A First Course in Continuum Mechanics

Presenting a concise account of various classic theories of fluids and solids, this book is designed for courses in continuum mechanics for graduate students and advanced undergraduates. Thoroughly class-tested in courses at Stanford University and the University of Warwick, it is suitable for both applied mathematicians and engineers. The only prerequisites are an introductory undergraduate knowledge of basic linear algebra and differential equations.

Unlike most existing works at this level, this book covers both isothermal and thermal theories. The theories are derived in a unified manner from the fundamental balance laws of continuum mechanics.

Intended both for classroom use and for self-study, each chapter contains a wealth of exercises, with fully worked solutions to odd-numbered questions. A complete solutions manual is available to instructors upon request. Short bibliographies appear at the end of each chapter, pointing to material which underpins or expands upon the material discussed.

OSCAR GONZALEZ is an Associate Professor of Mathematics at the University of Texas. His research interests cover computational and applied mathematical problems related to the large-scale deformations of thin rods and ribbons, and more general three-dimensional bodies. He has contributed articles to numerous journals across mathematics, engineering and chemistry. His current research efforts are directed toward understanding the mechanical properties of DNA at various length scales.

ANDREW M. STUART is Professor of Mathematics at the University of Warwick. His general research interests cover computational stochastic processes and dynamical systems and his current research efforts are directed mainly towards problems at the interface of applied mathematics and statistics. He has contributed articles to numerous journals across mathematics, engineering and physics and is the recipient of six prizes for his work in applied mathematics.

Cambridge Texts in Applied Mathematics

All titles listed below can be obtained from good booksellers or from Cambridge University Press. For a complete series listing, visit http:publishing.cambridge.org/uk/series/sSeries.asp?code=CTAM

Thinking about Ordinary Differential Equations
ROBERT E. O'MALLEY JR

A Modern Introduction to the Mathematical Theory of Water Waves
ROBIN STANLEY JOHNSON

Rarefied Gas Dynamics: From Basic Concepts to Actual Calculations
CARLO CERCIGNANI

Symmetry Methods for Differential Equations: A Beginner's Guide
PETER E. HYDON

High Speed Flow
C. J. CHAPMAN

Wave Motion
J. BILLINGHAM, AND A. C. KING

An Introduction to Magnetohydrodynamics
P. A. DAVIDSON

Linear Elastic Waves
JOHN G. HARRIS

Vorticity and Incompressible Flow
ANDREW J. MAJDA, AND ANDREA L. BERTOZZI

Infinite-Dimensional Dynamical Systems
JAMES C. ROBINSON

Introduction to Symmetry Analysis
BRAIN J. CANTWELL

Bäcklund and Darboux Transformations
C. ROGERS, AND W. K. SCHIEF

Finite Volume Methods for Hyperbolic Problems
RANDALL J. LEVEQUE

Introduction to Hydrodynamic Stability
P. G. DRAZIN

Theory of Vortex Sound
M. S. HOWE

Scaling
GRIGORY ISAAKOVICH BARENBLATT

Complex Variables: Introduction and Applications (2nd Edition)
MARK J. ABLOWITZ, AND ATHANASSIOS S. FOKAS

A First Course in Combinatorial Optimization
JONE LEE

Practical Applied Mathematics: Modelling, Analysis, Approximation
SAM HOWISON

An Introduction to Parallel and Vector Scientific Computation
RONALD W. SHONKWILER, AND LEW LEFTON

A First Course in Continuum Mechanics

OSCAR GONZALEZ AND ANDREW M. STUART

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521886802

First published 2008

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

ISBN 978-0-521-88680-2 hardback
ISBN 978-0-521-71424-2 paperback

Dedicated to our parents,
our mentors, Juan Simo and John Norbury,
and our partners, Martha and Anjum.

Contents

Preface

This book is designed for a one- or two-quarter course in continuum mechanics for first-year graduate students and advanced undergraduates in the mathematical and engineering sciences. It was developed, and continually improved, by over four years of teaching of a graduate engineering course (ME 238) at Stanford University, USA, followed by over four years of teaching of an advanced undergraduate mathematics course (MA3G2) at the University of Warwick, UK. The resulting text, we believe, is suitable for use by both applied mathematicians and engineers. Prerequisites include an introductory undergraduate knowledge of linear algebra, multivariable calculus, differential equations and physics.

This book is intended both for use in a classroom and for self-study. Each chapter contains a wealth of exercises, with fully worked solutions to odd-numbered questions. A complete solutions manual is available to instructors upon request. A short bibliography appears at the end of each chapter, pointing to material which underpins, or expands upon, the material discussed here. Throughout the book we have aimed to strike a balance between two classic notational presentations of the subject: coordinate-free notation and index notation. We believe both types of notation are helpful in developing a clear understanding of the subject, and have attempted to use both in the statement, derivation and interpretation of major results. Moreover, we have made a conscious effort to include both types of notation in the exercises.

Chapters 1 and 2 provide necessary background material on tensor algebra and calculus in three-dimensional Euclidean space. Chapters 3–5 cover the basic axioms of continuum mechanics concerning mass, force and motion, the balance laws of mass, momentum, energy and entropy, and the concepts of frame-indifference and material constraints. Chapters 6–9 cover various classic theories of inviscid and viscous fluids, and linear and nonlinear elastic solids. Chapters 6 and 7 cover isothermal theories, whereas Chapters 8 and 9 cover corresponding thermal theories. We emphasize the formulation of typical initial-boundary value problems for the various material models, study important qualitative properties, and, in several cases, illustrate how the technique of linearization can be used to simplify problems under appropriate assumptions.

For reasons of space and timeliness the scope of material covered in this book is necessarily limited. For further general reading we recommend the books by Gurtin (1981), Chadwick (1976), Mase (1970) and Malvern (1969). For further reading in the area of fluid mechanics we suggest the texts by White (2006), Anderson (2003), Chorin and Marsden (1993) and Temam (1984). In the area of solid mechanics we suggest the texts by Antman (1995), Ciarlet (1988), Ogden (1984) and Marsden and Hughes (1983). For a wealth of information on both the subject and history of various classic field theories of continuum mechanics we recommend the encyclopedia articles by Gurtin (1972), Truesdell and Noll (1965), Truesdell and Toupin (1960) and Serrin (1959).

We are indebted to many of our colleagues at Stanford and Warwick, especially to Huajian Gao who gave a version of the course which we sat through in the 1993-94 and 1994-95 academic years, to Tom Hughes who gave us considerable encouragement to develop the notes into a book, as well as guidance on the choice of material, and to Robert Mackay who read and commented upon an early draft of the book. It is also a pleasure to thank the many students at Stanford and Warwick who helped produce and type solutions. Special thanks go to Nuno Catarino, Doug Enright, Gonzalo Feijoo, Liam Jones, Teresa Langlands, Matthew Lilley and Paul Lim.

Note to instructors Chapters 1–5, together with selected topics from Chapters 6–9, can form a standard one-semester or one-quarter course. This format has been used as the basis for an advanced undergraduate course in applied mathematics. Chapters 1–9 in their entirety, together with supplemental material from outside sources, can form a standard two-semester or two-quarter course. This format has been used as the basis for a graduate engineering course sequence. Example supplemental material might include the finite element method, fluid-solid interactions, the balance laws of electro-magnetism and their coupling to the thermo-mechanical balance laws, and topics such as piezo-electricity. Fully worked solutions to all exercises are available to instructors who are using the book as part of their teaching. A copy of the solutions manual can be obtained via the web site www.cambridge.org/9780521714242 or by emailing solutions@cambridge.org.

Notational conventions Mathematical statements such as definitions, axioms, theorems and results are numbered consecutively within each chapter. Thus Definition 2.3 precedes Result 2.4 in Chapter 2. The symbol □ is used to denote the end of each such statement, including proofs, remarks and examples. Equation numbers, when assigned, are also in consecutive order within each chapter. Thus equations (3.6) and (3.7) are the sixth and seventh numbered equations in Chapter 3. Often, a single number is assigned to a displayed group of equations, and in this case, subscripts are used to refer to independent equations from the group. Thus $(3.7)_1$ and $(3.7)_2$ denote the first and second independent equations from the equation group (3.7). The ordering of independent equations within a group will always follow the standard reading order: left to right, top to bottom.