

CHAPTER

1

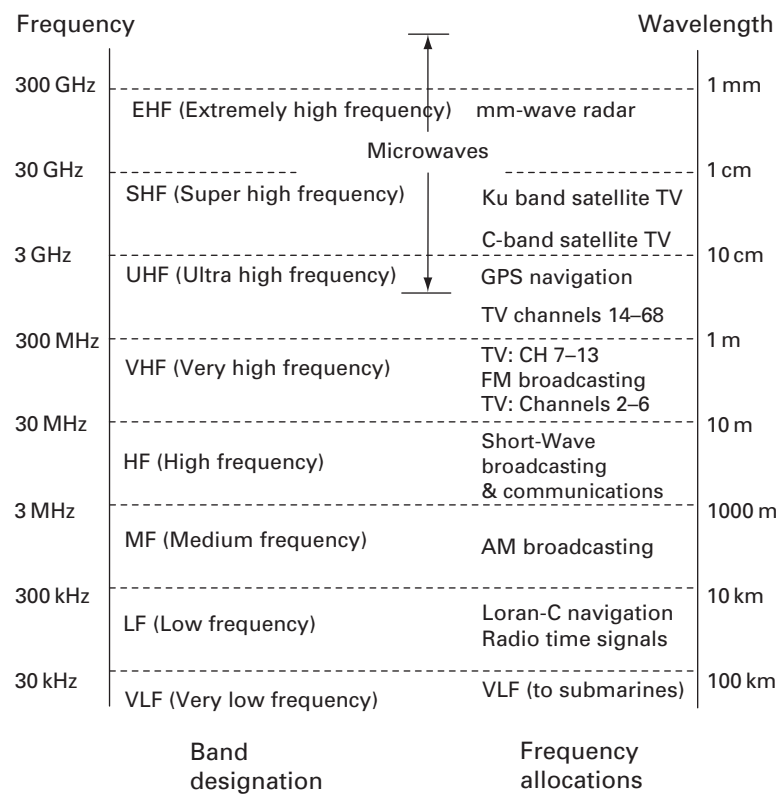
Introduction

Consider the magic of radio. Portable, even hand-held, short-wave transmitters can reach thousands of miles beyond the horizon. Tiny microwave transmitters riding on spacecraft return data from across the solar system. And all at the speed of light. Yet, before the late 1800s, there was nothing to suggest that telegraphy through empty space would be possible even with mighty dynamos, much less with insignificantly small and inexpensive devices. The Victorians could extrapolate from experience to imagine flight aboard a steam-powered mechanical bird or space travel in a scaled-up Chinese skyrocket. But what experience would have even hinted at *wireless* communication? The key to radio came from theoretical physics. Maxwell consolidated the known laws of electricity and magnetism and added the famous displacement current term, $\partial D/\partial t$. By virtue of this term, a changing electric field produces a magnetic field, just as Faraday had discovered that a changing magnetic field produces an electric field. Maxwell's equations predicted that *electromagnetic waves* can break away from the electric currents that generate them and propagate independently through empty space with the electric and magnetic field components of the wave constantly regenerating each other.

Maxwell's equations predict the velocity of these waves to be $1/\sqrt{\epsilon_0\mu_0}$ where the constants, ϵ_0 and μ_0 , can be determined by simple measurements of the forces between static electric charges and between current-carrying wires. The dramatic result is, of course, the experimentally-known speed of light, 3×10^8 m/sec. The electromagnetic nature of light is revealed. Hertz conducted a series of brilliant experiments in the 1880s in which he generated and detected electromagnetic waves with wavelengths very long compared to light. The utilization of Hertzian waves (electromagnetic waves) to transmit information developed hand-in-hand with the new science of electronics.

Where is radio today? AM radio, the pioneer broadcast service, still exists, along with FM, television and two-way communication. But radio now also includes digital broadcasting formats, radar, surveillance, navigation and broadcast satellites, cellular telephones, remote control devices, and wireless data communications. Applications of radio frequency (RF) technology outside

Figure 1.1. The radio spectrum.



radio include microwave heaters, medical imaging systems, and cable television. Radio occupies about eight decades of the electromagnetic spectrum, as shown in Figure 1.1.

1.1 RF circuits

The circuits discussed in this book generate, amplify, modulate, filter, demodulate, detect, and measure ac voltages and currents at radio frequencies. They are the blocks from which RF systems are designed. They scale up and down in both power and frequency. A six-section bandpass filter with a given passband shape, for example, might be large and water-cooled in one application but subminiature in another. Depending on the frequency, this filter might be made of sheet metal boxes and pipes, of solenoidal coils and capacitors, or of piezo-electric mechanical resonators, yet the underlying circuit design remains the same. A class-C amplifier circuit might be a small section of an integrated circuit for a wireless data link or the largest part of a multi-megawatt broadcast transmitter. Again, the design principles are the same.

1.2 Narrowband nature of RF signals

Note that most frequency allocations have small fractional bandwidths, i.e., the bandwidths are small compared to the center frequencies. The fractional bandwidth of the signal from any given transmitter is less than 10 percent – usually much less. It follows that the RF voltages throughout a radio system are very nearly sinusoidal. An otherwise purely sinusoidal RF “carrier” voltage¹ must be *modulated* (varied in some way) to transmit information. Every type of modulation (audio, video, pulse, digital coding, etc.) works by varying the amplitude and/or the phase of the sinusoidal RF wave, called the “carrier” wave. An unmodulated carrier has only infinitesimal bandwidth; it is a pure spectral line. Modulation always broadens the line into a spectral band, but the energy clusters around the carrier frequency. Oscilloscope traces of the RF voltages in a transmitter or on a transmission line or antenna are therefore nearly sinusoidal. When modulation is present, the amplitude and/or phase of the sinusoid changes but only over many cycles. Because of this narrowband characteristic, elementary sine wave ac circuit analysis serves for most RF work.

1.3 AC circuit analysis – a brief review

The standard method for ac circuit analysis that treats voltages and currents in linear networks is based on the linearity of the circuit elements: inductors, capacitors, resistors, etc. When a sinusoidal voltage or current generator drives a circuit made of linear elements, the resulting steady-state voltages and currents will all be perfectly sinusoidal and will have the same frequency as the generator. Normally we find the response (voltage and current amplitudes and phases) of driven ac circuits by a mathematical artifice. We replace the given sinusoidal generator by a hypothetical generator whose time dependence is $e^{j\omega t}$ rather than $\cos(\omega t)$ or $\sin(\omega t)$. This source function has both a real and an imaginary part since $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$. Such a nonphysical (because it is complex) source leads to a nonphysical (complex) solution. But the real and imaginary parts of the solution are separately good physical solutions that correspond respectively to the real and imaginary parts of the complex source. The value of this seemingly indirect method of solution is that the substitution of the complex source converts the set of linear *differential* equations into a set of easily solved linear *algebraic* equations. When the circuit has a simple topology, as is often the case, it can be reduced to a single loop by combining obvious series and parallel branches. Many computer programs are available to

¹ There is no low-frequency limit for radio waves but the wavelengths corresponding to audio frequencies, hundreds to thousands of kilometers, make it inefficient to connect an audio amplifier directly to an antenna of reasonable size. Instead, the information is impressed on a carrier wave whose wavelength is compatible with practical antennas.

find the currents and voltages in complicated ac circuits. Most versions of SPICE will do this steady-state ac analysis (which is much simpler than the transient analysis which is their primary function). Special linear ac analysis programs for RF and microwave work such as Agilent’s ADS and MMICAD include circuit models for strip lines, waveguides, and other RF components. You can write your own program to analyze ladder networks (see Problem 1.3) and to analyze most filters and matching networks.

1.4 Impedance and admittance

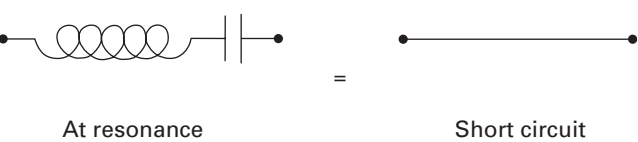
The coefficients in the algebraic circuit equations are functions of the complex *impedances* (V/I), or *admittances* (I/V), of the RLC elements. The voltage across an inductor is LdI/dt . If the current is $I_0e^{j\omega t}$, then the voltage is $(j\omega L)I_0e^{j\omega t}$. The impedance and admittance of an inductor are therefore respectively $j\omega L$ and $1/(j\omega L)$. The current into a capacitor is CdV/dt , so its impedance and admittance are $1/(j\omega C)$ and $j\omega C$. The impedance and admittance of a resistor are just R and $1/R$. Elements in series have the same current so their total impedance is the sum of their separate impedances. Elements in parallel have the same voltage so their total admittance is the sum of their separate admittances. The real and imaginary parts of impedance are called *resistance* and *reactance* while the real and imaginary parts of admittance (the reciprocal of impedance) are called *conductance* and *susceptance*.

1.5 Series resonance

A capacitor and inductor in series have an impedance $Z_s = j\omega L + 1/(j\omega C)$. This can be written as $Z_s = j(L/\omega)(\omega^2 - 1/[LC])$, so the impedance is zero when the (angular) frequency is $1/\sqrt{LC}$. At this *resonant frequency*, the series LC circuit is a perfect *short circuit* (Figure 1.2). Equal voltages are developed across the inductor and capacitor but they have opposite signs and the net voltage drop is zero.

At resonance and in the steady state there is no transfer of energy in or out of this combination. (Since the overall voltage is always zero, the power, IV , is always zero.) However, the circuit does contain *stored* energy which simply sloshes back and forth between the inductor and the capacitor. Note that this circuit, by itself, is a simple bandpass filter.

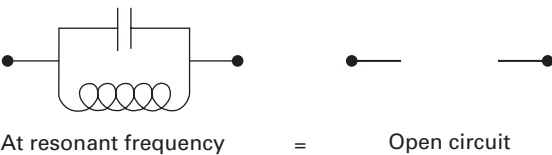
Figure 1.2. Series-resonant LC circuit.



1.6 Parallel resonance

A capacitor and an inductor in parallel have an admittance $Y_p = j\omega C + 1/(j\omega L)$ which is zero when the (angular) frequency is $1/\sqrt{LC}$. At this resonant frequency, the parallel LC circuit is a perfect *open circuit* (Figure 1.3) – a simple bandstop filter. Like the series LC circuit, the parallel LC circuit stores a fixed quantity of energy for a given applied voltage. These two simple combinations are important building blocks in RF engineering.

Figure 1.3. Parallel-resonant LC circuit.



1.7 Nonlinear circuits

Many important RF circuits, including mixers, modulators, and detectors, are based on nonlinear circuit elements such as diodes and saturated transistors used as switches. Here we cannot use the linear $e^{j\omega t}$ analysis but must use time-domain analysis. Usually the nonlinear elements can be replaced by simple models to explain the circuit operation. Full computer modeling can be used for accurate circuit simulations.

Problems

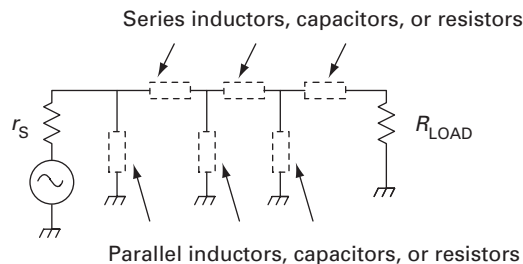
- Problem 1.1.** A generator has a source resistance r_S and an open circuit rms voltage V_0 . Show that the maximum power available from the generator is given by $P_{\max} = V_0^2/(4r_S)$ and that this maximum power will be delivered when the load resistance, R_L , is equal to the source resistance, r_S .
- Problem 1.2.** A passive network, for example a circuit composed of resistors, inductors, and capacitors, is placed between a generator with source resistance r_S and a load resistor, R_L . The *power response* of the network (with respect to these resistances) is defined as the fraction of the generator’s maximum available power that reaches the load. If the network is *lossless*, that is, contains no resistors or other dissipative elements, its power response function can be found in terms of the impedance, $Z_{\text{in}}(\omega) = R(\omega) + jX(\omega)$, seen looking into the network with the load connected. Show that the expression for the power response of the lossless network is given by

$$P(\omega) = \frac{4r_S R(\omega)}{(R(\omega) + r_S)^2 + X(\omega)^2}$$

where $R = \text{Re}(Z_{\text{in}})$ and $X = \text{Im}(Z_{\text{in}})$.

Problem 1.3. Most filters and matching networks take the form of the ladder network shown below.

Ladder network topology.



Write a program whose input data is the series and shunt circuit elements and whose output is the power response as defined in Problem 1.2.

Hints: One approach is to begin from the load resistor and calculate the input impedance as the elements are added, one by one. When all the elements are in place, the formula in Problem 1.2 gives the power response – as long as the load resistor is the only resistor. The process is repeated for every desired frequency.

A better approach, which is no more complicated and which allows resistors, is the following: Assume a current of $1 + j0$ ampere is flowing into the load resistor. The voltage at this point is therefore $R_L + j0$ volts. Move to the left one element. If this is a series element, the current is unchanged but the voltage is higher by IZ where Z is the impedance of the series element. If the element is a shunt element, the voltage remains the same but the input current is increased by IY where Y is the admittance of the shunt element. Continue adding elements, one at a time, updating the current and voltage. When all the elements are accounted for, you have the input voltage and current and could calculate the total input impedance of the network terminated by the load resistor. Instead, however, take one more step and treat the source resistance, r_s , as just another series impedance. This gives you the voltage of the source generator, from which you can calculate the maximum power available from the source. Since you already know the power delivered to the load, $(1)^2 R_L$, you can find the power response. Repeat this process for every desired frequency.

The ladder elements (and, optionally, the start frequency, stop frequency, frequency increment, and source and load resistances) can be treated as data, that is, they can be located together in a block of program statements or in a file so they can be changed easily. For now, the program only needs to deal with six element types: series and parallel inductors, capacitors, and resistors. Each element in the circuit file must therefore have an identifier such as “PL”, “SL”, “PC”, “SC”, “PR”, and “SR” or 1, 2, 3, 4, 5, 6, or whatever, plus the value of the component in henrys, farads, or ohms. Organize the circuit file so that it begins with the element closest to R_L and ends with some identifier such as “EOF” (for “End Of File”) or some distinctive number.

An example program, which produces both tabular and graphical output, is shown below, written in MATLAB, which produces particularly compact and readable code. The input data (included as program statements) is for the circuit shown below, of an LC network designed to connect a 50-ohm load to a 1000-ohm source. You will find this, or your own equivalent program, to be a useful tool when designing matching networks and filters. In the problems for Chapters 4, 10, 14, and 17, the program will be enhanced to plot phase response and to handle transmission lines, transformers, and transistors, making it a powerful design tool.

```
%MATLAB program to solve ladder networks
%Problem 1.3 in "Radio-Frequency Electronics"
%Save this file as "ladder.m" and run by typing "ladder" in command window
%INPUT DATA (circuit components from load end; 'SL' is series inductor, %etc.)
%_____
ckt={'SL',23.1e-6,'PC',463e-12,'EOF'};% 'EOF' terminates list
Rload=50;Rsource=1000;startfreq=1e6;endfreq=2e6;freqstep=5e4;
%_____

f=(startfreq:freqstep:endfreq);% frequency loop
w=2*pi.*f;% w is angular frequency
I=ones(size(w));V=ones(size(w))*Rload;% set up arrays for input I(f) and V(f)
ckt_index=0;morecompsflag=1;
while morecompsflag==1% loop through string of components
  ckt_index=ckt_index+1;% ckt_index prepared for next item in list
  component=ckt{ckt_index};
  morecompsflag=1-strcmp(component,'EOF');% zero after last component

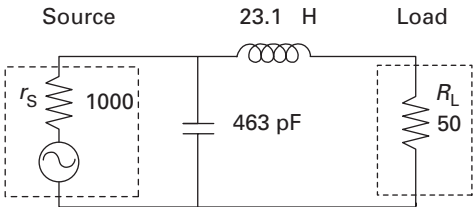
  if strcmp(component,'PC')==1
    ckt_index=ckt_index+1;capacitance=ckt{ckt_index};
    I=I+V.*(1j.*w.*capacitance);
  elseif strcmp(component,'SC')==1
    ckt_index=ckt_index+1;capacitance=ckt{ckt_index};
    V=V+I./(1j.*w.*capacitance);
  elseif strcmp(component,'PL')==1
    ckt_index=ckt_index+1;inductance=ckt{ckt_index};
    I=I+V./(1j.*w.*inductance);
  elseif strcmp(component,'SL')==1
    ckt_index=ckt_index+1;inductance=ckt{ckt_index};
    V=V+I.*(1j.*w.*inductance);
  elseif strcmp(component,'PR')==1
    ckt_index=ckt_index+1;resistance=ckt{ckt_index};
    I=I+V/resistance;
  elseif strcmp(component,'SR')==1
    ckt_index=ckt_index+1;resistance=ckt{ckt_index};
    V=V+I*resistance;
  end%components loop

end%frequency loop

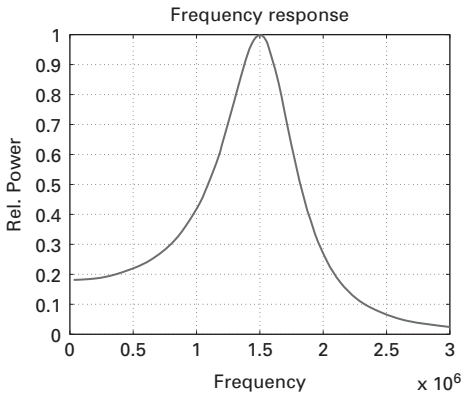
Z=V./I;V=V+I.*Rsource;
frac=Rload./((abs(V).^2)/(4.*Rsource));
db=10/log(10)*log(frac);
heading='freq(MHz) frac dB'% print heading in command window
A=[(1E-6*f)' frac' db] % print table of data in command window
plot(f,db);% graph the data
grid;xlabel('Frequency');ylabel('dB');title('Frequency response');
```

The circuit corresponding to the input data statements in the example program above is shown below, together with the analysis results produced by the program.

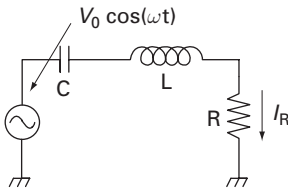
Example circuit.



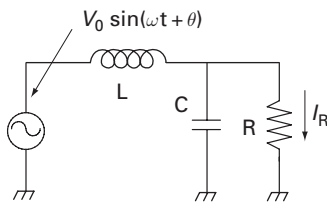
Analysis results.



Problem 1.4. (AC circuit analysis review problem.) For the circuit in the figure, derive an expression for $I_R(t)$. Use a complex source voltage, $V_0 e^{j\omega t}$, the real part of which is $V_0 \cos(\omega t)$. The impedances of C, L, and R are $(j\omega C)^{-1}$, $j\omega L$, and R, respectively. Find the complex current through the resistor. $I_R(t)$ will be the real part of this complex current.



Problem 1.5. (More AC circuit analysis review.) For the circuit in the figure, derive an expression for $I_R(t)$. Note that the source voltage, $V_0\sin(\omega t + \theta)$, is equal to the *imaginary* part of $V_0e^{j(\omega t + \theta)}$. Therefore, if we take the complex voltage to be $V_0e^{j(\omega t + \theta)}$, $I_R(t)$ will be the imaginary part of the complex current through R. Alternatively, you can let the complex voltage source have the value $-jV_0e^{j(\omega t + \theta)}$, the real part of which is $V_0\sin(\omega t)$. With this source, $I_R(t)$ is the real part of the complex current through R.



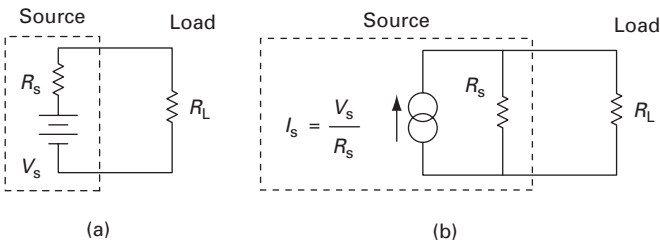
CHAPTER

2

Impedance matching

Matching normally means the use of a lossless (nonresistive) network between a signal source and a load in order to maximize the power transferred to the load. This presupposes that the source is not capable of supplying infinite power, i.e., that the source is not just an ideal voltage generator or an ideal current generator. Rather, the source is assumed to be an ideal voltage generator in series with a source impedance, i.e., a Thévenin equivalent circuit, or an ideal current generator in parallel with a source admittance, a Norton equivalent circuit. Note that these equivalent circuits are themselves equivalent; each can be converted into the form of the other. An antenna that is feeding a receiver is an example of an ac signal source connected to a load. Figure 2.1 shows the simplest situation, a dc generator driving a resistive load. The generator is represented in Thévenin style (a) and in Norton style (b).

Figure 2.1. DC generator driving a resistive load, Equivalent circuits.



You can see the equivalence by inspection: the generators have the same open-circuit voltage and the same short-circuit current. Maximum power is transferred when the load resistance is made equal to the source resistance. You can show this by differentiating the expression for the power, $P_{\text{load}} = [V_s R_L / (R_L + R_s)]^2 / R_L$. Figure 2.2 plots the relative transferred power ($P_{\text{wr}}/\text{MaxPwr}$) as a function of the normalized load resistance ($r = R_L/R_s$). In (a) the scales are linear and in (b) the scales are logarithmic so the relative power is expressed in dB. Note that R_L can differ by a factor of 10 from R_s and the power transferred is still 33% of the maximum value.