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978-0-521-88705-2 - A First Course in General Relativity: Second Edition
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A First Course in General Relativity
Second Edition

Clarity, readability, and rigor combine in the second edition of this widely used textbook to provide the first step into general relativity for undergraduate students with a minimal background in mathematics.

Topics within relativity that fascinate astrophysical researchers and students alike are covered with Schutz's characteristic ease and authority – from black holes to gravitational lenses, from pulsars to the study of the Universe as a whole. This edition now contains recent discoveries by astronomers that require general relativity for their explanation; a revised chapter on relativistic stars, including new information on pulsars; an entirely rewritten chapter on cosmology; and an extended, comprehensive treatment of modern gravitational wave detectors and expected sources.

Over 300 exercises, many new to this edition, give students the confidence to work with general relativity and the necessary mathematics, whilst the informal writing style makes the subject matter easily accessible. Password protected solutions for instructors are available at www.cambridge.org/Schutz.

Bernard Schutz is Director of the Max Planck Institute for Gravitational Physics, a Professor at Cardiff University, UK, and an Honorary Professor at the University of Potsdam and the University of Hannover, Germany. He is also a Principal Investigator of the GEO600 detector project and a member of the Executive Committee of the LIGO Scientific Collaboration. Professor Schutz has been awarded the Amaldi Gold Medal of the Italian Society for Gravitation.

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Max Planck Institute for Gravitational Physics (Albert Einstein Institute)
and
Cardiff University

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University Printing House, Cambridge CB2 8BS, United Kingdom

Published in the United States of America by Cambridge University Press, New York

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9780521887052

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First published 2009
6th printing 2015

Printed in the United Kingdom by TJ International Ltd, Padstow, Cornwall

A catalog record for this publication is available from the British Library

ISBN 978-0-521-88705-2 hardback

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Preface to the second edition

In the 23 years between the first edition of this textbook and the present revision, the field of general relativity has blossomed and matured. Upon its solid mathematical foundations have grown a host of applications, some of which were not even imagined in 1985 when the first edition appeared. The study of general relativity has therefore moved from the periphery to the core of the education of a professional theoretical physicist, and more and more undergraduates expect to learn at least the basics of general relativity before they graduate.

My readers have been patient. Students have continued to use the first edition of this book to learn about the mathematical foundations of general relativity, even though it has become seriously out of date on applications such as the astrophysics of black holes, the detection of gravitational waves, and the exploration of the universe. This extensively revised second edition will, I hope, finally bring the book back into balance and give readers a consistent and unified introduction to modern research in classical gravitation.

The first eight chapters have seen little change. Recent references for further reading have been included, and a few sections have been expanded, but in general the geometrical approach to the mathematical foundations of the theory seems to have stood the test of time. By contrast, the final four chapters, which deal with general relativity in the astrophysical arena, have been updated, expanded, and in some cases completely re-written.

In Ch. 9, on gravitational radiation, there is now an extensive discussion of detection with interferometers such as LIGO and the planned space-based detector LISA. I have also included a discussion of likely gravitational wave sources, and what we can expect to learn from detections. This is a field that is rapidly changing, and the first-ever direct detection could come at any time. Chapter 9 is intended to provide a durable framework for understanding the implications of these detections.

In Ch. 10, the discussion of the structure of spherical stars remains robust, but I have inserted material on real neutron stars, which we see as pulsars and which are potential sources of detectable gravitational waves.

Chapter 11, on black holes, has also gained extensive material about the astrophysical evidence for black holes, both for stellar-mass black holes and for the supermassive black holes that astronomers have astonishingly discovered in the centers of most galaxies. The discussion of the Hawking radiation has also been slightly amended.

Finally, Ch. 12 on cosmology is completely rewritten. In the first edition I essentially ignored the cosmological constant. In this I followed the prejudice of the time, which assumed that the expansion of the universe was slowing down, even though it had not yet been accurately enough measured. We now believe, from a variety of mutually consistent observations, that the expansion is accelerating. This is probably the biggest challenge to

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theoretical physics today, having an impact as great on fundamental theories of particle physics as on cosmological questions. I have organized Ch. 12 around this perspective, developing mathematical models of an expanding universe that include the cosmological constant, then discussing in detail how astronomers measure the kinematics of the universe, and finally exploring the way that the physical constituents of the universe evolved after the Big Bang. The roles of inflation, of dark matter, and of dark energy all affect the structure of the universe today, and even our very existence. In this chapter it is possible only to give a brief taste of what astronomers have learned about these issues, but I hope it is enough to encourage readers to go on to learn more.

I have included more exercises in various chapters, where it was appropriate, but I have removed the exercise solutions from the book. They are available now on the website for the book.

The subject of this book remains classical general relativity; apart from a brief discussion of the Hawking radiation, there is no reference to quantization effects. While quantum gravity is one of the most active areas of research in theoretical physics today, there is still no clear direction to point a student who wants to learn how to quantize gravity. Perhaps by the third edition it will be possible to include a chapter on how gravity is quantized!

I want to thank many people who have helped me with this second edition. Several have generously supplied me with lists of misprints and errors in the first edition; I especially want to mention Frode Appel, Robert D'Alessandro, J. A. D. Ewart, Steve Fulling, Toshi Futamase, Ted Jacobson, Gerald Quinlan, and B. Sathyaprakash. Any remaining errors are, of course, my own responsibility. I thank also my editors at Cambridge University Press, Rufus Neal, Simon Capelin, and Lindsay Barnes, for their patience and encouragement. And of course I am deeply indebted to my wife Sian for her generous patience during all the hours, days, and weeks I spent working on this revision.

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Preface to the first edition

This book has evolved from lecture notes for a full-year undergraduate course in general relativity which I taught from 1975 to 1980, an experience which firmly convinced me that general relativity is not significantly more difficult for undergraduates to learn than the standard undergraduate-level treatments of electromagnetism and quantum mechanics. The explosion of research interest in general relativity in the past 20 years, largely stimulated by astronomy, has not only led to a deeper and more complete understanding of the theory, it has also taught us simpler, more physical ways of understanding it. Relativity is now in the mainstream of physics and astronomy, so that no theoretical physicist can be regarded as broadly educated without some training in the subject. The formidable reputation relativity acquired in its early years (Interviewer: 'Professor Eddington, is it true that only three people in the world understand Einstein's theory?' Eddington: 'Who is the third?') is today perhaps the chief obstacle that prevents it being more widely taught to theoretical physicists. The aim of this textbook is to present general relativity at a level appropriate for undergraduates, so that the student will understand the basic physical concepts and their experimental implications, will be able to solve elementary problems, and will be well prepared for the more advanced texts on the subject.

In pursuing this aim, I have tried to satisfy two competing criteria: first, to assume a minimum of prerequisites; and, second, to avoid watering down the subject matter. Unlike most introductory texts, this one does not assume that the student has already studied electromagnetism in its manifestly relativistic formulation, the theory of electromagnetic waves, or fluid dynamics. The necessary fluid dynamics is developed in the relevant chapters. The main consequence of not assuming a familiarity with electromagnetic waves is that gravitational waves have to be introduced slowly: the wave equation is studied from scratch. A full list of prerequisites appears below.

The second guiding principle, that of not watering down the treatment, is very subjective and rather more difficult to describe. I have tried to introduce differential geometry fully, not being content to rely only on analogies with curved surfaces, but I have left out subjects that are not essential to general relativity at this level, such as nonmetric manifold theory, Lie derivatives, and fiber bundles.[1] I have introduced the full nonlinear field equations, not just those of linearized theory, but I solve them only in the plane and spherical cases, quoting and examining, in addition, the Kerr solution. I study gravitational waves mainly in the linear approximation, but go slightly beyond it to derive the energy in the waves and the reaction effects in the wave emitter. I have tried in each topic to supply enough

[1] The treatment here is therefore different in spirit from that in my book *Geometrical Methods of Mathematical Physics* (Cambridge University Press 1980*b*), which may be used to supplement this one.

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foundation for the student to be able to go to more advanced treatments without having to start over again at the beginning.

The first part of the book, up to Ch. 8, introduces the theory in a sequence that is typical of many treatments: a review of special relativity, development of tensor analysis and continuum physics in special relativity, study of tensor calculus in curvilinear coordinates in Euclidean and Minkowski spaces, geometry of curved manifolds, physics in a curved spacetime, and finally the field equations. The remaining four chapters study a few topics that I have chosen because of their importance in modern astrophysics. The chapter on gravitational radiation is more detailed than usual at this level because the observation of gravitational waves may be one of the most significant developments in astronomy in the next decade. The chapter on spherical stars includes, besides the usual material, a useful family of exact compressible solutions due to Buchdahl. A long chapter on black holes studies in some detail the physical nature of the horizon, going as far as the Kruskal coordinates, then exploring the rotating (Kerr) black hole, and concluding with a simple discussion of the Hawking effect, the quantum mechanical emission of radiation by black holes. The concluding chapter on cosmology derives the homogeneous and isotropic metrics and briefly studies the physics of cosmological observation and evolution. There is an appendix summarizing the linear algebra needed in the text, and another appendix containing hints and solutions for selected exercises. One subject I have decided not to give as much prominence to, as have other texts traditionally, is experimental tests of general relativity and of alternative theories of gravity. Points of contact with experiment are treated as they arise, but systematic discussions of tests now require whole books (Will 1981).[2] Physicists today have far more confidence in the validity of general relativity than they had a decade or two ago, and I believe that an extensive discussion of alternative theories is therefore almost as out of place in a modern elementary text on gravity as it would be in one on electromagnetism.

The student is assumed already to have studied: special relativity, including the Lorentz transformation and relativistic mechanics; Euclidean vector calculus; ordinary and simple partial differential equations; thermodynamics and hydrostatics; Newtonian gravity (simple stellar structure would be useful but not essential); and enough elementary quantum mechanics to know what a photon is.

The notation and conventions are essentially the same as in Misner *et al., Gravitation* (W. H. Freeman 1973), which may be regarded as one possible follow-on text after this one. The physical point of view and development of the subject are also inevitably influenced by that book, partly because Thorne was my teacher and partly because *Gravitation* has become such an influential text. But because I have tried to make the subject accessible to a much wider audience, the style and pedagogical method of the present book are very different.

Regarding the use of the book, it is designed to be studied sequentially as a whole, in a one-year course, but it can be shortened to accommodate a half-year course. Half-year courses probably should aim at restricted goals. For example, it would be reasonable to aim to teach gravitational waves and black holes in half a year to students who have already

[2] The revised second edition of this classic work is Will (1993).

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studied electromagnetic waves, by carefully skipping some of Chs. 1–3 and most of Chs. 4, 7, and 10. Students with preparation in special relativity and fluid dynamics could learn stellar structure and cosmology in half a year, provided they could go quickly through the first four chapters and then skip Chs. 9 and 11. A graduate-level course can, of course, go much more quickly, and it should be possible to cover the whole text in half a year.

Each chapter is followed by a set of exercises, which range from trivial ones (filling in missing steps in the body of the text, manipulating newly introduced mathematics) to advanced problems that considerably extend the discussion in the text. Some problems require programmable calculators or computers. I cannot overstress the importance of doing a selection of problems. The easy and medium-hard ones in the early chapters give essential practice, without which the later chapters will be much less comprehensible. The medium-hard and hard problems of the later chapters are a test of the student's understanding. It is all too common in relativity for students to find the conceptual framework so interesting that they relegate problem solving to second place. Such a separation is false and dangerous: a student who can't solve problems of reasonable difficulty doesn't really understand the concepts of the theory either. There are generally more problems than one would expect a student to solve; several chapters have more than 30. The teacher will have to select them judiciously. Another rich source of problems is the *Problem Book in Relativity and Gravitation*, Lightman *et al.* (Princeton University Press 1975).

I am indebted to many people for their help, direct and indirect, with this book. I would like especially to thank my undergraduates at University College, Cardiff, whose enthusiasm for the subject and whose patience with the inadequacies of the early lecture notes encouraged me to turn them into a book. And I am certainly grateful to Suzanne Ball, Jane Owen, Margaret Vallender, Pranoat Priesmeyer, and Shirley Kemp for their patient typing and retyping of the successive drafts.

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