}$	pore surface area per unit volume of the porous medium of porosity $\varepsilon$
<b>Bold</b>	vector quantity	$a_v, a_{vc}$	surface area of a particle per unit particle volume, value of $a_v$ in a cake (6.3.135j)
<b>Overlines</b>			
—	quantity averaged over time or a specific coordinate direction, multicomponent system, Laplace transformed dependent variable	$a_{\pm}$ $A; A_1, A_2, A_3$	mean electrolyte activity (3.3.119d) amplitude (7.3.18), pure-water permeability constant in reverse osmosis and diffusive ultrafiltration; constants in (7.1.90b), three surface areas in control volume of Figure 7.2.6(b)
— =	averaged quantity		cross-sectional area of a cone in centrifugal elutriation
^	quantity in a mixture, per unit mass of bulk phase	$A(r)$	cross-sectional area of duct
<b>Underlines</b>			
—	hypothetical binary system quantity (2.4.23), (2.4.24)	$A_c$	heat exchanger surface area; constant in equations for activity coefficients (4.1.34d); surface area of membrane; total membrane surface area; modified equilibrium constant (7.1.66); constant (7.1.72b)
~ , =	vector quantity, tensor quantity	$A_{hexi}; A_{ij}; A_{mi}; A_m^t; A_i(T); A_i^o$	constant in crystal growth rate equation (6.4.27)
<b>Brackets</b>			
$\langle v_{ij} \rangle$	average value of $v_{ij}$ over surface area $S_j$	$A_{1p}$	projected area of a particle (3.1.64); transport coefficient in solution-diffusion-imperfection model (3.4.60a)
$a$	ellipsoid semiaxis dimension; see also (3.1.10a); constant in relations (3.1.49), (4.3.7) and (4.3.43a); interfacial area per unit volume, defined by (7.2.191), (7.3.25)	$A_p$	cumulative crystal surface area distribution fraction (6.4.17)
$a_1; a_2, a_3, \dots$	constant in (3.3.105), (4.1.42b), (4.3.29) and (5.2.147); constants in (7.2.73), (7.3.50) and (7.3.139)	$A_p^+(x)$	total particle surface area per unit volume of total mixture (Example 2.4.2); total crystal surface area per unit liquid volume (6.4.16)
$a_1$	constant in (7.2.198a)	$A_T$	three forms of amino acid
$a_A$	stoichiometric coefficient for species A	$Am^+, Am^-, Am^{\pm}$	
$a_H$	Hamaker constant (3.1.16)	$b$	ellipsoid semiaxis dimension; proportionality constant in osmotic pressure relation (3.4.61b); half of channel gap; width of region of gas flow completely cleaned up by a fiber (6.3.42a); constant in crystal growth rate expression (6.4.35); parameter (7.2.18); liquid envelope radius (7.2.208)
$a_i; a_i(T); a_{ij}; a_{il}; \hat{a}_i; \hat{a}_m; a_{msl}$	activity of species $i$ ; equilibrium constant (7.1.63); atom fraction of $i$ th isotope of an element in region $j$ (1.3.6), value of $a_i$ in region $j$ and liquid phase, respectively; amplitude (7.1.72b); constant in (4.3.43c); membrane surface per unit channel length (7.2.70)		
$a_p$	surface area of a particle		
$a_{s1}, a_{s2}; a_{sw}$	activity of solvent $s$ in regions 1 and 2, respectively; activity of salt in water		

$b_1, b_2, b_3$	constants in relations (3.1.143f)	$C_{ij}^-, \bar{C}_i^-, C_i^-, C_i^*$	molar concentration of species $i$ ; an average of the molar concentration of species $i$ in the feed and the permeate (5.4.74), (6.3.158b); total molar concentration of species $i$ in the porous medium per unit volume (3.1.118b); nondimensional species $i$ concentration (5.3.35j), hypothetical gas-phase species $i$ concentration (8.1.47); initial bed concentration of solute $i$ in phases 1 and 2
$b_1$	constant in (3.3.105), (5.2.147)	$C_{ig}^*, C_{i1}^-, C_{i2}^-$	
$b_{if}, b_{ij}, b_{if}, b_{ie}, b_1^i$	membrane feed channel height (Figure 7.2.3(a)); constant in equilibrium relation (3.3.81); constant in (7.1.73), constant in Freundlich isotherm (3.3.112c); constant in Langmuir isotherm (3.3.112a); constant in equation (4.1.42a)		values of $C_{i1}, C_{i2}$ at $z = 0$
$b'_{i-k}, b'_k$	proportionality constant in (5.2.154), equilibrium constant in (5.2.155)	$C_{i10}, C_{i20}$	liquid-phase concentration of pure solute $i$ at spreading pressure $\pi$ and temperature $T$ providing the same surface phase concentration of $i$ as the mixture
$b_m, b'_m$	constants in (4.3.43a,c)	$C_i^o(\pi)$	
$\mathbf{B}$	magnetic induction vector (3.1.19)		
$B; B^o; B_i^o; B_p$	constant, density function of the birth rate of new particles (6.2.50g), value of $B$ as $r_p \rightarrow 0$ ; value of $B^o$ for crystal growth rate $g_{r,i}$ ; duct perimeter (7.3.41)	$C_{igel}$	molar species $i$ concentration in gel
$Bi$	Biot number (3.4.35)	$C_{ij}$	molar concentration of species $i$ in region $j$ or location $j$ or stream $j$ ; $j = b$ , bulk; $j = E$ , extract; $j = f$ , feed region; $j = g$ , gas phase; $j = k$ , $k$ th phase; $j = \ell$ , liquid; $j = \ell'$ , adhering liquid phase on crystal; $j = m$ , membrane; $j = o$ , organic; $j = p$ , permeate, product; $j = r$ , raffinate; $j = R$ , ion exchange resin phase, raffinate; $j = s$ , solution, solid phase or pore surface; $j = w$ , water, mol/liter
$B_{1p}, B_{2p}$	second virial coefficients for interaction between polymers 1 and 2 (4.1.34p)		total molar concentration of species $i$ in region $j$ including complexed or dissociated forms; value of $C_{ij}$ on $n$ th plate/stage
$c; c_C$	velocity of light; gap between plates at entrance (Figure 7.3.10), stoichiometric coefficient for species C		species $i$ concentrations in phase $j$ at locations 0 (or initial concentration), $\ell$ and $\delta$
$C$	clearance of a solute (8.1.390)		
$C_1, C_2; C_2(x)$	integration constants (3.3.10b); molar concentration of species 2 at location $x$	$C_{ij}^-, C_{ijn}$	
$C_{2f}$	molar concentration of species 2 in initial mixture, mol/liter		
$C^{(2)}$	defined in (6.3.75)	$C_{ij}^o, C_{ij}^\ell, C_{ij}^\delta$	
$C_A^o, C_A^\delta$	molar concentration of A at $z = 0$ , $\delta$ (3.1.124)		
$C_{Aib}, C_{Aei}$	molar concentration of A in phase bulk and phase interface and in liquid at gas-liquid interface (3.4.1b)	$C_{itb}, C_{ite}, C_{iti}$	molar concentrations of species $i$ in bulk solution, at the end of concentration process and at the beginning of concentration process (6.3.173)
$C_{Awb}, C_{Awib}, C'_{Awb}$	molar concentration of A in bulk water and in water at phase interface (3.4.45e); critical value of $C_{Awb}$ for maximum enhancement (5.3.53)	$C'_{ig}, C'_{igf}$	molar species $i$ concentration per unit gas phase volume in a pore
$C_{Bob}, C_{Boi}$	molar concentration of species B in organic-phase bulk and interface, respectively	$\bar{C}_{ik}, \bar{\bar{C}}_{ik}, \hat{C}_{ik}; \bar{C}_{it}$	intrinsic phase average, phase average and deviation in $C_{ik}$ for species $i$ concentration in phase $k$ (6.2.24a,b), (6.2.28); defined by (7.1.94)
$C_{ci}; C'_{Cb}; C_D$	slip (Cunningham) correction factor (3.1.215); critical value of bulk concentration of C for maximum enhancement (5.3.29b); particle drag coefficient (3.1.64), (6.3.4)	$C_{im}^d, C_{im}^H, C_{im}^p$	species $i$ concentration in membrane: for Henry's law and Langmuir species, respectively, in dual sorption model (3.3.81); membrane pore liquid
$C_{FC}$	molar concentration of fixed charges in ion exchange resin	$C_{imb}, C_{imoi}, C_{iob}$	molar concentrations of species $i$ at various locations in Figure 3.4.11
$C'_{H1}, C'_{H2}$	dual mode sorption constants for species 1 and 2 (3.3.82a,b)	$C_{iwb}, C_{iwb}$ $C_{prb}, C_{pw}$	molar protein concentrations in resin phase and aqueous solution

## Notation

xv

$C_{sf}$ , $C_{sm}$ , $C_{sp}$	molar solvent concentration in feed, membrane and permeate, respectively	$D_{im}$ ; $D_{imoi}$ ; $D_{ip}$	diffusion coefficient of $i$ in membrane (6.3.149); value of $D_{im}$ for $C_{im} = 0$ (3.4.67b), $D_i$ in a pore (3.4.89c), (6.3.145a); effective $D_i$ in the pores of a particle
$C_{ij}$ , $C_{ij}$ , $C_{vp}$	total molar concentration; $C_t$ in a mixture in region $j$ ; volume of particles per unit fluid volume (7.2.176)	$\bar{D}_{ik}$ , $\bar{D}_{im}$	multicomponent diffusion coefficient of species pair ( $i$ , $k$ ) and ( $i$ , $m$ ) in Maxwell–Stefan approach
$d$ ; $d_h$ ; $d_i$ ; $d_{lm}$	diameter of tube/pipe/vessel; hydraulic diameter (Table 3.1.8); effective diameter of a molecule of gas species $i$ (3.3.90a); logarithmic mean diameter (8.1.417)	$D_{iK}$ $D_{iM}$	Knudsen diffusion coefficient for species $i$ (3.1.115c) effective binary diffusivity of species $i$ in a mixture (3.1.184), (3.1.185)
$d_{ij}$ ; $d_{gr}$ ; $d_{imp}$	force-type term (3.1.178), (3.1.181); grain diameter; diameter of an impeller	$D_{is}$ ; $D_{is}^0$ ; $D_{is}^N$	binary diffusion coefficient for solute $i$ /solvent $s$ ; value of $D_{is}$ at infinite dilution; Nernst–Planck binary diffusion coefficient for species $i$ /solvent $s$
$d_{ion}$ ; $d_p$ ; $d_w$ ; $d_{p1}$ , $d_{p2}$	mean diameter of a molecular ion; mean diameter of a particle (6.1.4b); wire diameter (3.1.23); diameters of particles 1 and 2	$D_{12}$	binary diffusion coefficient for species 1 and 2
$d_{32}$	Sauter mean diameter of a drop or particle (6.4.88), (6.4.89)	$De$ ; $(De)_{mv}$	density function of particles which disappear (die) (6.2.50h); see (7.2.170a) and (7.2.172)
$D$	diffusion coefficient of species in countertransport through liquid membrane	$Df$ ; $DF$	decontamination factor (2.2.1c); dilution factor (6.4.106), (7.2.91c)
$D_A$ , $D_C$ ; $D_B$	diffusion coefficient of species A and C, respectively; dialysance in hemodialysis (8.1.389)	$e$	charge of an electron, $1.60210 \times 10^{-19}$ coulomb
$D_{eff}$	effective diffusion coefficient (5.4.64a)	$e_i$	constant in adsorption isotherm for solute $i$ (3.3.113d)
$D_{gr}$	crystal growth diffusivity (6.4.45)	$en_b, en_M, en_{MN}$ , $en_{Mp}, en_p$	molecular energy in the bulk, due to intramolecular interactions, due to intermolecular interactions, due to interaction between molecules and pores and total energy for molecules in the pore (3.3.89d)
$D_p$ ; $D_p(\phi)$	diffusion coefficient for particle (3.1.68), (6.2.52); shear-induced particle diffusivity (3.1.74), (7.2.126), (7.2.131a)	$E$ ; $E$ ; $E_c$ ; $E_{D_i}$ ; $E_e$	electrical force field; its magnitude (3.1.8), (6.1.22), (6.3.8f), extraction factor (8.1.281), stage efficiency (6.4.72); electrical field strength $E_c$ (7.3.32a); activation energy for diffusion of species $i$ in polymer (4.3.46b); extraction factor for extraction section (8.1.303)
$D_r$	desalination ratio (1.4.25), (2.2.1a)	$E_{ij}$ , $E_o$ ; $E_{oG}$ ; $E_s$ ; $E_y$ ; $E_{BRS}$ , $E_{GrS}$	enrichment of species $i$ by pervaporation (6.3.193b), particle collection efficiency (7.2.200b); overall column efficiency (8.1.195); point efficiency (8.3.13); extraction factor for the scrubbing section; electrical field strength in $y$ -direction (7.3.48); particle collection efficiency (7.2.219), (7.2.214)
$D_{i,eff}$ ; $D_{i,eff,r}$ $D_{i,eff,z}$ ; $D_{i,eff,1}$	effective diffusion coefficient of $i$ in liquid (6.2.18), (6.3.16b); value of $D_{i,eff}$ in $r$ - and $z$ -directions; value of $D_{i,eff}$ in phase/region 1	$E_{IS}$ ; $E_{IS}$	inertial impaction based single fiber capture efficiency (6.3.42a); particle capture efficiency by interception (7.2.224)
$D_{i,eff,k}$ ; $\underline{D}_{i,eff,k}$	effective diffusion coefficient of $i$ in phase $k$ (6.2.33); dispersion tensor (6.2.31)		
$D_{ij}$ ; $D_{il}$ ; $D_{is}$	diffusion coefficient of species $i$ in region $j$ ; $j = 1$ , liquid; $j = s$ , solvent		
$D_{A'}^T$ , $D_{B'}^T$ , $D_{is}^T$	thermal diffusion coefficient for species A and B (3.1.44); for species $i$ in solution		
$D_{AB}$ ; $D_{BR}$	binary diffusion coefficient for mixture of gases A and B; diffusion coefficient of particles due to Brownian motion (7.2.216)		
$D_{iD}$ , $D_{iH}$	diffusion coefficients in dual sorption-dual transport model (3.4.78)		
$D_{ie}$	effective diffusion coefficient of $i$ in a porous medium (3.1.112d)		

$E_{ME}, E_{MR}, E_{MV}, E_N$	Murphree extract stage efficiency, Murphree raffinate stage efficiency (6.4.70), (6.4.71); Murphree vapor separation efficiency (8.1.198); Newton particle separation efficiency (2.4.14a)	$F; \hat{F}, \hat{F}_p; F$	force on a particle; value of $F$ per unit particle mass; degrees of freedom (4.1.22)
$E_T; E_T^1; E_{T_i}$	total efficiency in solid–fluid separation (2.4.4a), overall filter efficiency (6.3.45), (7.2.201); reduced efficiency of Kelsall (2.4.16a); $E_T$ for ith solid–fluid separator (2.4.17c,d)	$F(r_p)$	probability distribution function corresponding to $f(r_p)$ (2.4.1c), crystal size distribution function (6.4.11)
$f$	friction factor (6.1.3a), fractional consumption of chemical adsorbent (5.2.19d)	$F_i; F_i^{\text{ext}}$	electrostatic force on 1 gmol of a charged species in solution (6.3.8a); magnitude of external force on 1 gmol of species $i$
$f_2$	fraction of the solute in ionized form ( $i = 2$ ) in RO (5.4.4)	$F_{\text{acrx}}$	acoustic radiation force in $x$ -direction (3.1.48)
$f(r); f(r_p); f(e)$	molar density function in a continuous/semi-continuous mixture with characteristic property $r$ ; particle size probability density function (2.4.1a), pore size distribution function in a membrane; defined by (7.2.222a,c)	$F_{\text{rad}}$	radiation pressure force (3.1.47), (7.3.267)
$f_f(r_p), f_i(r_p), f_2(r_p)$	value of $f(r_p)$ for feed stream, overflow and underflow based on particle weight fraction in a given size range (2.4.1b)	$F^{\text{BR}}$	force on very small particle due to random Brownian motion (3.1.43)
$f_N, f_i, f_i^0$	fugacity of species A, species $i$ ; standard state fugacity of species $i$	$F_i^{\text{ELK}}$	electrokinetic force on particle in double layer (3.1.17)
$f_g(\text{gr})$	probability density function of crystal growth rate (6.4.41a)	$F_i^{\text{ELS}}$	electrostatic force on particle $i$ , Coulomb's law (3.1.15)
$f_{ii}$	fugacity of pure species $i$ in liquid phase	$F_i^{\text{Lret}}$	London attraction force (3.1.16)
$f_{mv}, f_m$	quantities characteristic of a membrane polymer (4.3.46a,d)	$F_k^{\text{m}}$	force on species $k$ in mass flux $j_b$ , force relation (3.1.202)
$f_{ij}^0, f_{ig}^0, f_{il}^0$	standard state fugacity of species $i$ in region $j$ ; $j = g$ , gas phase; $j = l$ , liquid phase	$F_{\text{net}}^{\text{ext}}$	net external force; for gravity see (3.1.5)
$f_{i,p}^d, f_p^d$	frictional coefficient for species $i$ and spherical particle	$F_p^{\text{drag}}; F_p^{\text{iner}}; F_{pz}^{\text{ext}}$	frictional force on a particle; inertial force on a particle (6.2.45); external force on a particle in $z$ -direction
$\hat{f}_{ij}, \hat{f}_{ij}, \hat{f}_{ij}, \hat{f}_{ijpl}$	value of fugacity of $i$ in a mixture in gas phase, phase $j$ and liquid phase, respectively; value of $\hat{f}_{ij}$ for a planar surface	$F_{ti}, F_{ti}^{\text{ext}}$	total force and total external force on 1 gmol of species $i$ (3.1.50)
$f_{io}^d$	value of $f_{ij}^d$ for a sphere of equivalent volume (3.1.91e)	$F_{ip}^{\text{ext}}; F_{ipx}^{\text{ext}}; F_{ipy}^{\text{ext}}; F_{ipz}^{\text{ext}}$	total external force on a particle (3.1.59), (6.2.45); components of $F_{ip}^{\text{ext}}$ in $x$ -, $y$ - and $z$ -directions
$f_{im}^d, f_{sm}^d$	frictional coefficient for solute $i$ and solvent $s$ in a membrane	$F_{TA}$	force on 1 gmol of species A due to a temperature gradient (3.1.44)
$f_f(M), f_l(M), f_v(M); f_\infty, f_\lambda$	value of $f(r)$ , where $r = M$ , molecular weight, for the feed mixture, liquid fraction and the vapor fraction, respectively; defined by (7.1.59a), (7.2.187), respectively	$\mathcal{F}$	Faraday's constant, 96 485 coulomb/gm-equivalent
$f_{Mi}, f_{Qm}, f_{yo}$	fraction of the total metal ion concentration in the aqueous phase present as $M^{n+}$ (5.2.97); probability density functions (7.3.79), (7.3.80)	$g; g_c; g_m$	acceleration due to gravity; conversion factor; a quantity characteristic of a membrane polymer (4.3.46a)
		$g_z^{\text{ext}}; g_x^{\text{ext}}; g_y^{\text{ext}}; g_z^{\text{ext}}$	external body force per unit mass; its components in $x$ -, $y$ - and $z$ -directions
		$gr_i$	intrinsic growth rate of $i$ th crystal (6.4.41a)
		$G; G_g$	superficial mass average velocity based on empty flow cross section, $G$ for gas phase
		$G; G_a, G_b; G_i; G_o$	growth rate of crystal (6.4.25), (6.4.3b); value of $G$ under condition a, condition b; factor representing contribution of species $i$ properties to $Q_{im}$ (4.3.56a); constant

## Notation

xvii

$\overline{G}_c; \overline{G}_{Dr}; \overline{G}_{Du}$	convective hindrance factor (3.1.113); drag factor reducing solute diffusion by hindrance (3.1.112e); function of particle volume fraction in hindered settling (4.2.61)	$\mathbf{j}$ $j_x, j_y, j_z$	unit vector in positive y-direction mass flux vector of species $i$ , $M_i \mathbf{j}_i$ (3.1.98), Tables 3.1.3A, 3.1.3B, (6.2.5n); components of $\mathbf{j}_i$ in x-, y- and z-directions factor defined by (3.1.143g)
$\overline{G}_i; \overline{G}_{ij}$	partial molar Gibbs free energy of species $i$ , ratio of solute $i$ velocity to the averaged pore solvent velocity, convective hindrance factor ( $=\overline{G}_c$ ) (3.1.113), (3.4.89b); value of $\overline{G}_i$ in region $j$	$J_D$ $\mathbf{J}_i, J_i^*; (J_i)_k; J_1, J_1^T$	molar flux vector of species $i$ (3.1.98), (3.1.99), Tables 3.1.3A, 3.1.3B; value of $\mathbf{J}_i$ in region $k$ ; diffusive molar flux vector of species 1 (4.2.63); temperature gradient driven molar flux vector of species 1 (4.2.62)
$G_r(r_p)$ $G_{ij}^*; G_{crit}$	grade efficiency function (2.4.4b) total Gibbs free energy of all molecules in region $j$ (3.3.1), (4.2.23); defined by (7.1.58e)	$J_{iz}, J_{sz}; J_{iy}^*$ $J_{iz}^*, J_{iz}^{*'}, J_{sz}^*, J_{sz}^{*}'$	z-components of flux vectors $\mathbf{J}_i$ and $\mathbf{J}_s$ ; y-component of flux vector $\mathbf{J}_i^*$ z-components of flux vectors $\mathbf{J}_i^*, \mathbf{J}_j^*, \mathbf{J}_s^*$ and $\mathbf{J}_A^*$
$Gr; Gr(\beta_r, \sigma_v); Gz$	Grashof number (3.1.143e); function defined by (7.2.174); Graetz number (8.1.276)	$J_{Ay}^{*t}$ $J_{vz}$	total molar flux of species A in y-direction (5.4.51) volume flux through membrane in z-direction (3.4.60c), (6.3.155a)
$h; h_o$	membrane flow channel height, distance between particle and collector (3.1.17), constant in (4.1.9a) for Henry's law constant, $(1/h)$ is a characteristic thickness of double layer (6.3.31a), height of liquid in a capillary at any time $t$ ; value of $h$ as $t \rightarrow \infty$ (6.1.11)	$\mathbf{k}; k$ $k^B$ $k_{ad}, k_{d}$	unit vector in positive z-direction; region or phase, constant in (2.2.8a-c), $(2\pi/\lambda)$ (3.1.48) Boltzmann's constant (3.1.72), (3.3.90c) rate constants for adsorption and dissociation, respectively (4.1.77a)
$h_+, h_-, h_G$	contributions of different species to $h$ (4.1.9c)	$k_{br}, k_f$	backward and forward reaction rate constants (5.4.42)
$h_{min}$	minimum value of $h$	$k_{Ao}, k_{Aw}$	mass-transfer coefficient of species A in organic or water phase
$H; H_f; H_j$ $H_i^c, [H_i^c]'; H_i^p$ $H_{if}, H_{if}'$	plate height, stack height; molar enthalpy of feed; value of $H$ for species $i$ (6.3.22), Henry's law constants for species $i$ in gas-liquid equilibrium; (3.3.59) (4.1.7); (3.4.1b), (5.2.6); (5.2.7); (8.1.49), (3.4.1a); molar enthalpy of liquid fraction and vapor fraction, respectively, of the feed	$k_c, k_g, k_{xj}, k_y$ $k'_c, k'_g, k'_x, k'_y$ $k_d, k_s, k_\ell$	mass-transfer coefficients for species $i$ (3.1.139), (3.4.3) values of $k_c, k_g, k_x$ and $k_y$ for equimolar counterdiffusion (3.1.124) mass-transfer coefficients in crystallization (3.4.23a,b); liquid film mass transfer coefficient (5.3.3)
$H_{ot}, H_{otP}$	height of a transfer unit defined by (8.1.96)	$k_{igc}, k_{igx}, k_{igy}$	mass-transfer coefficients for species $i$ in gas phase when the concentration gradient is expressed in terms of $C$ , molar concentration of species $i$ in gas phase, $x$ , mole fraction of species $i$ in gas phase and similarly $y$ , mole fraction in gas phase, respectively
$\mathbf{H}^m$ $H_A, H_A^o, H_A^c$ $H_1, H_2$ $H_D, H_M, H_S, H_{SM}$ $\overline{H}_i$ HTU	magnetic field strength vector Henry's law constants for species A (3.4.1a,b), (3.4.8); defined by (7.1.20b) components of plate height (7.1.107e-i) partial molar enthalpy of species $i$ height of a transfer unit (6.4.85), (8.1.54b), (8.1.57b), (8.1.65e), (8.1.245b), (8.1.247a), (8.1.357b)	$k_T; k'_T$ $k_{gf}, k_{gs}$	thermal diffusion ratio (3.1.45); thermal diffusion constant (4.2.64) gas film mass-transfer coefficient on feed side and strip side of a liquid membrane (5.4.97a), (5.4.99a)
$\mathbf{i}$	unit vector in positive x-direction, current density (3.1.108c)	$k_{lf}, k_{ts}; k_{pp}$	liquid film mass-transfer coefficient on feed side and strip side of a liquid membrane (5.4.97b), (5.4.99b); particle mass-transfer coefficient (7.2.217b)
$I; \mathbf{I}; I_j; I(C_{isbL}^+)$	ionic strength of the solution (3.1.10c), (4.1.9b); purity index (1.4.3b), current; value of purity index for region $j$ (1.4.3b); integral (7.2.86)		

$k_{s^+}, k_s$	rate constants for forward and backward interfacial reactions (5.3.40)	$K_{ijc}; k_{ijc}; k'_{ijx}$	molar concentration based overall mass-transfer coefficient for phase $j$ (8.1.1c); $j$ phase mass-transfer coefficient (8.1.60); value of $k_{ijx}$ for equimolar counterdiffusion (8.1.62a)
$k'_{i1}; k''_{i1}$	distribution ratio of species $i$ between regions 1 and 2, also called capacity factor (1.4.1); distribution ratio defined by (2.2.19) for species $i$ between streams 1 and 2	$K_o, K_w$	overall mass-transfer coefficient based on organic or aqueous phase, ionization product for water (5.4.41c) values of $K$ for ion $i$ /protein (p)-salt (s) exchange on an ion exchange resin (7.1.109d), (3.3.122b)
$k_{cR}, k_{cE}$	mass-transfer coefficient in the continuous phase, raffinate based, extract based (6.4.97a,b)	$K_{is}, K_{ps}$	overall mass-transfer coefficient $K_x$ (3.4.5) based on extract phase and raffinate phase, respectively (6.4.77), (6.4.81)
$k_{gr}, k_{nu}$	rate constant for crystal growth and nucleation, respectively (6.4.51)	$K_{xE}, K_{xR}$	
$k_{ij}; k_{i13}, k_{i14}, k_{i1o}; k_{imo}$	mass-transfer coefficients for species $i$ in liquid phase; value of $k_{i1}$ for condition 3, condition 4, channel inlet; species $i$ mass-transfer coefficient through organic filled membrane pore	$l; \ell$	length of a device, length of molten zone in zone melting (6.3.109b), characteristic dimension of the separator; constant in (2.2.8a-c), length
$k_1$	first-order reaction rate constant (5.3.7)	$\ell_{ik}, \ell_{ki}$	phenomenological coefficients (3.1.203)
$k_{1m}$	membrane mass-transfer coefficient for species 1 (4.3.1)	$\ell_{loc}$	characteristic length of a local volume corresponding to a point in volume averaging Section 6.2.1.1
$K$	equilibrium constant for a chemical reaction (3.3.68), or an ion exchange process (3.3.121i), a constant (6.3.49)	$\ell_x, \ell_y, \ell_z$	dimensions of a rectangular separator, Figure 3.2.1
$K^x; K^\phi$	mole fraction based $K$ for a chemical reaction (5.2.35); defined by (5.4.100)	$L; L_f; L^+$	length of a separator, dimension of length, characteristic crystal size; molar feed flow rate; nondimensional $L$ (7.2.38)
$K_1, K_2, K_3$	constants in membrane transport (6.3.155a,b)	$L_{ii}, L_{is}, L_{ss}$	phenomenological coefficients for binary system ( $i, s$ ) (3.1.208), (3.1.209)
$K_{Ao}, K_{Aw}, K_{AB}$	overall mass-transfer coefficient of species A based on organic or water phase; equilibrium constant (7.1.42c)	$L_{ik}, L_{kb}, L_{iT}$	phenomenological coefficients (3.1.205)
$K_c, K_g, K_x, K_y$	overall mass-transfer coefficients (3.4.5), (3.4.6)	$L_p; L_p^a; L_p^b$	hydraulic transport parameter in Kedem-Katchalsky model (6.3.158a); value of $L_p$ in perfect region; value of $L_p$ in leaky region
$K_{cE}$	$K_c$ based on extract phase (6.4.80)	$L_T; L_{\min}; L_{MTZ}; LUB$	separator length ( $= L$ ); (7.1.60); Figure 7.1.5(b); (7.1.21g)
$K_C^A$	equilibrium constant for ion exchange reaction (5.2.122)	$m; m_B; m_i; m_1; m_i^0$	velocity profile constant (7.3.134); moles of B; moles of species $i$ in separator; moles of species 1 in separator; total number of moles of species $i$ in separator
$K_d$	equilibrium constant for protein-ion exchange resin binding (4.1.77c), ionization equilibrium constant (5.2.4)	$m_{ij}; m_{ij}^0; m_{ij}(n); m_{ij}^a$	moles of species $i$ in region $j$ , total number of moles of $i$ in region $j$ at $t = 0$ ; moles of $i$ in region $j$ after $n$ th contact; number of atoms of $i$ th isotope of the element in region $j$
$K_{d1}, K_{d2}$	dissociation constant for solutes 1 and 2, respectively (5.2.61a), (6.3.29)	$m_{i\sigma}$	moles of species $i$ in interfacial region $\sigma$
$K_i^a; K_1, K_2, K_3; K_i^a; K_i^\infty$	equilibrium ratio of species $i$ between regions 1 and 2 (1.4.1) or (3.3.61); value of $K_i$ for species 1, 2 and 3; values of $K_i$ in terms of activities (4.1.3); $f_{i1}^o/f_{i1}^s/P$ in dilute solution stripping (4.1.19b)		
$K_{i\ell}; K_\ell$	overall liquid-phase mass-transfer coefficient for species $i$ (7.1.5a); reaction equilibrium constant in the liquid phase based on molar concentrations (5.2.52a)		

## Notation

xix

$m_{bj}$	molality, moles of $i$ per kilogram of solvent in region $j$ : $j = R$ , resin; $j = w$ , aqueous phase	$n_{\max}$ ; $n_{\text{med}}$ , $n_{\text{par}}$	peak capacity (3.2.32) (6.3.26a); refractive index of medium and particle
$m_{F,R}$	molality of fixed charges in the resin phase	$n_p$ , $n_{pi}$ , $n_{py}$ ; $n_t$	particle number flux, (3.1.65), (3.1.66), (3.1.68); particle flux in $y$ -direction; number of turns by gas in a cyclone (7.3.146b)
$m_p$ ; $m_{sl}$	mass of particle; solvent moles in stationary liquid phase (7.1.104b)		
$m_j$	total moles of all species in region $j$ ( $j = f$ , feed; $j = 1$ , vapor phase; $j = 2$ , $\ell$ , liquid phase)	$N$ ; $N(r_p)$ ; $N(r_{\min}, r_p)$ ; $N(r_{p\max})$	total number of stages in a multistage device or in the enriching section of a cascade, anionic species in Donnan dialysis, a metal species, number concentration of molecules; numbers/cm <sup>3</sup> , number of particles per unit fluid volume in the size range of $r_{\min}$ to $r_p$ ; value of $N(r_{\min}, r_p)$ for $r_{p\max}$
$m_{11}(t)$ , $m_{21}(t)$	moles of species 1 and 2 in region 1 at time $t$		Avogadro's number ( $6.02 \times 10^{23}$ molecules/gmol); plate number for $i$ (6.3.27a)
$m'_{11}(t)$ , $m'_{21}(t)$	values of $m_{11}(t)$ and $m_{21}(t)$ in the case of a chemical reaction		
$m_o^{\text{mag}}$	magnetophoretic mobility (7.3.251)	$\tilde{N}$ ; $N_i$	molar flux of species A in a fixed reference frame without and with reaction; total molar flux of species A in facilitated transport or counter-transport or co-transport in $y$ -direction; normality of dilute solution (8.1.404)
$M$ ; $M_i$ , $M_s$ ; $M_{sl}$	molecular weight, a metal species, number of stages in stripping section of a cascade; value of $M$ for species $i$ , for solvent $s$ ; $M$ for coating liquid in stationary phase	$N_A$ ; $N_A^r$ , $N_{Ay}^t$ ; $N_{\text{dil}}$	species $i$ flux; $N_i$ through membrane, $N_j$ through membrane; components of $N_i$ in $x$ -, $y$ - and $z$ -directions; radial component of $N_i$ ; $N_i$ through surface area $S_j$ ; magnitude of $N_{iy}$
$M_t$	average molecular weight of solution (3.1.56)		$N_i$ ; $N_{im}$ , $N_{jm}$ ; $N_{ix}$ , $N_{iy}$ , $N_{iz}$ ; $N_{ij}$ ; $N_{ij}$ ; $ N_{iy} $
$M_w$	magnetization of wire (3.1.23)		$N_{iz}^p$
$M_{se}$	seed mass density per unit liquid volume (6.4.40a)		$N_{oj}$ , $N_{o\ell p}$ ; $N_p$
$M_T$ ; $M_{Ta}$ , $M_{Tb}$	suspension density of a crystal-containing solution (2.4.2f), (6.4.18); value of $M_T$ for cases $a$ and $b$		number of transfer units (8.1.92), (8.1.96); defined in (8.1.96); number of pores per unit area size $r_p$ (6.3.135d)
$Mo^{(n)}$ ; $Mo_f^{(i)}$ , $Mo_\ell^{(i)}$ , $Mo_v^{(i)}$	$n$ th moment of the density function (2.4.1g); $i$ th moment of molecular weight density function of feed, liquid and vapor, respectively (6.3.70)		flux ratio (3.1.129a); solvent flux number density of crystals having a size less than $r_p$ and growth rate $gr_i$ ; total number of crystals per unit volume having a growth rate of $gr_i$ (6.4.41c)
$Mo_{jr}^j$ , $Mo_{gr}^j$	$j$ th moment of crystal size density functions $f(r_p)$ and $f_g(gr)$		total number of particles per unit volume (6.4.10)
MWCO	molecular weight cut off of a membrane	$N_R$ ; $N_S$ ; $N_i(r_p)$ ; $N_{it}$	number of transfer units based on extract and raffinate phases, respectively (6.4.86a), (6.4.83)
$n$ ; $n_c$	number of species/components in a system, number of contacts, stage/plate number, number of positive charges in a metal ion, number of unit bed elements; number of collectors, number of channels (7.3.109)	$N_t$	number of particles of size $r_{pi}$ (2.4.2k)
$n(r_p)$ ; $n^o$ ; $\tilde{n}(r_p)$	population density function, particle number density function (2.4.2a); nucleation population density parameter (6.4.7); defined by (6.4.46b)	$N_{toE}$ , $N_{toR}$	number of transfer units (8.1.54c), (8.1.57c), (8.1.66b), (8.1.67d), (8.1.338)
$\mathbf{n}_i$ ; $n_{ix}$ , $n_{iy}$ , $n_{iz}$ ; $\mathbf{n}_{ij}$	mass flux vector of species $i$ , $M_i N_i$ ; its components in $x$ -, $y$ - and $z$ -directions; $\mathbf{n}_i$ through surface area $S_j$	$No_i$	number of transfer units (8.1.54c), (8.1.57c), (8.1.66b), (8.1.67d), (8.1.338)
$\mathbf{n}_k$	outwardly directed unit normal to the $k$ -phase surface (6.2.26b-d)	NTU	number of transfer units (8.1.54c), (8.1.57c), (8.1.66b), (8.1.67d), (8.1.338)
		$\mathbf{p}$	dipole moment of a dielectric particle
		$p$	stoichiometric coefficient for product P, kinetic order in the dependence of nucleation rate (6.4.30a)

$p_A; p_B; p_i p_j$	partial pressures of species A, species B; species $i$ and species $j$	$q_{fr}$	fraction of light reflected (3.1.47)
$p_{\Delta b}; p_{\Delta i}; p_{B,lm}$	value of $p_A$ in the gas bulk; value of $p_A$ at gas-liquid interface; logarithmic mean of $p_B$ (3.1.131b)	$q_{i1}(C_{i2}), q_{i1}(C_{i2}), \bar{q}_{i1}; q_{i1}^0, q_{i1}^s$	moles of species $i$ in solid phase 1 per unit mass of solid phase, cross-sectional average of $q_{i1}$ , initial value of $q_{i1}$ ; saturation value of $q_{i1}$
$p_{if}; p_{ij}; p_{ip}; p_{iv}$	value of $p_i$ in feed gas, region/stream $j$ , permeate gas, vapor phase	$q_{iR}, q_{is}$	moles of species $i$ per unit mass of ion exchange resin ( $R$ ) or solid adsorbent ( $s$ )
$\bar{p}_{ib}, \bar{p}_{ii}$	species $i$ partial pressure in bulk and particle interface, averaged over bed cross section, Figures 3.4.4(a), (b)	$q_{maxR}$	maximum molar fixed charge density per unit resin mass
$pH; pI$	indicator of hydrogen ion concentration (5.2.65a); isoelectric point for a protein/amino acid; at $pI = pH$ , net charge is zero	$q(\mathbf{r}, \psi, \lambda)$	probability that a molecule having configuration $(\mathbf{r}, \psi, \lambda)$ does not intersect pore wall (3.3.89f)
$pK_i$	$-\log_{10} K_{div}$ (5.2.65b) for $i = 1$ , (5.2.74b) for $i = 2$ , (6.3.29)	$Q$	volumetric fluid flow rate, product species in reaction (5.3.5), hydraulic permeability in Darcy's law (6.1.4g,h), heat transfer rate (6.4.47a,d)
$P; P^0; P_c$	total pressure, system pressure; standard state pressure; critical pressure	$Q_c; Q_d$	electrical charge of a collector; volume flow rate of dialysate solution
$\bar{P}; P_f; P_j, P_p; P'_p; P_\ell$	local solute permeability coefficient (6.3.157b); feed pressure; total pressure of $j$ th region and permeate, respectively; gas pressure (Figure 7.2.1(b)); gas pressure at the end of a capillary of length $l$ (6.1.5d)	$Q_f; Q_g; Q_h; Q_i; Q_b, Q_o, Q_p, Q_R, Q_1, Q_2$	volumetric feed flow rate to separator; Darcy permeability for gas through packed bed; amount of heat supplied at a high temperature; electrical charge on 1 gmol of charged species $i$ ; amount of heat rejected at a low temperature; volumetric flow rate at membrane channel inlet; electrical charge of a particle; excess particle flux (7.2.123), volumetric flow rate of product stream from separator; heat supplied at the reboiler per mole of feed; volume flow rate of overflow; volumetric rate of underflow/concentrate
$P_i^0(\pi); P_i^*$	equilibrium gas-phase pressure for pure $i$ adsorption at spreading pressure $\pi$ , which is the same for a mixture (3.3.111a); pressure at crossover point for solute $i$ in supercritical extraction	$Q_{Am}, Q_{Bm}, Q_{im}, Q_{jm}$	permeabilities of species A, B, $i$ and $j$ , respectively, through membrane in gas permeation and pervaporation, respectively
$P_{atm}; P_{liq}; P_1, P_2$	atmospheric pressure; pressure in the liquid (6.1.12); purification factors (7.2.97)	$Q_{ij}$	permeability of species $i$ through region $j$ ( $= A, B, C, D, 1, 2$ ) of the membrane
$Pe; Pe_i; Pe_i^m; Pe_{z,eff}; Pe_{zj}$	Péclet number (3.1.143g), (7.3.34d); $Pe$ number for dispersion of solute $i$ (6.3.23a); pore Péclet number (6.3.145a); ( $z v_z / D_{i,eff,z}$ ) (7.1.18h); $j$ phase $Pe_z$ (8.1.92)	$Q_{im}^{ov}$	overall permeability of species in membrane pervaporation for a membrane of thickness $\delta_m$
$P_i^{sat}, P_j^{sat}; P_{iPl}^{sat}, P_{i,curved}^{sat}$	vapor pressure of pure $i$ and pure $j$ , respectively, at system temperature; value of $P_i^{sat}$ on a plane surface; value of $P_i^{sat}$ on a curved surface	$Q_{crys}, Q_{sub}$	heat transfer rate of a solution during crystallization and subcooling, respectively
$P_M^{sat}$	value of $P_i^{sat}$ for pure species $i$ of molecular weight $M$	$Q_{im0}, Q_{sm}, Q_{sc}$	value of $Q_{im}$ for $C_{im} = 0$ solvent permeability through membrane and cake in cake filtration
$P_0$	amplitude of pressure wave (3.1.48)		
$P_R, P_w$	pressure of resin phase and external aqueous solution, respectively		
$Po, Pr$	power number (6.4.976), power (3.1.47)		
$q$	number of variables in a problem, stoichiometric coefficient for species $Q$ , heat flux in a heat exchanger attached to a cooling crystallizer (6.4.47a), the power of $M_T$ in expression (6.4.39a) for $B^0$ , factor (8.1.150)	$\mathbf{r}$	vector of molecular position, radius vector, unit vector in radially outward direction

## Notation

xxi

$r$	radial coordinate, any characterizing property of a continuous mixture	$R_1, R_2$	intrinsic RO rejection of the unionized species 1 and ionized species 2
$r_1, r_2; r_{1b}, r_{2b}; r_c; r_f$	radii of curvature of interface (3.3.47); liquid outlet radii in tubular centrifuge; critical size of a nucleus (3.3.100b), cyclone radius; free surface radius in tubular centrifuge	$R^+ (R^-)$	ion exchange resin with fixed positive (negative) charge; cation (anion) of a surface active solute
$r_g; r_h; r_b; r_{in}$	radius of gyration of a macromolecule; hydrodynamic viscosity based radius (3.3.90f); radius of spherical solute molecule (3.3.90a); radius of liquid-liquid interface in tubular centrifuge	$R_{ij}; R_{im}; R_{i,reqd}$	permeation resistance of region $j$ ( $j = A, B, C, D$ ) in the membrane to species $i$ ; membrane permeation resistance of species $i$ ; solute $i$ rejection required in a RO membrane
$r_o$	radius of a sphere whose volume is equal to that of an ellipsoid (3.1.91g), radial location of the center of solute peak profile, radius of a cylindrical centrifuge	$R_{ik}, R_{ki}; R_{min}$	phenomenological coefficients (3.1.207); minimum reflux ratio (8.1.170)
$r_p; r_{p,a}; r_{p1}, r_{p2}; r_{p1}, r_{s1}; r_b; r_w$	radius of particle, pore radius; analytical cut size (2.4.18); particles of two different sizes; dimension of particle of a certain size; particle size for classification (2.4.8); cyclone exit pipe radius; radius of wire	$R_{obs}, R_{true}; Re; Re_L; Re_{imp}$	observed and true solute rejection in ultrafiltration and RO; Reynolds number; $Re$ for a plate of length $L$ (3.1.143a), Table 3.1.5; $Re$ for an impeller (Table 3.1.7), (6.4.96)
$r_{max}, r_{min}; r_{p,50}$	maximum and minimum radius of membrane pores or particle sizes; equiprobable size	$s$	fractional supersaturation (3.3.98b), power of the $\Delta P$ dependence of $\hat{R}_{c\delta}$ (6.3.138j), Laplace transform complex variable (5.4.35), solution volume fed to bed per unit empty bed cross section (7.1.17a), eluent/salt (7.1.109b), salinity (10.1.13)
$\sqrt{r_p^2}$	hydraulic mean pore radius (3.4.87)	$s_m; s_{mp}; s_{ms}; s_p$	pore surface area/membrane volume (6.3.135e); value of $s_m$ for membrane pore volume or membrane solids volume (6.3.135f,g); sedimentation coefficient (4.2.16b)
$\bar{r}_p; \bar{r}_{p,1,0}; \bar{r}_{p,i+1,i}$	mean of particle size distribution based on $f(r_p)$ and $n(r_p)$ , respectively (2.4.1e), (2.4.2g), (2.4.2h)	$s_1, s_2$	two different solvents
$\bar{r}_{p,3,2}$	Sauter mean radius of a drop, bubble or particle (6.4.89)	$S; S_c$	solute transmission/sieving coefficient (6.3.141e), supersaturation ratio (3.3.98c), stripping factor (8.1.135), (8.1.189b); bed/column cross-sectional area
$ra; re; re _{flow}$	rate of arrival of cells; fractional water recovery; value of $re$ in a cell (6.3.172b)	$S_f, S_k, S_M; S_j; S_M, S_d$	cross-sectional area vector of feed entrance, $k$ th feed exit and membrane surface area vector in a separator; cross-sectional area of flow for $j$ th stream; selectivities (7.3.219a,b)
$R; R$	universal gas constant; radius of a tube or capillary, reflux ratio (2.3.5), (2.3.7), (8.1.137), solute rejection in reverse osmosis, solute retention in ultrafiltration	$S_{ij}; \bar{S}_{ij}$	molar entropy of species $i$ ; partial molar entropy of species $i$ (3.3.17b)
$R_c; \hat{R}_{c\delta}, \hat{R}_{cw}$	cake resistance; specific cake resistance per unit cake thickness and unit cake mass, respectively (6.3.135l)	$S_{im}; S_{im0}$	solubility coefficient of species $i$ in membrane; value of $S_{im}$ for $C_{im} = 0$
$R_{ij}; R_j; R'_j; R_L$	hydraulic radius (6.1.4c); solute rejection/retention by membrane for species $i$ , retention ratio for species $i$ (7.3.211); fraction of solute $i$ in mobile phase (7.1.16c); largest radius of a conical tube	$S_\infty; \bar{S}_\sigma; S_1, S_2$	value of $S$ when $Pe_i^m \gg 1$ (6.3.145b) surface area of interfacial region (Figure 3.3.2A); molar surface area in gas-solid adsorption (3.3.107); sieving coefficients for species 1 and 2
$R_m; R_s; R_\infty$	membrane resistance; resolution between neighboring peaks (2.5.7) in chromatography; value of $R_i$ at large $Pe_i^m$	$S_{ij}; S_{mag}$	total entropy for region $j$ (3.3.3); magnetic field force strength (7.3.251)
$R_A, R_i$	molar rate of production of A per unit volume (5.3.7), (6.2.2d), for species $i$		

$S_{\text{obs}}$ ; $S_{\text{true}}$	observed solute transmission/sieving coefficient; true value of $S$	$u_i$ ; $u_k$ ; $u_{ij}$ ; $u_{il}$ ; $u_{io}$ ; $u_{is}$ ; $u_{ilb}$	mass fraction of impurity species $i$ ; mass fraction of species $k$ ; mass fraction of species $i$ in region $j$ (1.3.5) or $j$ th stream (2.1.20); value of $u_i$ in melt; value of $u_i$ initially in the solid; value of $u_i$ in the recrystallized solid; value of $u_i$ in the bulk melt
$Sc$ ; $Sc_c$	Schmidt number (3.1.143a); $Sc$ for continuous phase	$u_o$ ; $u_{rc}$ ; $u_{ro}$	mass fraction of solute in the solution charged for crystallization; $u_c/(1 - u_c)$ ; $u_o/(1 - u_o)$ (6.4.47b)
$Sh$ ; $Sh_c$ ; $Sh_D$ ; $Sh_p$ ; $Sh_z$	Sherwood number (3.1.143a); $Sh$ for continuous phase and dispersed phase, respectively; $Sh$ in a packed bed (7.2.218a); $Sh$ at location $z$ (7.2.64)	$u_i^m$ $u_{iEn}$ ; $u_{iRn}$	ionic mobility (3.1.108j) mass fraction of species $i$ in extract and raffinate streams, respectively, from stage $n$
$St$	Stanton number (3.1.143g), Stokes' number (6.3.41)	$ur_{ig}$ ; $ur_{is}$ $ur_{ij}$	defined by (8.1.349) and (8.1.350) weight of solute $i$ per unit weight of phase $j$ (9.1.32)
$t$ ; $t_{br}$ ; $t_c$ ; $t_{res}$ ; $t_\sigma$	time; breakthrough time; time for cut point in the chromatographic separator output (2.5.1); residence time; thickness of interfacial region (Figure 3.3.2A)	$\mathbf{U}_i$ ; $\mathbf{U}_p$	migration velocity vector for species $i$ (3.1.84b); particle velocity vector
$t_i^+$ ; $t_1^+$ ; $t_2^+$ ; $\bar{t}$ ; $\bar{t}_1$ ; $\bar{t}_2$ ; $\bar{t}_3$	nondimensional time variable for species $i$ (6.3.12); value for $t_i^+$ for species 1, 2 (3.2.9), (3.2.20); breakthrough time (7.1.15c); value of $\bar{t}$ for species 1, 2 and 3	$\bar{U}_i$ $\bar{U}_i^{\text{new}}$ $U_{ix}$ ; $U_{iy}$ ; $U_{iz}$	averaged velocity vector of $i$ th molecules due to all forces defined by (3.1.103) components of migration velocity of species $i$ in $x$ -, $y$ - and $z$ -directions, (3.1.82)
$t_i^{\text{in}}$ ; $t_i^0$	times when species $i$ appears and disappears, respectively, from a chromatographic separator output	$U_{ik}$ ; $\langle U_{ik} \rangle^k$ ; $\hat{U}_{ik}$	value of $\mathbf{U}_i$ in the $k$ th phase/region; average of $\mathbf{U}_{ik}$ in the $k$ th phase/region; defined as a fluctuation by (6.2.28)
$t_{im}$ ; $t_{is}$	transport number of $i$ in membrane or solution (3.1.108d)	$U_p^{\text{int}}$ ; $U_{px}^{\text{int}}$ ; $U_{py}^{\text{int}}$	internal particle velocity vector (6.2.50d); its component in the direction of the internal coordinate $x^i$ ; internal particle velocity vector for all particles of size $r_p$
$t_{s_1}$ ; $t_{s_2}$	time required for solvents, $s_1$ , $s_2$ (3.2.24)	$\mathbf{U}_{pb}$ ; $U_{pr}$ ; $U_{prb}$ $U_{pzv}$ ; $U_{pyt}$	terminal velocity vector of particle (3.1.62); radial particle velocity; terminal value of $U_{pr}$ ; value of $\mathbf{U}_{pt}$ in $z$ -direction (6.3.1), (7.2.211), $y$ -direction (7.3.154)
$t_{R_i}$ ; $t_{R_M}$ ; $t_{R_0}$	retention time for species $i$ in capillary electrophoresis (6.3.18a), (7.1.99d); retention time for the mobile phase; retention time based on $v_{z,\text{avg}}$ (7.3.207)	$\mathbf{v}$ ; $\mathbf{v}_i$ ; $\mathbf{v}^*$ ; $\mathbf{v}_i^*$ ; $\mathbf{v}^+$ , $\mathbf{v}_i^{\text{ref}}$ ; $\mathbf{v}_{ij}$ ; $\mathbf{v}_{ij}^*$	mass averaged velocity vector of a fluid (also $\mathbf{v}_i$ ); molar average velocity vector of a fluid (also $\mathbf{v}_i^*$ ); nondimensional $\mathbf{v}$ (6.3.39); reference $\mathbf{v}_i$ ; values of $\mathbf{v}_i$ and $\mathbf{v}_i^*$ on surface area $\mathbf{S}_j$ (2.1.1), (2.1.2)
$T$ ; $T_1$ ; $T_2$ ; $T_c$	absolute temperature; temperature of cooled plate; temperature of the heated plate; critical temperature	$\mathbf{v}_i$ ; $\mathbf{v}_{ij}$	averaged velocity vector of $i$ th species; value of $\mathbf{v}_i$ on surface area $\mathbf{S}_j$ (2.1.1)
$T_f$ ; $T_g$ ; $T_i$ ; $T_p$	feed temperature; glass transition temperature of a glassy polymer; value of absolute temperature $T$ of region $i$ ; product temperature	$v_k$ ; $u_{kz}$ ; $v_k^{\text{int}}$	velocity of region $k$ ; $z$ -component of velocity $\mathbf{v}_k$ of region $k$ ; mass average velocity of the interface of two phases
$T_C$ ; $T_H$ ; $T_L$ ; $T_R$	temperature of condenser; two temperatures in supercritical extraction; reboiler temperature	$v_p^r$	particle diffusion velocity relative to that of the fluid phase (3.1.43)
$T_{cf}$ ; $T_{ci}$ ; $T_{mi}$	temperature of cooling fluid, critical temperature of species $i$ ; melting temperature for species $i$		
$T_{\text{sat}}$	temperature at which the solution is saturated		
$T_{\text{sol}}$	temperature of the solution due to undergo crystallization		
$Th$	dimensionless group (3.1.46b)		
$u$	number of fundamental dimensions (Section 3.1.4.1)		
$u_c$	mass fraction of solute in the crystallized solution on a solid-free basis		

## Notation

xxiii

$u_{ci}^*$ ; $u_{ci}^+$ ; $u_{oi}$ ; $v^0$ ; $u_{ci}^+$ ; $u_{og}$	average velocity of liquid zone carrying species $i$ (7.3.208); concentration wave velocity of species $i$ (7.1.12a); velocity of micelles, superficial velocity (3.1.176), in a packed bed (6.1.4a); amplitude of square-wave velocity (7.1.72a); nondimensional concentration wave velocity (7.1.105o); superficial velocity for gas phase	$V_k$ ; $V_b$ ; $V_\ell^p$ ; $V_o$ ; $V_o^t$ ; $V_o^o$	volume of region $k$ ; liquid volume; specific pore volume of a microporous adsorbent (4.1.64a); volume of organic solvent; total amount of $V_o$ (6.3.99); sample volume (7.1.101b)
$u_r$ ; $u_s$ ; $u_{so}$ ; $u_{tw}$ ; $u_{tz}$ $u_{vz}$	radial velocity; shell-side velocity (3.1.175), volume flux of solvent/permeate (7.2.71); value of $v_s$ at membrane channel inlet; tangential fluid velocity in wall region (7.3.134); same as $v_z$ ; vapor velocity in $z$ -direction (6.3.47a)	$V_M$ ; $V_{pi}$ ; $V_s$ ; $V_{SE}$ ; $V_{sp}$	mobile-phase volume in a column (7.1.99g); volume of a particle, cumulative permeate volume (7.2.87); stationary adsorbent-phase volume in a column; suspension volume; stationary liquid-phase volume; mobile-phase volume present inside the pores of a particle (7.1.110c)
$u_x$ ; $u_y$ ; $u_{yw}$ ; $u_{ywf}$	local convective velocity in $x$ -direction, $y$ -direction; value of $v_y$ at wall; value of $v_{yw}$ at feed entrance (7.2.39)	$\bar{V}_s$ ; $\bar{V}_t$ ; $V_w$ ; $V_\sigma$	partial molar volume of solvent; averaged partial molar volume (3.1.56); volume of water; volume of interfacial region, Figure 3.3.2A
$u_z$ ; $u_{zmax}$ ; $u_{z,avg}$ ; $u_{z,avg,f}$	velocity of fluid in $z$ -direction; maximum value of $v_z$ , averaged value of $v_z$ over flow cross section; value of $u_{z,avg}$ at feed location (6.1.5f)	$V_{fi}$ ; $V_{fe}$	initial volume of feed solution; final volume of solution
$u_\infty$ $u_{fr}$ ; $u_{i,eff}^+$	uniform gas velocity far away from object velocity of freezing interface in zone melting (6.3.110c); nondimensional velocity of species $i$ in capillary electrophoresis (6.3.12)	$V_{f0}$ ; $V_{fR}$ ; $V_{Ni}$ ; $V_{Ri}$	volume of feed solution at time $t = 0$ ; volume of retentate; net retention volume for species $i$ (7.1.99j), retention volume for species $i$ (7.1.99e,f), (7.1.99h)
$u_z^{AA}$ ; $u_{zH}$ ; $u_{zL}$	velocity of interface AA between suspension and clarified liquid in $z$ -direction (4.2.52); interstitial gas velocity at high-pressure feed step and low-pressure purge step, respectively (7.1.53a,b)	$V_{cap}^{iex}$ ; $V_{eff}$ ; $V_{channel}$	volumetric ion exchange capacity of packed bed; liquid filled centrifuge volume; volume of channel
$u_{EOF}$ ; $u_{EOF,z}$	electroosmotic velocity (6.1.22); value in $z$ -direction	$\bar{V}_{AY}$	partial molar volume of electrolyte AY (3.3.119b)
$vr$ $V$ ; $V_b$ ; $V_d$ ; $V_f$ ; $V_h$ ; $\hat{V}_a$ ; $\hat{V}$ ; $\bar{V}$	defined by (7.2.167) volume of a region, volume of separator, volume of feed solution/eluent passed, volume of a sphere (3.3.52a), voltage between electrodes; volume of buffer; volume of the dialysate solution; volume of feed solution; hydrodynamic volume of a macromolecule/protein (3.3.90f); adsorbed monolayer phase volume/weight of adsorbent (3.3.114b); radial volume flow rate of solution (Figure 7.1.6); defined by (7.1.18i)	$\bar{V}_{ic}$ $V_{im}$ $\bar{V}_{ij}$ $VCR$	value of $\bar{V}_i$ for a crystal of $i$ molar volume of species $i$ at its normal boiling point value of $\bar{V}_i$ in region $j$ volume concentration ratio ( $V_{f0}/V_{fR}$ ) (6.4.98)
$\bar{V}_c$ ; $V_i$ ; $\bar{V}_i$ ; $V_j$	critical molar volume of a species; molar volume of pure species $i$ ; partial molar volume of species $i$ ; volume of region $j$ : $j = 1$ , stationary phase; $j = 2$ , mobile phase	$w$ ; $w_{crys}$ ; $w_i$ ; $w_{so}$  $w_{ij}$ ; $w_{ji}$ ; $w_{tz}$ ; $w_{ij}^s$ ; $w_{tE}$ ; $w_{ER}$  $w_E$ ; $w_M$ ; $w_R$  $w_j^l$ ; $w_j^s$ ; $w_p^s(r_p)$ ; $wa$	mass of adsorbent, quantity defined by (5.2.54); mass of crystals formed at any time, mass of solute $i$ charged; mass of solution charged to the crystallizer mass flow rate of species $i$ through $S_j$ (2.1.4); total mass flow rate through $S_j$ (2.1.5); total mass flow rate in $z$ -direction; total solids flow rate in $j$ th stream (2.4.3a); $w_{ij}$ for $j =$ extract, $E$ , $w_{ij}$ for $j =$ raffinate, $R$ weight of mixture at $E$ , $M$ and $R$ , respectively mass of stationary liquid phase used as coating; mass of particles per unit of fluid volume (2.4.2e); mass of a solid particle of size $r_p$ (2.4.2e); waist size of a light beam

$W, W_A$	width of rectangular channel walls, molar transfer rate of species A	$x_{s,lm}$ $X$	defined by (3.1.137) molar density of fixed charges in electrically charged system
$\underline{W}$	total reversible work done (3.1.25a,b)	$X_{ij}, X_{ijn}$	mole ratio of species $i$ in region $j$ (1.3.2) or stream $j$ (2.2.2d), $X_{ij}$ for state $n$
$W_{bb}, W_{ij}, W_{ij}$	band width of the chromatographic output of species $i$ (Figure 2.5.2), molar flow rate of species $i$ through $S_j$ (2.1.7), total molar flow rate through $S_j$ (2.1.8)	$X_{ij}^a$	abundance ratio of isotope $i$ in region $j$ (1.3.7)
$W_{tAm}, W_{tBm}$	molar rate of permeation of species A and B through membrane	$y, y_f$	coordinate direction, normal to gas-liquid, liquid-liquid or membrane-fluid interface, feed gas mole fraction (Figure 7.1.14)
$W_{if}, W_{ib}, W_{iv}$	molar flow rate of feed, liquid fraction and vapor fraction, respectively	$y_b, y_A, y_H, y_L$	mole fraction of species $i$ and A in vapor/gas phase, respectively, fluid mole fraction (7.1.62b), gas-phase mole fractions defined by (7.1.54a)
$W_{ilb}, W_{ild}$	total molar flow rate of bottoms product and distillate product from a distillation column	$Y_A^*, Y^*, Y_{tim}$	hypothetical gas-phase mole fraction of species A in equilibrium with $x_{Ab}$ , nondimensional $y$ -coordinate (5.3.5j), $y$ -coordinate of limiting trajectory (7.3.263a)
$W_{t1}, W_{t2}, W_{tp}$	total molar flow rate of streams $j = 1, j = 2$ and $j =$ permeate	$Y_{Ab}, Y_{Ai}$	bulk gas mole fraction of species A, value of $y_A$ at a two-phase interface
$W_{tjn}^t$	total molar flow rate of stream $j$ for stage $n$ in enriching section of cascade	$Y_{ij}, Y_{iR}, Y_2, Y_{2f}$	segregation fraction of species $i$ in region $j$ (1.3.8a), segregation fraction of solute $i$ in retentate in a membrane process (6.4.107), defined by (7.2.19) and (7.2.20)
$We$	Weber number, $(\rho_c v_c^2 d_p / \gamma)$ (6.4.91)	$Y_{11}(t), Y_{21}(t)$	segregation fraction of species 1 and 2 in region 1 at time $t$
$x, x_A$	coordinate direction, mole fraction of species A in liquid phase	$Y_{ij}^a, \dot{Y}_{ij}, \dot{Y}_{rj}$	segregation fraction for an isotopic mixture (1.3.9a), of species $i$ stream $j$ (2.2.12), of particles of size $r$ in stream $j$ (2.4.5)
$x_A^*$	hypothetical liquid phase mole fraction of species A in equilibrium with $y_{Ab}$	$z, z_{cr}$ (or $z_{crit}$ ), $z_H,$ $z_L, z_{AA}, z_{BB}$	coordinate direction, critical distance (7.2.133), characteristic locations at pressures $P_H$ and $P_L$ (7.1.55e), vertical coordinates of interfaces AA and BB, respectively
$x_{Ab}, x_{Ai}$	bulk liquid mole fraction of species A, value of $x_A$ at a two-phase interface	$z_{HL}, z_{HS}$	height of the liquid level of a dilute suspension, height of sludge layer
$x_b, x_{ij}, x_{is}, x_{i,j}$	mole fraction of species $i$ , value of $x_i$ ( $i = A, B, 1, 2, s$ (solvent), etc.) in region $j$ or stream $j$ ( $j = 1, 2, f$ (feed), $g =$ gas, $l =$ liquid, $p =$ permeate, $R =$ resin, $s =$ solution, $v =$ vapor, $w =$ water), $x_{ij}$ where $j = \sigma$ , the surface adsorbed phase, mole fraction of species $i$ in region $j$ when molecular formula is substituted for species $i$	$z^+, z_i^+$	nondimensional $z$ -coordinate (Figure 3.2.2), (6.3.12), value of $z^+$ for center of mass of concentration profile of species $i$ (3.2.22a,b)
$x_{iE}, x_{iE}^*, x_{iE}^e$	mole fraction of species $i$ in extract phase, value of $x_{iE}$ in equilibrium with $x_{iR}$ , value of $x_{iE}$ if both streams leave stage in equilibrium	$z_\alpha, z_\beta, z_0(\bar{C}_{i2})$	$z$ -coordinate locations of regions $\alpha$ and $\beta$ , respectively, defined by (7.1.17e,f)
$x_{ilb}, x_{ild}$	bulk value of $x_{ib}$ , mole fraction of $i$ in liquid product from reboiler in a distillation column, $x_{il}$ in liquid product from condenser at the top of a distillation column	$Z_b, Z_{i,eff}, Z_p$	electrochemical valence of species $i$ , effective charge on ion $i$ due to the diffuse double layer ( $< Z_i$ ) (6.3.31a), value of $Z_i$ for a protein
$x_{iR}, x_{iR}^*, x_{iR}^e$	mole fraction of species $i$ in raffinate phase, value of $x_{iR}$ in equilibrium with $x_{iE}$ , value of $x_{iR}$ if both streams leave stage in equilibrium		
$x_{if}, x_{jpf}, x_{ip}$	mole fraction of species $j$ in feed and permeate, respectively, mole fraction (Figure 7.2.1(b))		

## Notation

xxv

## Greek letters

$\alpha$	constant in (3.1.215), separation factor between two species 1 and 2 in capillary electrophoresis (6.3.27e)	$\gamma_1, \gamma_2, \gamma_3, \gamma_A, \gamma^{12}; \gamma^{SL}, \gamma^{LG}, \gamma^{SG}$	factors/parameters in plate height (7.1.107f–i); interfacial tension between two bulk phases 1 and 2; interfacial tension between phases S and L, L and G, S and G, respectively (S = solid, L = liquid, G = gaseous)
$\alpha_f, \alpha_l, \alpha_v$	parameters for feed, liquid fraction, vapor fraction in a continuous chemical mixture described by a $\Gamma$ distribution function (4.1.33f), (6.3.71)	$\dot{\gamma}; \dot{\gamma}_w$	shear rate (3.1.74), (6.1.31); wall shear rate
$\alpha_{ij}; \alpha_{ij}^{hf}, \alpha_{ij}^{ft}, \alpha_{ij}^{hl}, \alpha_{ijn}$	separation factor between two species $i$ ( $i = 1, A, s$ (solvent)) and regions $j$ ( $j = 2, B$ (species), $i$ ), value of $\alpha_{ij}$ between two streams (2.2.3), (2.2.4), (2.2.2a); value of $\alpha_{ij}$ for $n$ th plate multicomponent separation factor between species $i$ and $n$ (1.6.6)	$\gamma_{ii}^s; \gamma_{ii,u}^c$	infinite dilution activity coefficient for species $i$ in the liquid; activity coefficient of species $i$ in liquid phase on a mass fraction basis at infinite dilution (6.3.83)
$\alpha_{in}$	separation factor of Sandell (1.4.15)	$\Gamma_{is}^E$	molar excess surface concentration of species $i/\text{cm}^2$ of pore surface area (3.1.117a)
$\alpha'$ $\alpha_{AB}^*; \alpha_{ij}^{\text{evap}}; \alpha_{ij}^{\text{perm}^s}$	value of $\alpha_{AB}$ under ideal condition of zero pressure ratio (6.3.198), (8.1.426); value of $\alpha_{ij}$ for evaporation only (6.3.180); ideal separation factor in vapor permeation (6.3.180)	$\Gamma_{io}; \Gamma_{io}^E$	surface concentration of species $i$ (3.3.34); algebraic surface excess of species $i$ (3.3.40b), (5.2.145a)
$\beta$	defined by (2.2.8a,b), exponent in (3.3.89a), indicator of macromolecular shape for a given $r_g$ (3.3.90e), constant in (3.1.215), defined by (7.1.52b), (7.2.58), (7.2.138)	$\delta; \delta(x), \delta(z), \delta(z^+); \delta_c$	falling film thickness; delta function in $x$ -, $z$ - and $z^+$ -directions (3.2.14b); cake layer thickness
$\beta_f, \beta_l, \beta_v$	compressibility of fluid (3.1.48), parameters for feed, liquid fraction and vapor fraction in a continuous chemical mixture (4.1.33f), (6.3.71)	$\delta_g; \delta_i; \delta_\ell; \delta_m; \delta_s; \delta_\ell$	thickness of gas film; solubility parameter of species $i$ , characteristic thickness of concentration profile (7.3.202); thickness of liquid film; membrane thickness, pore length; sorbed surface phase thickness (3.1.118b); thickness of liquid film from phase interface to reaction interface (5.3.22)
$\beta_i; \beta_p; \beta_N, \beta_B; \bar{\beta}; \bar{\beta}_c$	exponent in (3.3.112c), parameter (7.2.18); compressibility of particle (3.1.48); defined by (7.1.52a); coefficient of volume expansion (6.1.9); a parameter (6.4.129); coefficient of thermal expansion of solution density (7.3.233)	$\delta_A, \delta_B, \delta_C$ $\delta_1, \delta_2, \delta_n$ $\Delta C; \Delta E_s; \Delta G; \Delta G_t; \Delta H_s; \Delta P; \Delta S; \Delta T$	membrane thickness in regions A, B, C of a membrane (Figure 6.3.35(b)) thickness of layer 1, layer 2, layer $n$ in a composite membrane molar supersaturation (3.3.98a); activation energy for crystal growth rate (3.4.29); Gibbs free energy change, e.g. for forming a crystal (3.3.100a); change in Gibbs free energy for all molecules; enthalpy of solution of a gas species in a polymer; pressure difference ( $P_f - P_p$ ), etc.; selectivity index (7.2.93); extent of supercooling (3.3.98d), temperature difference ( $T_2 - T_1$ ) heat of crystallization standard Gibbs free energy change, enthalpy change and entropy change for a reaction
$\gamma; \gamma_{cr}; \gamma_A; \gamma_R; \gamma_{R_2}$	surface tension, interfacial tension, pressure ratio (6.3.197), (6.4.123), (8.1.426), parameter (7.1.18f); critical surface tension for a polymer; Damkohler number ( $k_b \delta_m^2 / D_A$ ) (5.4.100); ( $D_A k_1 / k_t^2$ ) (5.3.11); defined by (5.3.35j)	$\Delta H_{\text{crys}}; \Delta G^0, \Delta H^0, \Delta S^0$	enthalpy of solution of a gas species in a polymer; pressure difference ( $P_f - P_p$ ), etc.; selectivity index (7.2.93); extent of supercooling (3.3.98d), temperature difference ( $T_2 - T_1$ ) heat of crystallization standard Gibbs free energy change, enthalpy change and entropy change for a reaction
$\gamma_f, \gamma_l, \gamma_v$	parameters for feed, liquid and vapor fraction (4.1.33f), (6.3.69)	$\Delta d_p; \Delta r_p; \Delta w^s$	particle diameter difference between two consecutive sieves; ( $\Delta d_p/2$ ) or the net growth in size of all seed crystals (6.4.40i); mass of crystals retained on a sieve of given size
$\gamma_i; \gamma_{if}; \gamma_{ij}; \gamma_{jf}$	activity coefficient for species $i$ , dimensionless steric correction factor; value of $\gamma_i$ in feed stream, in phase $j$ ; value of $\gamma_j$ in feed stream (species $j$ )		

$\Delta x, \Delta y, \Delta z,$ $\Delta z_L, \Delta z_H$	lengths of a small rectangular volume element in three coordinate directions, defined by (7.1.53a,b)	$\theta$	cut (2.2.10a), stage cut, fraction of adsorbent sites occupied (3.3.112a), contact angle, gas-liquid-solid system (Figure 3.3.16), contact angle between membrane surface and liquid (6.3.140)
$\Delta\mu_i^0; \Delta v_1, \Delta v_+,$ $\Delta v_-, \Delta\pi$	$\mu_{i1}^0 - \mu_{i2}^0$ , number of water molecules, cations and anions, respectively released during binding; osmotic pressure difference ( $\pi_f - \pi_p$ )	$\theta_i$	component cut for species $i$ (2.2.10b), Langmuir isotherm (7.1.36a), nondimensional mobile phase concentration (7.1.18b)
$\Delta\rho_i; \Delta\rho_{i,\text{sat}}; \Delta\phi$	mass concentration change of species $i$ in crystallizer (6.4.24); change in saturation concentration due to $\Delta T$ (6.4.51); potential difference between two phases	$\theta_{id}, \theta_{kd}$	component cut for species $i$ and $k$ , respectively, in the distillation column top product (8.1.224)
$\Delta_R$ $\nabla^i$	defined by (8.1.328) del operator in internal coordinates $x^i, y^i$ and $z^i$	$\kappa_{Ao}; \kappa_d; \kappa_i$	distribution coefficient of species A between aqueous and organic phases (3.4.16); reciprocal Debye length (3.1.17); distribution coefficient of species $i$
$\varepsilon$	void volume fraction in a packed bed, electrical permittivity of the fluid ( $= \varepsilon_0 \varepsilon_d$ )	$\kappa_{i1}, \kappa_{i1}^N$	distribution coefficient of species $i$ between regions 1 and 2 (1.4.1), its Nernst limit (3.3.78), (3.3.79)
$\varepsilon_a; \varepsilon_b$	phase angle; fractional cross-sectional area of membrane which is defective	$\kappa_{ij}; \kappa_{ij}; \kappa_{im}$	partition coefficient of solute $i$ between feed solution and feed side of the membrane; partition/distribution coefficient of solute $i$ between two immiscible phases; $\kappa_{ij}$ for feed solution and the membrane (3.3.87), (3.3.89a)
$\varepsilon_d; \varepsilon_f; \varepsilon_k$	dielectric constant of a fluid, porosity of particle deposit on filter media; Lennard-Jones force constant for species $i$ ; volume fraction of phase $k$ in a multiphase system	$\kappa'_{ij}, \kappa''_{ij}$	effective value of $\kappa_{ij}$ when there is no reaction or there is reaction
$\varepsilon_m; \varepsilon_p; \varepsilon_0; \varepsilon_0$	porosity of membrane, pellet, bead or particle; dielectrical constant of a particle; electrical permittivity of vacuum; porosity of spacer in spiral-wound module (3.1.170), porosity of clean filter	$\kappa_{io}$	partition coefficient of $i$ between an organic and an aqueous phase (6.3.101a)
$\varepsilon_B; \varepsilon_C$	fractional area of defects in glassy skin region B (Figure 6.3.35(b)); value of fractional membrane cross section in region C	$\kappa_{ip}$	$\kappa_{im}$ at permeate-membrane interface (3.4.58)
$\varepsilon_{12}$	enrichment factor for species 1 and 2 (1.4.10), (2.2.2e)	$\kappa_{i,\text{loc}}(\mathbf{r})$	local equilibrium partition constant (3.3.89g)
$\zeta$	zeta (electrokinetic) potential (3.1.11a)	$\kappa_{is}, \kappa'_{is}$	impurity distribution coefficient (6.3.109a)
$\eta$	recycle ratio (2.2.22), nondimensional variable (3.2.11), (6.3.14b), (6.3.114), fraction of particles collected (7.3.37), (7.3.38), (7.3.46), intrinsic viscosity variable defined by (3.2.17)	$\kappa'_{is, \text{eff}}$	effective partition coefficient in zone melting for species $i$ (6.3.119a)
$\eta^+$	variable (2.5.5), $\eta$ for species $i$ (3.2.25); impurity ratio for region $j$ (1.4.3a); impurity ratio for $i$ th species in region $j$ (1.6.4); defined by (1.4.4); intrinsic viscosity (3.3.90f); its value for polymer $i$ (7.1.110f); current utilization factor (8.1.406)	$\kappa_{iE}$	distribution (partition) coefficient for species $i$ in solvent extraction
$\eta_i; \eta_j; \eta_{ij}; \eta'_j,$ $[\eta]; [\eta]_i; \eta_{iF}$	variable (2.5.5), $\eta$ for species $i$ (3.2.25); impurity ratio for region $j$ (1.4.3a); impurity ratio for $i$ th species in region $j$ (1.6.4); defined by (1.4.4); intrinsic viscosity (3.3.90f); its value for polymer $i$ (7.1.110f); current utilization factor (8.1.406)	$\kappa_{iR}$	distribution coefficient of $i$ in ion exchange system (3.3.115)
$\langle \eta_i \rangle; \langle \eta_i^2 \rangle$	first moment of $C_i(\eta_i, t_i^+)$ (3.2.26); defined by (3.2.29)	$\kappa_{p1}; \kappa_{p1}^0$	protein partition coefficient in aqueous two-phase extraction; protein distribution coefficient in the absence of a charge gradient at interface
		$\kappa_{\text{salt}}^m$	distribution coefficient of salt, molal basis (4.1.34o)

## Notation

xxvii

$\lambda$	Debye length (3.1.10b), mean free path of a gas molecule (3.1.114), filter coefficient (7.2.187), parameter for a dialyzer (8.1.399), parameter for a distillation plate/stage (8.3.38), latent heat of vaporization/condensation	$\pi; \pi$	constant (3.1416...); osmotic pressure, spreading pressure (3.3.41a)
$\lambda_i; \lambda_{\infty}; \lambda_{zi}; \lambda_i; \lambda_{+}; \lambda_{-}; \lambda_i^0$	molecular conformation coordinate (3.3.89c); electrode spacings (7.3.18); retention parameter for species $i$ (7.3.213), ionic equivalent conductance of ion $i$ (3.1.108r); value of $\lambda_i$ for a cation; value of $\lambda_i$ for an anion; value of $\lambda_i$ at infinite dilution (Table 3.A.8)	$\pi_f; \pi_i; \pi_p, \pi_w$	osmotic pressure of feed solution; spreading pressure in adsorption of solute $i$ from a solution (4.1.63a); osmotic pressures of permeate solution and the solution at the membrane wall, respectively
$\lambda_1, \lambda_2$	defined by (5.4.100)	$\pi_1, \pi_2$	osmotic pressure of solution in regions 1 and 2
$A$	equivalent conductance of a salt (an electrolyte) (3.1.108s)	$\rho; \rho_b; \rho_c; \rho_e;$ $\rho_f; \rho_{fm}; \rho_i; \rho_{ij}$	fluid density; bulk density of the packed bed; density of continuous phase (also fluid density at critical point); electric charge density per unit volume; density of feed fluid; moles/fluid volume; mass concentration of species $i$ (also density of solute $i$ (3.3.90b)); value of $\rho_i$ in region $j$ (1.3.4)
$\mu; \mu_0; \mu_{dr}$	dynamic fluid viscosity; value of $\mu$ at inlet; viscosity of drop fluid	$\rho_s; \rho_i; \rho_{ij}$	density of liquid (melt); mass density of particle material; particle mass concentration in gas; density of solid material; total density of fluid; total mass density of mixture in region $j$ (1.3.5)
$\mu_C, \mu_D$	fluid viscosity of continuous and dispersed phases, respectively	$\rho_v; \rho_{i1}, \rho_{if}$	density of vapor phase; mass concentration of species $i$ in crystallizer outlet solution and feed solution, respectively
$\mu_{EOF}^m$	electroosmotic mobility (6.3.10c)	$\rho_{i\ell}; \rho_{ip}; \rho_p, r_p$	mass density of species $i$ in liquid (melt); mass density of crystals of species $i$ ; mass density contribution of particles of size $r_p$ to $r_p + dr_p$ (6.2.55)
$\mu_i, \mu_i^0, \mu_i^m;$ $\mu_i^m; \mu_{i,eff}^m; \mu_{ion,g}^m$	species $i$ chemical potential and its standard state value; ionic mobility of ion $i$ (3.1.108m); effective value of $\mu_i^m$ (6.3.8d,g); ionic mobility of ion in gas phase	$\rho_R; \rho_1, \rho_2$	density of resin particle, reduced density of fluid ( $= \rho/\rho_c$ ); density of liquids 1 and 2
$\mu_{ij}; \mu_{ijn}; \mu_{ij}^{\ell}$	value of $\mu_i$ in region $j$ ; value of $\mu_{ij}$ for plate/stage $n$ ; value of $\mu_{ij}$ in a system with electrical charge (3.3.27)	$\rho_{dR}; \rho_{pR}$ $\rho_{avg}; \rho_{isat}$	fluid resistivity; particle resistivity average gas density (6.1.5e); mass density of solute $i$ in solution at saturation
$\mu_{ijPl}$	value of $\mu_{ij}$ for a planar interface (3.3.50)	$\bar{\rho}_{ik}, \bar{\bar{\rho}}_{ik}$	intrinsic phase average and phase average, respectively, of $\rho_{ik}$ (6.2.24a,b)
$\mu_{\ell}(M), \mu_v(M)$	chemical potential of species of molecular weight $M$ in liquid and vapor phases, respectively, in continuous chemical mixtures	$\rho_{ik, avg}$ $\langle \rho_i \rangle$	mass density of solute $i$ in solution averaged over flow cross section (6.2.16b)
$\mu_0^m, \mu_p^m, \mu_s^m; \mu_{\infty}$	magnetic permeabilities of vacuum, particle and solution, respectively; chemical potential of a crystal	$\sigma$	electrical conductivity of the solution (3.1.108q), particle sticking probability
$v; v(z); v_0$	kinematic viscosity; $v$ at axial location $z$ ; $v$ at inlet	$\sigma_i$	standard deviation of any profile (3.2.21a,b), Lennard-Jones parameter, ionic equivalent conductance (3.1.108r), solute $i$ reflection coefficient through membrane (6.3.157a,b)
$v_i$	stoichiometric coefficient for species $i$ in any chemical reaction		
$v_A, v_Y$	moles of ions A and Y produced by dissociation of 1 mole of electrolyte AY		
$\zeta$	extent of separation for a binary system (1.4.16), (2.2.11), (6.3.105)		
$\zeta_{12}, \zeta_{r_{p1}}, r_{p2}$	extent of separation for components 1 and 2 in particle classification (2.4.9), for particles of size $r_{p1}$ and $r_{p2}$ (2.4.6)		

$\sigma_p; \sigma_v, \sigma_v^i; \sigma_x, \sigma_y$	steric factor for protein SMA model (3.3.122d); specific volume based deposit in filter (7.2.172), (7.2.189); standard deviations (7.3.173)	$\phi_i^{\text{ext}}, \phi_{i\alpha}^{\text{ext}}, \phi_{i\beta}^{\text{ext}}$ $\Phi; \hat{\Phi}_{ig}; \Phi_i^{\text{sat}}$	value of $\phi$ or $\phi^{\text{ext}}$ for any species $i$ ; values of $\phi_i^{\text{ext}}$ in regions $\alpha$ and $\beta$ fugacity coefficient (3.3.56); fugacity coefficient of species $i$ in a mixture in gas phase; value of $\hat{\Phi}_{ig}$ for pure $i$ at $P_i^{\text{sat}}$ at system temperature
$\sigma_{AB}; \sigma_{ix}$	average of $\sigma_A$ and $\sigma_B$ (3.1.91b); standard deviations (7.3.12a)		defined by (3.2.4); defined by (3.2.5b), defined by (3.2.6); defined by (3.2.9) and (6.3.14a)
$\sigma_i^+; \bar{\sigma}_m$	nondimensional standard deviation of a profile (3.2.21a,b); average electrical conductivity of the ionic and electronic species in a mixed conducting membrane	$\phi_i^{\text{tot}}, \phi_i^*, (\phi_i^*)_{\text{max}};$ $\phi_i^+; (\phi_i^+)$ $\phi_{ii}^{\text{ext}}$	$\sum \phi_i^{\text{ext}}$ , summation over different external forces (3.2.4)
$\sigma_{ti}, \sigma_{zi}; \sigma_{zi}^+; \sigma_{Vi}$	standard deviation in the output profile of species $i$ in $t$ -coordinate and $z$ -coordinate, respectively (2.5.3), (6.3.18c,b); nondimensionalized $\sigma_{zi}$ (6.3.16b); standard deviation in volume flow units (7.1.102b)	$\chi, \chi_p, \chi_s$ $\chi_{ij}, \chi_{ip}$	volume susceptibility of fluid, particle and the solution Flory interaction parameter between species $i$ and $j$ and species $i$ and polymer
$\tau; \tau_m; \tau_w; \tau_{wc}$	tortuosity factor; value of $\tau$ for porous membrane; wall shear stress (7.2.118); $\tau_w$ in cake region (7.2.136a)	$\psi; \psi$	molecular orientation coordinate vector (3.3.89c); extent of facilitation of flux (5.4.58) (5.4.59a), selectivity (7.2.92)
$\tau_{yx}, \tau_{yz}$	components of tangential stress $\tau_y$ in $x$ - and $z$ -directions (6.1.24)	$\psi_k$	any property or characteristic of the $k$ th phase
$\phi; \phi_A, \phi_B$	potential of any external force field (also $\phi^{\text{ext}}$ ), angle in spherical polar coordinate system, enhancement factor due to reaction, solids volume fraction profile in suspension boundary layer (7.2.109); enhancement factors for species A and B	$\psi_s, \psi_v$ $\psi_{As}$ $\psi_{el}$ $\langle \psi_k \rangle$	shape factor for particle surface and volume (Example 2.4.1), (2.4.2e) (3.4.26), (6.4.15), (6.4.19) association factor (3.1.91a) electrical potential in the double layer phase average of $\psi$ in the $k$ th phase (6.2.25a) = $\psi_{k,\text{avg}}$
$\phi_c; \phi_D, \phi_s$	volume fraction of solids in cake; volume fraction of continuous, dispersed phase (Table 3.1.7), (6.4.88); volume fraction of particles and solids in a suspension	$\langle \psi_k \rangle^k$	intrinsic phase average of $\psi$ in the $k$ th phase (6.2.25b) = $\psi_{k,\text{avg}i}$
$\phi_j; \phi_m$	electrical potential of phase $j$ ; defined in (7.1.107h)	$\omega$	angular velocity, solute permeation parameter (6.3.158b), sign of fixed charge in (3.3.30b)
$\phi_{il}; \phi_{im}; \phi_m;$ $\phi_p; \phi_{pl}$	volume fraction of species $i$ in liquid phase; volume fraction of species $i$ in membrane; polymer volume fraction in membrane; voltage drop over a cell pair in electrodialysis; volume fraction of polymer in liquid phase	$\Omega_{D,AB}$	quantity in diffusion coefficient expression (3.1.91b)
<b>Superscripts</b>			
$\phi_N$	correction factor for nonequimolar counterdiffusion (3.1.136b)	$a; b; \text{Br}$	activity based; bottom/stripping section of a column; Brownian motion
$\phi_0$	arbitrary value of centrifugal potential at $r = 0$ (3.1.6d)	$d, \text{drag}$ $\text{eff}; \text{eph}; \text{ext}$ $ft; G$	drag effective; electrophoretic; external between the feed stream and the tails stream; gas phase
$\phi_w$	volume fraction of water in resin, particle volume fraction at wall	$hf$	between the heads stream and the feed stream
$\phi_{\text{max}}$	maximum enhancement due to an instantaneous reaction, maximum particle volume fraction	$ht$	between the heads stream and the tails stream
		$i; \text{iner}; \text{int}$	species $i$ ; inertial; internal

## Notation

xxix

$L$	liquid phase	in; ion	at inlet; ionic species
$m$	magnetic, mobility, based on molal quantities, maximum number	$j$	region/phase $j$ , where $j = 1, 2, E, f, g, l, \ell, m, o, p, R, s, t, v, w, T, \sigma$
N	related to Nernst-Plank equation (3.1.106)	$k$	species $k$
0, o	location $z = 0$ , standard state, infinite dilution, original quantity	$l, \text{liq}; \text{loc}; \ell; \ell; L$	liquid phase; local; adhering liquid phase on crystal; low; at end of separator of length $L$
oo, ov	(7.1.101c), overall	$m; mc; \text{max}; \text{min}$	membrane phase; micellar; maximum; minimum
$p$	pore, pure species	$M$	mixture, molecular weight based, metallic species
$P$	mole fraction based in Henry's law	$ME, MR$	Murphree based on extract phase, raffinate phase
$r$	quantity in a system with recycle or reflux, for the case of a chemical reaction	$n, (n + 1), (n - 1)$	stage number $n, (n + 1)$ and $(n - 1)$ , respectively
$s$	related to solids only in solid-fluid separation, at saturation, scrubbing cascade	$N; \text{nu}$	metallic species; nucleation
S	solid phase	$o; \text{obs}; og; ol; \text{out}$	organic phase; observed; overall gas phase based; overall liquid phase based; at outlet
$t, T$	total value, top/enriching section of a column, thermal diffusion	$p; pd; P$	permeate side, product side, particle; dominant particle; planar interface
$v\ell$	vapor liquid	$r; r_{p_1}, r_{p_2}; R$	radial direction; related to particles of size $r_1$ or $r_2$ ; ion exchange resin phase, resistive, raffinate phase
$x, \delta$	mole fraction based, location $z = \delta$ or $\delta_m$	$s$	solvent, surface integration step in crystallization, surface adsorption site, salt/eluent counterion, stripping section, stage number
$\infty$	infinite dilution condition	$s^+, s^-; se$	forward and reverse surface reaction; seed crystal
'	equimolar counterdiffusion case, first derivative, feed side of membrane	$S$	location $S$ , stationary phase
"	permeate side of membrane	$t; tOE; tOR$	total, top product stream in a column; transfer units (overall) based on extract phase; transfer units (overall) based on raffinate phase
+	nondimensional quantity	$tj$	total quantity in $j$ th stream, $j = f$ for a single feed stream, $j = f_1, f_2$ for two feed streams entering separator, $j = 1, 2$ for product streams rich in species 1 and 2, respectively
*	hypothetical quantity	$true; T$	true value; thermal diffusive, total vapor phase, volumetric; aqueous phase, water
<b>Subscripts</b>		$x$	liquid mole fraction based, coordinate direction
atm; $b$	atmospheric; bulk phase value, bottom product stream from a column, backward reaction	$y$	gas phase mole fraction based, coordinate direction
A,B,C	species A, B and C	Y	as in species AY
$c$	based on molar concentration, continuous phase, cake, critical quantity	$z$	coordinate direction
$d$	diffusive, dialysate phase, dissociation, distillate	1, 2	species 1 and species 2, phase 1 and phase 2
$D$	drag related, Henry's law species related, dispersed phase	3, 4	species 3 and species 4, phases 3 and 4
$e$	enriching section	$\alpha, \beta$	phase $\alpha$ and phase $\beta$
eff; ex; ext	effective value; exit location; external	$\sigma, s$	surface phase
$f; fr$	feed side, feed based; during formation of drop, size based		
$g; gr$	gas phase; growth based		
$H; \text{hex}$	high; heat exchanger		
$i$	species $i$ , where $i = 1, 2, 3, A, B, M, s$ , phase interface, $i$ th module/tube/stage		
$ij$	$i$ th species in $j$ th stream, $j = f$ for a single feed stream, $j = f_1, f_2$ for two feed streams entering separator, $j = 1, 2$ for product streams rich in species 1 and species 2, respectively		
imp; iner	impeller; inertial		

**Abbreviations and acronyms**

Å	angstrom	HFCLM	hollow fiber contained liquid membrane
AEM	anion exchange membrane	HGH	human growth hormone
atm	atmosphere	HGMS	high-gradient magnetic separation
avg	average	HK	heavy key
AVLIS	atomic vapor laser isotope separation	HPCE	high-performance capillary electrophoresis
BET	Brunauer-Emmet-Teller	HPLC	high-performance liquid chromatography
bar	10 <sup>5</sup> pascal	HPTFF	high-performance tangential-flow filtration
barrer	unit for permeability coefficient of gases through membrane, $1\text{ barrer} = 10^{-10} \frac{\text{cm}^3(\text{STP})\text{-cm}}{\text{cm}^2\text{-s-cmHg}}$	IEF	isoelectric focusing
Btu	British thermal unit	ILM	immobilized liquid membrane
CAC	continuous annular chromatograph	IMAC	immobilized metal affinity chromatography
CACE	counteracting chromatographic electrophoresis	ITM	ion transport membrane
CCEP	countercurrent electrophoresis	ITP	isotachopheresis
CD(s)	cyclodextrin(s)	LDF	linear driving force approximation
CDI	capacitive deionization	LK	light key
CE	capillary electrophoresis	LLC	liquid-liquid chromatography
CEC	capillary electrochromatography	LM	logarithmic mean
CEDI	continuous electrodeionization	LRV	log reduction value
CEM	cation exchange membrane	LSC	liquid-solid adsorption chromatography
CFD	computational fluid dynamics	LTU	length of transfer unit
CFE	continuous free-flow electrophoresis	mAB	monoclonal antibody
CGC	countercurrent gas centrifuge	MBE	moving boundary electrophoresis
CHO	Chinese hamster ovary	ME	multiple effect
CHOPs	Chinese hamster ovary cell proteins	MEKC	micellar electrokinetic chromatography
cm Hg	pressure indicated in the height of a column of mercury	MEUF	micellar enhanced ultrafiltration
CMC	critical micelle concentration	MSA	mass-separating agent
CMS	carbon molecular sieve	MSC	molecular sieve carbon
CSC	continuous-surface chromatography	MSF	multistage flash
CSTR	continuous stirred tank reactor	MSMPR	mixed suspension, mixed product removal
CSTS	continuous stirred tank separator	MTZ	mass-transfer zone
CV	control volume	NEA	nitrogen-enriched air
CZE	capillary zone electrophoresis	OEA	oxygen-enriched air
ED	electrodialysis	PAC	powdered activated carbon
ELM	emulsion liquid membrane	PBE	population balance equation
EOF	electroosmotic flow	PSA	pressure-swing adsorption
ESA	energy-separating agent	psia	pound force per square inch absolute
FACS	fluorescence-activated cell sorting	psig	pound force per square inch gauge
FFE	free-flow electrophoresis	RO	reverse osmosis
FFF	field-flow fractionation	SCF	supercritical fluid
FFM	free-flow magnetophoresis	SEC	size exclusion chromatography
GAC	granular activated carbon	SLM	supported liquid membrane
GLC	gas-liquid chromatography	SMA	steric mass action
GPC	gel permeation chromatography	SMB	simulated moving bed
HDPs	high-density particles	SPs	structured packings
HETP	height of an equivalent theoretical plate	TDS	total dissolved solids
		TFF	tangential-flow filtration
		TOC	total organic carbon