Part I

Basics

1

Review of basic magnetostatics

> Mention magnetics and an image arises of musty physics labs peopled by old codgers with iron filings under their fingernails.
>
> John Simonds, Magnetoelectronics today and tomorrow, *Physics Today*, April 1995

Before we can begin our discussion of magnetic materials we need to understand some of the basic concepts of magnetism, such as what causes magnetic fields, and what effects magnetic fields have on their surroundings. These fundamental issues are the subject of this first chapter. Unfortunately, we are going to immediately run into a complication. There are two complementary ways of developing the theory and definitions of magnetism. The "physicist's way" is in terms of circulating currents, and the "engineer's way" is in terms of magnetic poles (such as we find at the ends of a bar magnet). The two developments lead to different views of which interactions are more fundamental, to slightly different-looking equations, and (to really confuse things) to two different sets of units. Most books that you'll read choose one convention or the other and stick with it. Instead, throughout this book we are going to follow what happens in "real life" (or at least at scientific conferences on magnetism) and use whichever convention is most appropriate to the particular problem. We'll see that it makes most sense to use Système International d'Unités (SI) units when we talk in terms of circulating currents, and centimeter–gram–second (cgs) units for describing interactions between magnetic poles.

To avoid total confusion later, we will give our definitions in this chapter and the next from *both* viewpoints, and provide a conversion chart for units and equations at the end of Chapter 2. Reference [1] provides an excellent light-hearted discussion of the unit systems used in describing magnetism.

1.1 Magnetic field

1.1.1 Magnetic poles

So let's begin by defining the magnetic field, $\boldsymbol{H}$, in terms of magnetic poles. This is the order in which things happened historically – the law of interaction between magnetic poles was discovered by Michell in England in 1750, and by Coulomb in France in 1785, a few decades before magnetism was linked to the flow of electric current. These gentlemen found empirically that the force between two magnetic poles is proportional to the product of their pole strengths, p, and inversely proportional to the square of the distance between them,

$$F \propto \frac{p_1 p_2}{r^2}. \tag{1.1}$$

This is analogous to Coulomb's law for electric charges, with one important difference – scientists believe that single magnetic poles (magnetic monopoles) do not exist. They can, however, be approximated by one end of a very long bar magnet, which is how the experiments were carried out. By convention, the end of a freely suspended bar magnet which points towards magnetic north is called the north pole, and the opposite end is called the south pole.[1] In cgs units, the constant of proportionality is unity, so

$$F = \frac{p_1 p_2}{r^2} \qquad \text{(cgs)}, \tag{1.2}$$

where r is in centimeters and F is in dynes. Turning Eq. (1.2) around gives us the definition of pole strength:

A pole of unit strength is one which exerts a force of 1 dyne on another unit pole located at a distance of 1 centimeter.

The unit of pole strength does not have a name in the cgs system.

In SI units, the constant of proportionality in Eq. (1.1) is $\mu_0/4\pi$, so

$$F = \frac{\mu_0}{4\pi} \frac{p_1 p_2}{r^2} \qquad \text{(SI)}, \tag{1.3}$$

where μ_0 is called the permeability of free space, and has the value $4\pi \times 10^{-7}$ weber/(ampere meter) (Wb/(A m)). In SI, the pole strength is measured in ampere meters (A m), the unit of force is of course the newton (N), and 1 newton $= 10^5$ dyne (dyn).

[1] Note, however, that if we think of the earth's magnetic field as originating from a bar magnet, then the *south* pole of the earth's "bar magnet" is actually at the magnetic north pole!

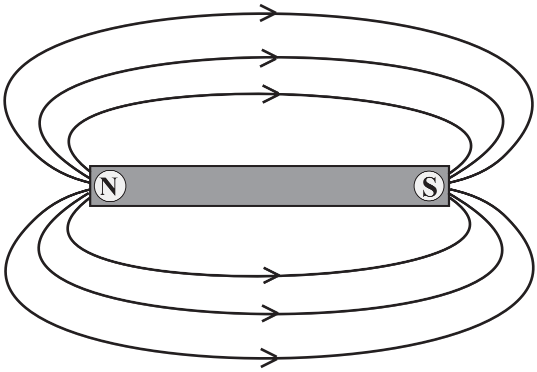

Figure 1.1 Field lines around a bar magnet. By convention, the lines originate at the north pole and end at the south pole.

To understand what causes the force, we can think of the first pole generating a magnetic field, $\boldsymbol{H}$, which in turn exerts a force on the second pole. So

$$F = \left(\frac{p_1}{r^2}\right) p_2 = \boldsymbol{H} p_2, \tag{1.4}$$

giving, by definition,

$$\boldsymbol{H} = \frac{p_1}{r^2}. \tag{1.5}$$

So:

A field of unit strength is one which exerts a force of 1 dyne on a unit pole.

By convention, the north pole is the *source* of the magnetic field, and the south pole is the *sink*, so we can sketch the magnetic field lines around a bar magnet as shown in Fig. 1.1.

The units of magnetic field are oersteds (Oe) in cgs units, so a field of unit strength has an intensity of 1 oersted. In the SI system, the analogous equation for the force one pole exerts on another is

$$F = \frac{\mu_0}{4\pi}\left(\frac{p_1}{r^2}\right) p_2 = \frac{\mu_0}{\boldsymbol{H}} p_2, \tag{1.6}$$

yielding the expression for $\boldsymbol{H} = \frac{1}{4\pi}\frac{p_1}{r^2}$ in units of amperes per meter (A/m); 1 Oe = $(1000/4\pi)$ A/m.

The earth's magnetic field has an intensity of around one-tenth of an oersted, and the field at the end of a typical kindergarten toy bar magnet is around 5000 Oe.

1.1.2 Magnetic flux

It's appropriate next to introduce another rather abstract concept, that of *magnetic flux*, Φ. The idea behind the term "flux" is that the field of a magnetic pole is conveyed to a distant place by something which we call a flux. Rigorously the flux is defined as the surface integral of the normal component of the magnetic field. This means that the amount of flux passing through unit area perpendicular to the field is equal to the field strength. So the field strength is equal to the amount of flux per unit area, and the flux is the field strength *times* the area,

$$\Phi = \boldsymbol{H} A. \tag{1.7}$$

The unit of flux in cgs units, the oersted cm^2, is called the maxwell (Mx). In SI units the expression for flux is

$$\Phi = \mu_0 \boldsymbol{H} A \tag{1.8}$$

and the unit of flux is called the weber.

Magnetic flux is important because a *changing* flux generates an electric current in any circuit which it intersects. In fact we define an "electromotive force" ε, equal to the rate of change of the flux linked with the circuit:

$$\varepsilon = -\frac{d\Phi}{dt}. \tag{1.9}$$

Equation (1.9) is Faraday's law of electromagnetic induction. The electromotive force provides the potential difference which drives electric current around the circuit. The minus sign in Eq. (1.9) shows us that the current sets up a magnetic field which acts in the opposite direction to the magnetic flux. (This is known as Lenz's law.)[2]

The phenomenon of electromagnetic induction leads us to an alternative definition of flux, which is (in SI units):

A flux of 1 weber, when reduced to zero in 1 second, produces an electromotive force of 1 volt in a one-turn coil through which it passes.

1.1.3 Circulating currents

The next development in the history of magnetism took place in Denmark in 1820 when Oersted discovered that a magnetic compass needle is deflected in the neighborhood of an electric current. This was really a huge breakthrough because it unified two sciences. The new science of electromagnetism, which dealt with

[2] We won't cover electromagnetic induction in much detail in this book. A good introductory text is [2].

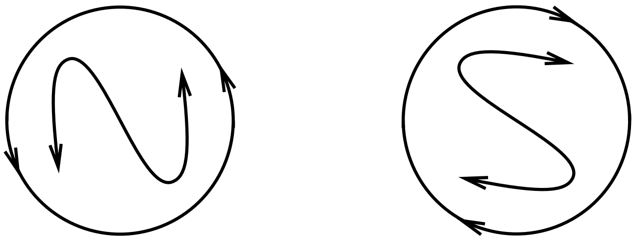

Figure 1.2 Relationship between direction of current flow and magnetic pole type.

forces between moving charges and magnets, encompassed both electricity, which described the forces between charges, and magnetism, which described the forces between magnets.

Then Ampère discovered (again experimentally) that the magnetic field of a small current loop is identical to that of a small magnet. (By small we mean small with respect to the distance at which the magnetic field is observed.) The north pole of a bar magnet corresponds to current circulating in a counter-clockwise direction, whereas clockwise current is equivalent to the south pole, as shown in Fig. 1.2. In addition, Ampère hypothesized that *all* magnetic effects are due to current loops, and that the magnetic effects in magnetic materials such as iron are due to so-called "molecular currents." This was remarkably insightful, considering that the electron would not be discovered for another 100 years! Today it's believed that magnetic effects are caused by the orbital and spin angular momenta of electrons.

This leads us to an alternative definition of the magnetic field, in terms of current flow:

A current of 1 ampere passing through an infinitely long straight wire generates a field of strength $1/2\pi$ amperes per meter at a radial distance of 1 meter.

Of course the next obvious question to ask is what happens if the wire is *not* straight. What magnetic field does a *general* circuit produce? Ampère solved this one too.

1.1.4 Ampère's circuital law

Ampère observed that the magnetic field generated by an electrical circuit depends on both the *shape* of the circuit *and* the amount of current being carried. In fact the total current, I, is equal to the line integral of the magnetic field around a closed path containing the current. In SI units,

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = I. \tag{1.10}$$

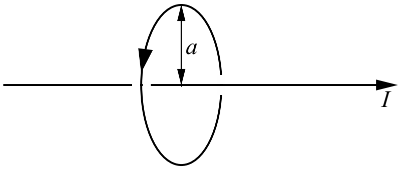

Figure 1.3 Calculation of the field from a current flowing in a long straight wire, using Ampère's circuital law.

This expression is called Ampère's circuital law, and it can be used to calculate the field produced by a current-carrying conductor. We will look at some examples later.

1.1.5 Biot–Savart law

An equivalent statement to Ampère's circuital law (which is sometimes easier to use for particular symmetries) is given by the Biot–Savart law. The Biot–Savart law gives the field contribution, $\delta \boldsymbol{H}$, generated by a current flowing in an elemental length $\delta \boldsymbol{l}$, of a conductor:

$$\delta \boldsymbol{H} = \frac{1}{4\pi r^2} I \delta \boldsymbol{l} \times \hat{\boldsymbol{u}}, \tag{1.11}$$

where r is the radial distance from the conductor, and $\hat{\boldsymbol{u}}$ is a unit vector along the radial direction.

1.1.6 Field from a straight wire

To show that these laws are equivalent, let's use them both to calculate the magnetic field generated by a current flowing in a straight wire.

First let us use Ampère's law. The geometry of the problem is shown in Fig. 1.3. If we assume that the field lines go around the wire in closed circles (by symmetry this is a fairly safe assumption) then the field, $\boldsymbol{H}$, has the same value at all points on a circle concentric with the wire. This makes the line integral of Eq. (1.10) straightforward. It's just

$$\oint \boldsymbol{H} \cdot d\boldsymbol{l} = 2\pi a \boldsymbol{H} = I \qquad \text{by Ampère's law,} \tag{1.12}$$

and so the field, $\boldsymbol{H}$, at a distance a from the wire is

$$\boldsymbol{H} = \frac{I}{2\pi a}. \tag{1.13}$$

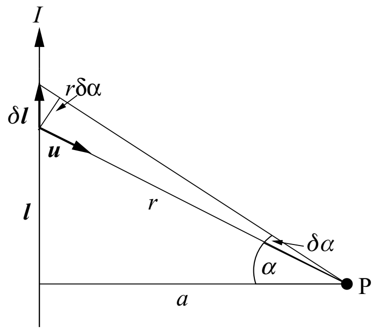

Figure 1.4 Calculation of the field from a current flowing in a long straight wire, using the Biot–Savart law.

For this particular problem, the Biot–Savart law is somewhat less straightforward to apply. The geometry for calculating the field at a point P at a distance a from the wire is shown in Fig. 1.4. Now

$$\begin{aligned}\delta \boldsymbol{H} &= \frac{1}{4\pi r^2} I \delta \boldsymbol{l} \times \hat{\boldsymbol{u}} \\ &= \frac{1}{4\pi r^2} I |\delta \boldsymbol{l}||\hat{\boldsymbol{u}}| \sin\theta, \qquad (1.14)\end{aligned}$$

where θ is the angle between $\delta \boldsymbol{l}$ and $\hat{\boldsymbol{u}}$, which is equal to $(90^\circ + \alpha)$. So

$$\begin{aligned}\delta \boldsymbol{H} &= \frac{I}{4\pi r^2} \delta \boldsymbol{l} \sin(90^\circ + \alpha) \\ &= \frac{I}{4\pi r^2} \frac{r\delta\alpha}{\cos\alpha} \sin(90^\circ + \alpha), \qquad (1.15)\end{aligned}$$

since $\delta \boldsymbol{l} = r\delta\alpha/\cos\alpha$.

But $\sin(90^\circ + \alpha) = \cos\alpha$, and $r = a/\cos\alpha$. So

$$\begin{aligned}\delta \boldsymbol{H} &= \frac{I}{4\pi} \frac{\cos^2\alpha}{a^2} \frac{a\delta\alpha}{\cos^2\alpha} \cos\alpha \\ &= \frac{I \cos\alpha\, \delta\alpha}{4\pi a} \qquad (1.16)\end{aligned}$$

and

$$\begin{aligned}\boldsymbol{H} &= \frac{I}{4\pi a} \int_{-\pi/2}^{\pi/2} \cos\alpha \, d\alpha \\ &= \frac{I}{4\pi a} [\sin\alpha]_{-\pi/2}^{\pi/2} \\ &= \frac{I}{2\pi a}. \qquad (1.17)\end{aligned}$$

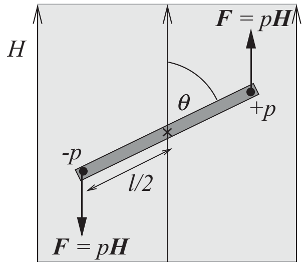

Figure 1.5 Calculation of the moment exerted on a bar magnet in a magnetic field.

The same result as that obtained using Ampère's law! Clearly Ampère's law was a better choice for this particular problem.

Unfortunately, analytic expressions for the field produced by a current can only be obtained for conductors with rather simple geometries. For more complicated shapes the field must be calculated numerically. Numerical calculation of magnetic fields is an active research area, and is tremendously important in the design of electromagnetic devices. A review is given in [3].

1.2 Magnetic moment

Next we need to introduce the concept of magnetic moment, which is the moment of the couple exerted on either a bar magnet or a current loop when it is in an applied field. Again we can define the magnetic moment either in terms of poles or in terms of currents.

Imagine a bar magnet is at an angle θ to a magnetic field, $\boldsymbol{H}$, as shown in Fig. 1.5. We showed in Section 1.1.1 that the force on each pole, $\boldsymbol{F} = p\boldsymbol{H}$. So the torque acting on the magnet, which is just the force times the perpendicular distance from the center of mass, is

$$pH \sin\theta \frac{l}{2} + pH \sin\theta \frac{l}{2} = pHl \sin\theta = mH \sin\theta, \tag{1.18}$$

where $\boldsymbol{m} = pl$, the product of the pole strength and the length of the magnet, is the *magnetic moment*. (Our notation here is to represent vector quantities by bold italic type, and their magnitudes by regular italic type.) This gives a definition:

The magnetic moment is the moment of the couple exerted on a magnet when it is perpendicular to a uniform field of 1 oersted.

Alternatively, if a current loop has area A and carries a current I, then its magnetic moment is defined as

$$\boldsymbol{m} = IA. \tag{1.19}$$

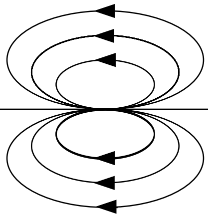

Figure 1.6 Field lines around a magnetic dipole.

The cgs unit of magnetic moment is the emu. In SI units, magnetic moment is measured in A m^2.

1.2.1 Magnetic dipole

A magnetic dipole is defined as either the magnetic moment, $\boldsymbol{m}$, of a bar magnet in the limit of small *length* but finite moment, or the magnetic moment, $\boldsymbol{m}$, of a current loop in the limit of small *area* but finite moment. The field lines around a magnetic dipole are shown in Fig. 1.6. The energy of a magnetic dipole is defined as zero when the dipole is perpendicular to a magnetic field. So the work done (in ergs) in turning through an angle $d\theta$ against the field is

$$\begin{aligned} dE &= 2(pH \sin\theta)\frac{l}{2} d\theta \\ &= mH \sin\theta \, d\theta, \end{aligned} \tag{1.20}$$

and the energy of a dipole at an angle θ to a magnetic field is

$$\begin{aligned} E &= \int_{\pi/2}^{\theta} mH \sin\theta \, d\theta \\ &= -mH \cos\theta \\ &= -\boldsymbol{m} \cdot \boldsymbol{H}. \end{aligned} \tag{1.21}$$

This expression for the energy of a magnetic dipole in a magnetic field is in cgs units. In SI units the energy is $E = -\mu_0 \boldsymbol{m} \cdot \boldsymbol{H}$. We will be using the concept of magnetic dipole, and this expression for its energy in a magnetic field, extensively throughout this book.

1.3 Definitions

Finally for this chapter, let's review the definitions which we've introduced so far. Here we give all the definitions in cgs units.