1

Synopsis

The book consists of three major themes. Any one of the three components can be read without many gaps in the analysis.

1 The introductory chapters are primarily intended for readers who are unfamiliar with the fundamental concepts of finance. The principles and mathematical expressions for debt instruments, which are analyzed in later chapters, are reviewed in Chapter 2, 3, and 4. Options are briefly discussed and the Black–Scholes option theory is given a path integral formulation.

2 A major subject matter of the book is the theory of coupon bonds. A quantum field theory of the bond forward interest rates \( f(t, x) \) is developed in Chapter 5 and forms a core chapter. It provides a model for the study of coupon and zero coupon bonds. Many of the derivations in later chapters are based on the quantum finance model of bond forward interest rates.

3 The quantum finance formulation of Libor interest rates is another major topic. The Libor Market Model is formulated in Chapter 6; the nonlinear Libor forward interest rates \( f_L(t, x) \) that it is based upon are transformed into logarithmic Libor interest rates \( \phi(t, x) \). In Chapter 7 some empirical properties of the Libor Market Model are studied and in Chapter 8 the prices of Libor options are obtained by using techniques of quantum field theory. A derivation of the Libor Market Model’s nonlinear drift term is given in Chapter 15, based on the Libor Hamiltonian and state space of \( \phi(t, x) \).

The inter-connection of the various chapters is shown in the flowchart given overleaf.
Chapter dependency flowchart

1. Interest rates & coupon bonds
2. Options & option theory
3. Interest rates & coupon bond options
4. Quantum field theory of bond forward interest rates
5. Libor Market Model of interest rates
6. Numeraires for bond forward interest rates
7. Empirical analysis of forward interest rates
8. Libor Market Model of interest rate options
9. Empirical analysis of interest rate caps
10. American options for coupon bonds & interest rates
11. Correlation of coupon bond options
12. Empirical analysis of interest rate caps
13. Correlation of coupon bond options
14. Hedging interest rate options
15. Interest rate Hamiltonian & option theory
16. American options for coupon bonds & interest rates
17. Hamiltonian derivation of coupon bond options

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Excerpt
More information
Interest rates, coupon bonds, and their derivatives are the main instruments of the debt markets, which constitute well over 60% of the entire capital markets. A brief discussion locates the debt markets in the general framework of finance and points to the growing importance of the debt markets in the global economy. Interest rates are a measure of the returns on cash deposits, whereas coupon bonds are a measure of the present value of future cash flows. From this intuitive and apparently simple idea flow all the various definitions of interest rates and coupon bonds. The fundamental concept of forward interest rates that describe the bond market is introduced. The interest rate markets are driven by Libor and Euribor; these two instruments are defined and a few of their important features are discussed.

2.1 Introduction

Finance is the discipline that studies the borrowing, lending, and investing of money capital. The main form of money capital is paper issued by various governments and private organizations, which includes corporations and individuals. The three pillars of finance are equity, debt, and foreign exchange and are the basis of all financial instruments. Financial markets, collectively known as the capital markets, trade in these instruments [31].

Capital in economics represents the collection of productive assets required for carrying out economic activities. Financial ‘paper’ is not merely ordinary paper, but, rather, the preferred form of money capital that is used for representing value: a value based not on how it has been generated but, rather, on its present day and future value in the capital markets – and in economic activity in general. Money capital carries an intrinsic risk since expectations of what can be realized in the present and future are always subject to uncertainties inherent in any form of forecasting. Unlike traditional economies – where finance is a passive force and auxiliary to the real economy – the capital markets today are one of the most
Interest rates and coupon bonds

powerful and dynamic components of the modern global economy and a potent force for economic growth and expansion. The capital markets are expected to become increasingly important with the increasing inter-connections of the global economy. However, there is a downside to the increasing importance of finance. Due to the inherently uncertain and random nature of money capital, the capital markets have an uncontrollable and unpredictable component that can wreak havoc on the real economy. Advanced theories of money capital are required for creating financial instruments that can be used for managing risk and reducing instabilities of the capital markets – and thus help to tame the destabilizing spikes, bubbles, meltdowns, and crashes of the financial markets.

Money capital comes in many forms with the main three forms being stocks and shares of companies, debt instruments, and cash of various currencies. Money, or more precisely money capital that is seeking returns from the economy, is a dynamic quantity – with opportunities for money to yield profit constantly changing with time. Interest rates reflect the relation of the value of money with time and quantify the time-dependent and dynamic aspect of money.

Debtors pay a return – the amount depending on the interest rate – to the providers of credit. Debt and surplus capital are two sides of the same coin, since debt for one party is the complement of the credit that the other party has provided. The world’s debt market is an expression of the net savings that the world economy has generated.

One needs sophisticated and effective models of interest rates to manage and expand the net global savings so as to maximize its returns. It is from this perspective – of optimizing the management of the international debt markets – that quantum finance models of interest rates and coupon bonds have been developed and form the main content of this book.

Optimizing the management of international liquidity will result in better allocation and returns on investments as well as create conditions for the prosperity of society at large. In particular, managing flows of international capital to developing and other higher risk economies, using customized financial instruments, would result in a larger fraction of mankind having access to investment capital – leading to the betterment of people’s lives and wealth.

2.2 Expanding global money capital

The nature of finance has undergone a radical change in the last 30 years, with the financial sector of the economy becoming increasingly more important. There are many indicators that point to this fundamental change in the financial superstructure of economically advanced countries.
2.2 Expanding global money capital

In 2006, the world economy generated about US$65 trillion worth of goods and services, of which raw materials (taken directly from nature) constituted about two-thirds (US$43 trillion) of the total value. The remaining one-third (US$22 trillion) was the value added by human labor. For example, in 2007 – based on daily production of 85 million barrels (about 31 billion barrels a year) – the sale of petroleum at around US$100 per barrel generated a revenue worth about US$3.1 trillion, with a large part of this revenue being invested in the capital markets.

In general, a substantial fraction of the net profit generated by the world economy as well as the savings and net accumulated surplus capital of many individuals, organizations, and countries is held in the form of money capital. In particular, cash rich oil and gas producers as well as East Asian economies (with substantial national reserves) have created ‘sovereign funds’ for investing their surplus in the capital markets. Money capital is bound to be increasingly important; due to the enormous scale of the global economy and the net savings it generates, there is not enough gold or other precious commodities that can hold this value. Paper seems the only way to represent and store the generated global surplus value.

Risk management, based on models that quantify the degree of risk, allows many institutional investors to convert net savings into money capital. Better risk management instruments have drawn risk-averse investors, such as insurance companies and pension and sovereign funds amongst others, to place their assets in the capital markets, contributing to the current explosion of the money capital.

IMF estimates that in 2005 the total value of the stocks, bonds, and bank loans worldwide was about US$165 trillion. The global bond (debt) market’s share was close to US$104 trillion – by far the largest component of the global capital markets – accounting for over 63% of the total; banking credit in 2008 amounted to about US$23 trillion. In 2005, cross-border money flows (stocks, bonds, real estate, and so on) amounted to about US$6 trillion. The foreign exchange markets have also undergone a phenomenal increase, with about US$3 trillion being traded daily in 2007.

In 2007, global stocks were worth about US$56 trillion – about 35% of the capital markets – with the US and Eurozone each accounting for US$18 trillion and the rest of world accounting for US$20 trillion. The US capital markets had a total worth of US$42 trillion of which US$24 trillion was in the bond (debt) market and US$18 trillion in stocks (equity).

In 2006 global debt issuance rose to a record US$6.9 trillion with the global syndicated loan volume exceeding US$3.2 trillion. During the period of 2000–2005 nonfinancial companies worldwide issued $19.3 trillion worth of debt, in the form of corporate medium-term notes (MTNs), with the biggest issuers being the automotive industry, issuing 70 MTNs worth US$4.54 trillion followed by insurance companies issuing 26 MTNs worth US$4.49 trillion.
Market liquidity and risk management – two of the current lynch pins of the financial system – require the participation of speculators. A speculator, who can be an individual, a corporation, or a financial institution, makes an estimate of the future and if right profits and if wrong loses. Speculating on the capital markets usually means taking high risks since the future is always uncertain. Speculative positions create market liquidity as well as provide a mechanism for sharing risk, which, for example, a (manufacturing) business, not having in-house expertise in risk management, may want to dispense with.

Although the term ‘speculator’, to some, carries a negative connotation, the market needs both informed and uninformed, traders. Speculators are not inside traders but, instead, should be called uninformed traders, in contrast to informed traders who buy or sell a specific instrument. If only informed traders were market players, any move to buy or sell would lead to slippage in the offered prices, leading to the informed traders being held to ransom by the market. Uninformed traders provide the ‘veil’, a background of ‘noise’, that allows informed traders to enter the market without causing major slippages in prices. One needs both the informed and uninformed traders for the market to function efficiently.

### 2.2.1 Securitization

Another reason for the expansion of the capital markets is that financial engineering has created instruments that allow diverse forms of future cash flows to be used for issuing vast amounts of securitized debt. Securitization is the consolidation and structuring of cash-flow producing financial instruments, called asset-backed securities, that can then be traded in the capital markets. For example, the securitization of cash flows, such as mortgage payments and rentals, has allowed these to be traded in the capital markets – adding to the depth and liquidity of the capital markets.

Securitization is a relatively new concept in finance, having gained acceptance only over the last 20 years. Securitized debt has grown in the issuance of new loans and covers such diverse sectors as residential mortgages, commercial real estate, corporate loans, auto loans, student loans, and so on. In 1990, just 10% of mortgages in the United States were securitized, compared to 70% in 2007. It is estimated that by the middle of 2008 there were asset-backed securities worth US$10.2 trillion in the US and US$2.3 trillion in Europe. In 2007, new issues of asset-backed securities amounted to US$3.5 trillion in the US and US$650 billion in Europe. Securitization has had a major setback due to the 2008 US economic crisis, with the issuance of new mortgage-backed securities dropping by almost 85% in the first half of 2008 compared to the same period in 2007. The 2008 subprime crisis...
2.2 Expanding global money capital

Crisis in US home mortgages is claimed by some critics to be a negative example of securitization; this is not entirely correct and is discussed in Appendix B.2.

The lack of securitization can be a formidable barrier to economic development. It has been argued by Soto [91] that the securitization of third world developing countries’ real estate, and of property in general, into tradable financial instruments could release vast amounts of capital. It was estimated that, in 1997, capital worth about US$9.3 trillion was locked up due to lack of securitization, an amount twice of the then total US public debt [91]. This ‘dead capital’, if securitized, could play a major role in the economic growth of the developing countries. Mortgages are fungible (a commodity that is freely interchangeable with another in satisfying an obligation) only in countries where the rule of law is well established and the legal system guarantees ownership. To securitize real estate assets in third world countries, hence, requires a stable political system that is accountable and relatively free from corruption. For these reasons, third world countries will have to overcome many major hurdles before they are in a position to create mortgage and other asset-backed securities, which would in turn release presently inert capital.

2.2.2 Profitability of the financial sector

At present, the rate of return of the financial sector and services in general is about 20% for the advanced economies of the US, Europe, and Japan – much higher than the 8–10% returns from manufacturing. For example, from 2002 to 2006 five leading US investment banks – Goldman Sachs, Merrill Lynch, Morgan Stanley, Lehman Brothers, and Bears Stern – had an average return on equity of about 22%, amounting to US$30 billion – rivaling returns for such profitable industrial sectors as pharmaceuticals and energy.

The increasing volume of financial money capital reflects the overall expansion of the world economy, with vast amounts of surplus finding its way to the capital markets. The high rates of return from finance capital is one of the reasons for the immense infusion of savings and other assets into the global capital markets. The higher rate of return is thought to be due to the finance industry not being as mature as manufacturing and is taken to indicate a shift of the global economy to a new regime. There is, however, a contrarian view that the high returns from finance are primarily the result of the formation of an asset bubble – and hence intrinsically unstable and not sustainable.

The September–October 2008 global financial meltdown seems to provide strong evidence in support of the contrarian view. By the end of the September–October

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1 The rate of return on manufacturing is thought to be low due to the increasingly large capital investment required for setting up and upgrading modern industries.
2008 US financial meltdown, all the five US investment banks had ceased to exist – with Goldman Sachs and Morgan Stanley having converted themselves into bank holding companies. The consistently high returns of 22% from 2002 to 2006 shown by the five investment banks, with hind sight, is seen to completely coincide with the formation and expansion of the US subprime mortgage loans’ financial bubble and may have simply been a result of this bubble.

Finance may still give a return higher than manufacturing due to the creation of new financial instruments, but in the current climate of financial turmoil and contraction it will be a while before such innovations find acceptance in the capital markets.

2.3 New centers of global finance

The United States (US) capital markets, since 1945, have been the most important component of the global capital markets, playing a central role in shaping and developing the international financial system. In Appendix B, the structure of the US debt markets is briefly reviewed.

The US is losing its pre- eminent position in the global capital markets due to the following reasons: (a) massive financial losses caused by the 2008 economic meltdown – in the US stock market, for example stocks on the Dow Jones lost 34% of their value in 2008 (the largest drop since 1931), and in the bankruptcy of major US financial institutions; (b) the rise of other capital markets and centers of wealth. The year 2007 saw a sea change in the distribution of global wealth. Largely due to the rise of China and India and investments by oil and gas producing countries, for the first time since the Second World War (1945) London displaced New York to become the center of the global capital markets. Over 40% of the world’s foreign equities were traded in London, more than New York. Over 30% of the world’s foreign currency trading took place in London, being larger than New York and Tokyo combined.

The US capital markets, in 2007, were worth US$42 trillion of which US$7.3 trillion was owned by foreigners, namely 17%, who also held 44% of the US national debt. In contrast to both New York and Tokyo, which depend largely on their domestic and East Asian markets, 80% of London’s business comes from international sources, spread widely over many regions and countries.

The shift away from a US-centered global financial system can also be seen in the emergence of the Euro as an international reserve currency, as can be seen from Table 2.1. The Euro was introduced in 1999 and by 2008 had appreciated over 50% against the US Dollar. International reserves are now held in both the US Dollar and the Euro, with estimates that by 2010 about 34% will be held in Euro and 54% in the US Dollar, in contrast to 2000 when 71% of world reserves were held in
2.4 Interest rates

Table 2.1 International reserve in Euros and US Dollars, and the projected currency distribution of these reserves by the year 2010.

<table>
<thead>
<tr>
<th>Currency of international reserve</th>
<th>2000</th>
<th>2007</th>
<th>2010 (projected)</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Dollar $</td>
<td>71%</td>
<td>63%</td>
<td>54%</td>
</tr>
<tr>
<td>Euro €</td>
<td>18%</td>
<td>26%</td>
<td>34%</td>
</tr>
</tbody>
</table>

US Dollars. Some economists have predicted that, by as early as 2015, the Euro may overtake the US Dollar as the main international reserve currency provided two conditions hold: (a) more countries, including the UK, join the Eurozone countries and (b) the 2008 US economic crisis causes a deterioration in the value of the US Dollar.

With the increasing pace of globalization, one can expect the emergence of new international centers of finance in Shanghai, Hong Kong, Singapore, Mumbai, Dubai, Sao Paolo, and so on.

2.4 Interest rates

Interest rates, in essence, represent the interplay of time with economic activity, money capital, and real (tangible) assets.

The money form of capital represents real productive assets of society that can erode over time; furthermore, other factors like inflation, currency devaluations, new technologies, and so on make the value represented by financial assets a variable quantity that responds to changing circumstances. Financial assets represent the ability to initiate or facilitate economic activities, opportunities for which are tied to many social factors. For these and many other reasons, the effective value of money is strongly dependent on time.

How does one estimate the time value of money? From economic theory, the sum total of all the endogenous and exogenous factors that affect the time value of money are contained in the interest rates that one earns on cash deposits or on Treasury Bonds. Money invested in other financial instruments is more complicated to value as risk premiums are involved, perceptions of which differ between investors. Ultimately, the time value of money involves discounting expected future cash flows from bonds to obtain its present-day value; or, inversely, compounding present-day cash deposits for obtaining its expected future value.

Interest rates fix the cost of borrowing capital, the ‘cost’ of money, and are determined by both, the supply and demand for money – which depend on the prevailing interest rates – and by the macroeconomic policies of central banks.
Central banks would, ideally, like to hold down inflation while at the same time engendering economic growth; central banks balance inflation against the rate of economic growth by regulating the supply of money. One of the major tools for influencing the supply and demand for money is by setting interest rates.

Market forces of supply and demand and central banks’ setting of interest rates are in a state of constant tension. Market forces sometimes force the central banks to change the interest rates so as to bring them in line with the market; at other times, central banks intervene by changing the interest rates and thus affecting the market’s demand for money.

The concepts of discounting and compounding are fundamental to finance. However, contrary to what one intuitively expects, the relation turns out to be far more complex than discounting and compounding simply being the inverse of each other. The different forms of compounding (discounting) present (future) cash flows provide different ways by which interest rates are defined.

Consider the future value of a fixed deposit that is rolled over continuously; a constant interest rate leads to an exponential compounding of the value of the initial fixed deposit. Discounting, on the other hand, is the procedure that yields the present-day value of a fixed future cash flow and is exponentially smaller for constant interest rates. In essence, all measures of interest rates arise by either discounting expected future cash flows to obtain their present-day value or by compounding the present-day value of fixed deposits to obtain the value of future cash flows.

### 2.5 Three definitions of interest rates

The following procedures for defining interest rates are widely used in the financial markets, with an interest rate ‘yield curve’ for each case.

- Returns on cash deposits using simple interest rates. This is the basis of defining Libor and Euribor, the two fundamental market determined interest rates.
- Discrete compounding of cash deposits and discrete discounting of bonds. This procedure is the basis for the definition of the zero coupon yield curve (ZCYC), which is fundamental to the interest rates and bond markets.
- Instantaneous compounding and discounting future cash flows. This definition leads to the concept of instantaneous forward interest rates, the main theoretical construct of the bond market.

To simplify the discussion of the central concepts, all interest rates for now are taken to be constant. The more complex generalizations of these concepts are discussed in the later sections.