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978-0-521-88907-0 - Price and Quantity Index Numbers: Models for Measuring Aggregate Change and Difference

Bert M. Balk

Excerpt

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Price Indices through History

1.1 Introduction

Where people trade with each other, there are prices involved – either explicitly, when for the provision of goods or services has to be paid with money, or implicitly, when there is payment in kind. Over the course of history people have expressed concerns about fluctuations of prices, especially of daily necessities such as bread. Also, though to a lesser extent, regional price differences were a source of concern. Since sharp price fluctuations easily led to social unrest, authorities considered it their task to regulate prices. And price regulation presupposes price measurement. Though the systematic measurement of price changes and price differences had to wait until the emergence of official (national) statistical agencies around the turn of the 20th century, there are numerous examples of individuals and authorities who were engaged in price measurement and/or regulation.

A rather famous example is the *Edict on Maximum Prices* (Edictum de Pretiis Rerum Venalium), issued by the Roman emperor Diocletianus in the year 301. Along with a coinage reform, the Edict declared maximum prices for more than a thousand commodities, including food, clothing, freight charges, and wages. This turned out to be not very helpful, because the continued money supply increased inflation, and the maximum prices were apparently set too low.

An interesting case is the regulation on bread prices that was issued by the municipal council of Gdańsk in 1433 (see Kula 1986, chapter 8). Here the price of bread was fixed through time, while fluctuations in the supply of corn were to some extent accommodated by letting the weight of a loaf

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vary. Technically speaking, the unit of measurement was allowed to vary. Kula remarks that

The system of a constant price for bread coupled with a variable weight for the loaf must have accorded well with the pre-industrial mentality as well as with the social situation that obtained in urban markets, or else it would hardly have been found throughout Europe.

He goes on to observe that

Its ideological basis was St. Thomas's theory of the just price – just in the sense of being invariable, its invariability being dictated above all by its usefulness to man. The practice thus constituted a tolerable compromise between the theory of invariable price and the requirements of the commodity market, while preserving as constant the quantity of money paid. Technically, it would seem this method was favored by the frequent lack of small change and the limited divisibility of coinage.

In our view, however, the paramount importance of this system lay in the political sphere. For it made it possible to alter the price of the most basic article of diet in a manner that was not obvious, and therefore less offensive, to the urban plebs, whose wrath was often feared by the bakers' guild as well as by the municipal authorities and their feudal overlords. . . . It is thus reasonable to look upon the whole process, within limits, as a safety-valve or a buffer against social reaction to market developments.

In his historic overview entitled "Digressions concerning the variations in the value of silver during the course of the four last centuries," which is part of chapter 11 of book one of *An Enquiry into the Nature and Causes of the Wealth of Nations*, Adam Smith (1776) quotes numerous individuals and authorities who were engaged in price measurement and/or regulation. Among those Bishop Fleetwood figures as one of "the two authors who seem to have collected, with the greatest diligence and fidelity, the prices of things in ancient times."

Indeed, according to Edgeworth (1925a), "the earliest treatise on index numbers and one of the best" is Bishop William Fleetwood's *Chronicon Preciosum; Or an Account of English Money, the Price of Corn and Other Commodities for the Last 600 Years*, the first edition of which was published in 1704. Edgeworth (1925a), Ferger (1946), and Kendall (1969) all provide the relevant details. Based on their accounts the story can be summarized as follows. A certain Oxford college was founded between 1440 and 1460, and one of its original statutes required a person, when admitted to fellowship, to swear to vacate it if coming into possession of a personal estate of more than £5 per annum. The question was whether, in the year 1700, a man might conscientiously take his oath even if he possessed a larger estate, seeing that the value of money had fallen in the meantime.

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Fleetwood rightly decided that “the Founder intended the same ease, and favour to those who should live in his college 260 years after his decease, as to those who lived in his own time.” To answer the question, Fleetwood executed an extensive inquiry into the course of prices over the past 600 (!) years. In particular he considered how much money would be required to buy £5 worth (at 1440/60 prices) of four commodities – corn, meat, drink, and cloth, these being then, apparently, the necessities of academic life. He came to the conclusion that for these four, respectively, the present value of £5 was £30, £30, “somewhat above £25,” and “somewhat less than £25.” “And therefore I can see no cause, why £28, or £30 per annum should now be accounted, a greater estate, than £5 was heretofore, betwixt 1440, and 1460.” The inference was that an income of £30 or less “may be enjoyed, with the same innocence and honesty, together with a Fellowship, according to the Founder’s will.”

Fleetwood thus had four items in his basket-of-goods. As he found, in each case, the decrease in the purchasing power of money to be of more or less the same magnitude, he was relieved of the necessity of averaging his four price relatives, or of considering their weights. His formulation of the problem, however, is strikingly modern. Fleetwood tried to determine the amount of money that would guarantee “the same ease and favour” as could be obtained with £5 in 1440/1460.

Similar concerns led the government of the State of Massachusetts in 1780 to issue bonds whose value was indexed by means of a so-called Tabular Standard (see Fisher 1913). The goal here was to terminate unrest among the soldiers fighting in the independence war. Apart from incidents like this, however, it took about 200 years before Fleetwood’s problem was rediscovered and its central importance recognized.

Although there has not yet been written a complete history of the development of price measurement, it is not the purpose of this chapter to remedy this. Such a project would require one or more separate volumes.¹ The more modest purpose of this chapter is to give an impression of the genesis of the main types of price index theory as well as the various formulas that will be discussed in more detail in the remainder of this book.

There exist a number of (short) surveys about the history of the subject. Fisher’s (1922) *The Making of Index Numbers* contains a separate historical

¹ Interesting material can be found in a number of recent reviews, such as Reinsdorf and Triplett (2008). The Boskin Commission Report (1996) gave rise to a lot of (historical) research. See the Spring 2006 issue of the *International Productivity Monitor* on this report’s impact on price measurement.

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appendix, entitled “Landmarks in the History of Index Numbers” and has historical remarks scattered throughout the book. Walsh (1932) reviews the history up to 1920. The *Bibliography on Index Numbers* (compiled by R. G. D. Allen and W. R. Buckland), issued in 1956 by the International Statistical Institute, contains a brief but useful general survey of the literature up to 1954. Also, the paper by Ruggles (1967), though focussed on international price comparisons, contains a lot of information about the historical development. Kendall’s (1969) essay on the early history of index numbers reviews the progress of the subject up to 1900. There is an interesting note on the origins of index numbers by Chance (1966). Diewert (1988) surveyed the (early) history of price index research under five distinct headings: the fixed basket approach, the statistical approach, the test approach, the Divisia approach, and the economic approach. More recently, a brief review of the history was provided by Persky (1998).

This chapter will highlight the main events in a more or less chronological order.² The notation used therefore deviates from the notation systems in the various sources and complies with modern standards.

In line with most of the literature it is assumed that there are N commodities, labelled as $1, \dots, N$, which are available through a number of consecutive time periods t (usually but not necessarily of equal length). The period t vector of prices will be denoted by $p^t \equiv (p_1^t, \dots, p_N^t)$, and the associated vector of quantities by $x^t \equiv (x_1^t, \dots, x_N^t)$. All the prices and quantities are assumed to be positive real numbers.³

A bilateral comparison concerns two periods, which may or may not be adjacent, and is carried out by means of a price and/or quantity index. In its most general form, a *bilateral price index* is a certain positive function $P(p, x, p', x')$ of $4N$ variables, two price vectors and two quantity vectors, which shows “appropriate behavior” with respect to the prices that are the subject of comparison. Likewise, a *bilateral quantity index* is another positive function $Q(p, x, p', x')$ of the same $4N$ variables, that shows “appropriate behavior” with respect to the quantities.

Let the periods to be compared be denoted by 0, called the *base period*, and 1, called the *comparison period*. Then $P(p^1, x^1, p^0, x^0)$ and

² More detailed discussions and biographies of the people involved can be accessed via the references.

³ The term *commodity* serves as a primitive term that can refer to goods as well as services, tightly or loosely defined. It is assumed that there are no new or disappearing commodities. It is also (tacitly) assumed that the commodities do not exhibit quality change, or that quality change has been accounted for by making appropriate adjustments to the prices or quantities. For the history of quality adjustment, see Banzhaf (2001).

$Q(p^1, x^1, p^0, x^0)$ are price and quantity index numbers, respectively, for period 1 relative to period 0. Put otherwise, an index number (outcome) is a particular realisation of an index (function). In literature and daily talk the distinction between index and index number is often blurred. Although it is important to keep this distinction in mind, in the interest of readability an index is usually presented in the form of an index number for a certain period 1 relative to another period 0. The suggestion therefore is that period 0 precedes period 1.

1.2 The Fathers

All historians agree that the first genuine price index was constructed by the French economist Dutot (1738).⁴ His computation can be formalized as

$$P^D(p^1, p^0) \equiv \frac{\sum_{n=1}^N p_n^1}{\sum_{n=1}^N p_n^0} = \frac{(1/N) \sum_{n=1}^N p_n^1}{(1/N) \sum_{n=1}^N p_n^0}, \quad (1.1)$$

Dutot's price index can, according to the rightmost part of (1.1), be conceived as a ratio of arithmetic averages of prices coming from the two periods. Either average could be viewed as measuring the price level of a period. Hence, Dutot's price index can also be conceived as a ratio of price levels.

Next comes the Italian, more precisely Istrian, economist Carli (1764).⁵ The price index he computed was a simple arithmetic average of price relatives,

$$P^C(p^1, p^0) \equiv (1/N) \sum_{n=1}^N \frac{p_n^1}{p_n^0}. \quad (1.2)$$

Young (1812) appears to be one of the first who recognized, although rather implicitly, the necessity of introducing weights into a price index, to reflect the fact that not all the commodities are equally important. His proposal could be interpreted as a generalization of the Dutot index, namely $\sum_{n=1}^N a_n p_n^1 / \sum_{n=1}^N a_n p_n^0$, where a_n is some (positive, real-valued) measure of the importance of commodity n ($n = 1, \dots, N$). Walsh (1932), however, interpreted Young as proposing the following price index,

$$P^Y(p^1, p^0; a) \equiv \frac{\sum_{n=1}^N a_n (p_n^1 / p_n^0)}{\sum_{n=1}^N a_n}, \quad (1.3)$$

⁴ On Dutot and his work see Mann (1936).
⁵ Details on Carli's life can be found at the website www.istrianet.org.

which can be considered a generalization of the Carli index.⁶ A rather realistic system of weights was proposed by Lowe (1823). He suggested

$$P^{Lo}(p^1, p^0; x^b) \equiv \frac{\sum_{n=1}^N p_n^1 x_n^b}{\sum_{n=1}^N p_n^0 x_n^b}, \quad (1.4)$$

where x_n^b was a (rough) estimate of the quantity of commodity n ($n = 1, \dots, N$) consumed during a certain period of time b . Such a system of weights was called a Tabular Standard. Lowe's index, then, compares the cost of the commodity basket (x_1^b, \dots, x_N^b) at the two periods 0 and 1.⁷ The Tabular Standard employed by the State of Massachusetts during 1780–6 had a simple structure and used only four commodities, namely “Five Bushels of Corn, Sixty-eight Pounds and four-seventh Parts of a Pound of Beef, Ten Pounds of Sheeps Wool, and Sixteen Pounds of Sole Leather” (see Fisher 1913).

In the second half of the 19th century the interest in the construction of price indices increased gradually. Jevons (1863) was a sort of pioneer.⁸ He introduced what later came to be called the geometric mean price index,

$$P^J(p^1, p^0) \equiv \prod_{n=1}^N \left(\frac{p_n^1}{p_n^0} \right)^{1/N}, \quad (1.5)$$

and argued why this mean should be preferred to other kinds of mean.

Jevons, like other authors of the decades to come, was primarily concerned with the measurement of a concept called “the value of money,” “the purchasing power of money,” “the general price level,” and all this in connection with fluctuations in the quantity of gold. Since he was of the opinion that a change on the part of gold affected the prices of all commodities equiproportionately, he thought the geometric mean of the price relatives to be the appropriate measure (see also Jevons 1865). Laspeyres (1864) opposed this view and advocated instead the Carli index (1.2).

⁶ But note that by choosing $a_n = p_n^0$ ($n = 1, \dots, N$) one gets the Dutot index.
⁷ Essentially the same idea was proposed in 1828 by Phillips, though Jastram (1951) interprets Phillips's idea as being identical to Paasche's index. In the context of producing annual index numbers, Lowe suggested keeping the quantities fixed during five years.
⁸ On Jevons see Fitzpatrick (1960), Aldrich (1992), and Maas (2001). Jevons was regarded by Fisher (1922, p. 459) as “the father of index numbers.” According to Walsh (1932) he “opened the theory of the subject.” Edgeworth (1925c), Kendall (1969), and Diewert (1988), however, regarded Lowe as “father.” Actually, Fleetwood could be considered as the real “father.”

In 1871, Drobisch discussed a number of alternatives, among which was the formula

$$P^U(p^1, x^1, p^0, x^0) \equiv \frac{\sum_{n=1}^N p_n^1 x_n^1 / \sum_{n=1}^N x_n^1}{\sum_{n=1}^N p_n^0 x_n^0 / \sum_{n=1}^N x_n^0}. \quad (1.6)$$

This formula has since then become known as the “unit value” index (hence the superscript U). It admits two interpretations: first, as a ratio of weighted arithmetic averages of prices, and, second, as a value index divided by a Dutot-type quantity index.

Laspeyres (1871) took up the issue again.⁹ He showed the inadequacy of the unit value index to measure price change – if prices do not change, that is, $p_n^1 = p_n^0$ for $n = 1, \dots, N$, then formula (1.6) can nevertheless deliver an outcome different from 1¹⁰ – and again strongly advocated the use of the Carli index (1.2). In the course of his argument, however, he proposed the formula¹¹

$$P^L(p^1, x^1, p^0, x^0) \equiv \frac{\sum_{n=1}^N p_n^1 x_n^0}{\sum_{n=1}^N p_n^0 x_n^0} \quad (1.7)$$

as being superior to the Carli index. However, since Laspeyres thought that the quantities that are necessary for the computation could not be determined accurately enough, he rejected formula (1.7) for practical purposes. Obviously he failed to notice the identity

$$\frac{\sum_{n=1}^N p_n^1 x_n^0}{\sum_{n=1}^N p_n^0 x_n^0} = \sum_{n=1}^N \frac{p_n^0 x_n^0}{\sum_{n'=1}^N p_{n'}^0 x_{n'}^0} \frac{p_n^1}{p_n^0}, \quad (1.8)$$

that is, Laspeyres’ price index can be written as a weighted arithmetic mean of price relatives, with the base period value shares as weights. Thus knowledge of the base period quantities is not necessary. Only the value shares do matter. Irving Fisher was the first to recognize the operational significance of the identity (1.8). It is mainly because of this identity that the Laspeyres price index (1.7) gained such a widespread acceptance in later years.¹²

⁹ For biographical details about Laspeyres one should consult Rinne (1981). This paper is accompanied by a reprint of Laspeyres’ 1871 publication. See also Diewert (1987b) and Roberts (2000).

¹⁰ We see here the birth of the (strong) identity test.

¹¹ Actually, this formula was among the alternatives discussed by Drobisch (1871).

¹² See Fisher (1922, p. 60). In practice, however, value shares and price relatives usually come from different sources (for example, from a household expenditure survey and a price survey respectively). The problem whether the resulting statistic can still be interpreted as

Three years later, Paasche (1874) argued that aggregate price change should be measured neither by the Carli index nor by the value ratio, $\sum_{n=1}^N p_n^1 x_n^1 / \sum_{n=1}^N p_n^0 x_n^0$, as suggested by Drobisch, but by

$$P^P(p^1, x^1, p^0, x^0) \equiv \frac{\sum_{n=1}^N p_n^1 x_n^1}{\sum_{n=1}^N p_n^0 x_n^1}. \quad (1.9)$$

Though Paasche was aware of Laspeyres' 1871 paper, because he refers to it, he did not provide reasons why formula (1.9)¹³ should be preferred to Laspeyres' formula (1.7). In turn, Laspeyres (1883) took notice of Paasche's proposal, but, rather than discussing their difference, considered Paasche as an ally in his battle against a geometric mean price index.

Like Laspeyres, however, Paasche was apparently unaware of the fact that the index he favored, expression (1.9), can be written as a weighted mean of price relatives, the type of mean now being harmonic and the weights being the value shares of the comparison period. The recognition of the operational significance of this identity had also to wait for Fisher.

A very complicated formula was derived by Lehr (1885). Recast in modern notation, this formula reads

$$P^{Le}(p^1, x^1, p^0, x^0) \equiv \frac{\sum_{n=1}^N p_n^1 x_n^1 / \sum_{n=1}^N p_n^0 x_n^0}{\sum_{n=1}^N \bar{p}_n x_n^1 / \sum_{n=1}^N \bar{p}_n x_n^0}, \quad (1.10)$$

where

$$\bar{p}_n \equiv \frac{p_n^0 x_n^0 + p_n^1 x_n^1}{x_n^0 + x_n^1} \quad (n = 1, \dots, N).$$

There are two interesting features here. The first is that Lehr's price index is defined as value index divided by a Lowe-type quantity index. Thus expression (1.10) defines what is now called an *implicit* price index. Of course, Lehr himself did not see it this way. Central to his derivation is the argument that $\sum_{n=1}^N p_n^t x_n^t / \sum_{n=1}^N \bar{p}_n x_n^t$ must be seen as the average price of the "pleasure-units" of period t ($t = 0, 1$).

The second interesting feature is that in Lehr's quantity index \bar{p}_n is defined as the unit value of commodity n ($n = 1, \dots, N$) over the two periods 0 and 1. This is one of the earliest occurrences of weights that are averages over the two periods considered.

a Laspeyres index was discussed by Ruderman (1954) and Banerjee (1956). Walsh (1901, pp. 349–50) noticed already that the Lowe price index (1.4) can be written as a weighted arithmetic or harmonic mean of price relatives.

¹³ Actually, this formula was also among the alternatives discussed by Drobisch (1871).

Palgrave (1886) proposed what later would appear to be an obvious variant to the right-hand side of equation (1.8), namely

$$P^{Pa}(p^1, x^1, p^0, x^0) \equiv \sum_{n=1}^N \frac{p_n^1 x_n^1}{\sum_{n'=1}^N p_{n'}^1 x_{n'}^1} \frac{p_n^1}{p_n^0}, \quad (1.11)$$

that is, a weighted arithmetic mean of price relatives, where the weights are the comparison period value shares.

Also in 1886, in a note contributed to the first volume of *The Quarterly Journal of Economics*, a certain Coggeshall returned to Jevons' discussion of the type of mean to be used for averaging price changes. He expressed a preference for the (unweighted) harmonic mean of price relatives,¹⁴

$$P^{Co}(p^1, p^0) \equiv \left[(1/N) \sum_{n=1}^N \left(\frac{p_n^1}{p_n^0} \right)^{-1} \right]^{-1}. \quad (1.12)$$

However, he added immediately that "This is a very awkward mean to calculate, which renders it undesirable for general use." Therefore his advice was to use the geometric mean, that is, Jevons' index as defined in expression (1.5).

1.3 Early Price Statistics

As said, most of the authors in the second half of the 19th century were interested in price index numbers as measures of changes in "the value of money." However, there were no statistical offices to provide (reliable) price statistics. Thus all these authors had to search for suitable price data. Such data usually came from import, export, or trade authorities. Using such data, the London-based journal *The Economist* started in 1869 with the annual publication of a table with price index numbers for 22 commodities, four of which were varieties of cotton, which led Pierson (1894) to the conclusion that such index numbers were meaningless.

German authors, such as Laspeyres and Paasche, could use price (= unit value) and quantity data for more than 300 commodities as collected and published by the Chamber of Commerce at Hamburg. This rich database,

¹⁴ When he comes to discuss the harmonic mean, Walsh (1932) refers to Messedaglia, and the bibliography of Walsh (1901) refers to an article by Messedaglia (1880). Angelo Messedaglia (1820–1901) is considered as one of the fathers of statistical methodology in Italy (according to Zalin 2002), though Gini (1926) does not mention his name. Messedaglia (1880) discusses the calculation of averages in various situations, but there appears to be no particular mention of index number issues in this article.

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going back to 1847, had been founded by the German economist Soetbeer, who worked there from 1840 to 1872, first as librarian and later as secretary. Using this material, Soetbeer published in the second edition (Berlin, 1886) of his book *Materialien zur Erläuterung und Beurteilung der wirtschaftlichen Edelmetallverhältnisse und der Währungsfrage* price index numbers for 114 commodities. Using the same material, the German economist Kral published in his book *Geldwert und Preisbewegung im Deutschen Reiche* (1887) price index numbers for 265 commodities.

All these price index numbers were calculated according to what came later to be known as the Carli formula (1.2).

In 1886 the London wool merchant Sauerbeck published an article entitled “Prices of Commodities and the Precious Metals” in the September issue of the *Journal of the Statistical Society of London*. Sauerbeck was primarily concerned with the causes behind the unprecedented price decline in the United Kingdom that had occurred during the previous 12 years. Basically, Sauerbeck considered the supply side of the economy. His database was therefore “confined to the prices of general commodities, almost entirely raw produce. Of articles not comprised in my statistics, wine is the only important one which has risen” (p. 599). From various sources he could obtain annual prices (= unit values) for 45 produced and imported commodities that had a trade value larger than one million pounds; the more important of these commodities were represented by more than one variety. The tables show three groups of food commodities and three groups of materials commodities, respectively consisting of 19 and 26 items. The data cover the years 1848–85. Price index numbers for groups and the grand total were computed according to the Carli formula (though without referring to this or other names – Jevons and Newmarch, the architect of *The Economist* index numbers,¹⁵ were mentioned only in the data construction appendix), whereby 1867–77 was used as the base period and each of the years 1848–85 acted as comparison period.

Though for Sauerbeck “the price index” appeared to be identical to the Carli index, and alternatives were not considered, he was aware of the weighting issue:

It may be argued that index numbers do not in the aggregate give a correct illustration of the actual course of prices, as they take no notice of quantities, and estimate all articles as of equal importance. This is true to some extent, particularly if a comparison is made with very remote times, and if in the interval a radical change

¹⁵ On Newmarch, see Fitzpatrick (1960).