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Introduction

1.1 The subject matter: definition, history, study methods

Gravity-driven regional groundwater flow is induced by elevation differences in the water table and its pattern is self-organized into hierarchical sets of flow systems. Tóth (1963, p. 4806) defined a groundwater flow system as '*a set of flow lines in which any two flow lines adjacent at one point of the flow region remain adjacent through the whole region; they can be intercepted anywhere by an uninterrupted surface across which flow takes place in one direction only.*' While flow is generated by the relief of the water table, its patterns are modified by heterogeneities in the rock framework's permeability.

Topographic effects are ubiquitous and may cause water to move at depths of several kilometres beneath the Earth's terrestrial areas. Most of people's needs for subsurface water are met with water obtained from this depth range. However, in addition to satisfying this need, gravity-driven groundwater also generates and affects a wide variety of economically important natural processes at or below the land surface. It is of both economic and environmental importance, therefore, to understand the properties, controlling factors, effects and manifestations of this type of flow, as well as to develop methods and techniques for its study and possible modification. Furthermore, because of the relatively easily accessible depths, known and measurable controlling factors, observable natural effects and manifestations, in short, unique tractability, the study of gravity-driven groundwater flow is instructive and useful in the understanding and exploitation of groundwater motion generated by other sources of driving forces, such as differences in dissolved salt contents, thermal convection, sedimentary compaction and tectonic compression.

Owing to their practical relevance and scientific nature, the questions of driving forces, spatial patterns and controlling factors of natural groundwater flow have long interested hydrologists, hydrogeologists and, more recently, Earth scientists in general. Munn's hydraulic theory of oil and gas migration is one of the many

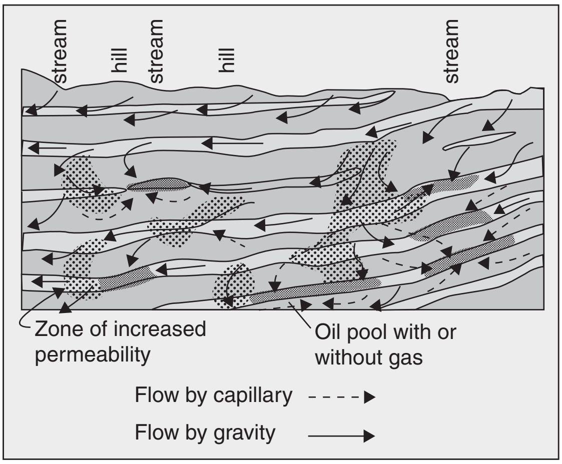

Fig. 1.1 Groundwater flow and hydrocarbon accumulation conceptualized by Munn (1909; modified from Fig. 77, p. 526).

possible examples that illustrate the point. Munn (1909) envisaged meteoric water to descend from the land surface across beds of sandstone and shale and, driven by capillary forces, to push ahead particles of oil and gas dispersed in these beds (Fig. 1.1).

Permeability differences in the rock would cause different parts of the fluid front to advance at different rates 'which would finally result in zones of conflicting currents of water between which the bodies of oil and gas would be trapped and held' (Munn, 1909, Figs. 77–79; p. 525). The idea of conflicting currents of groundwater appears to be a realistic mechanism for entrapment (its application to petroleum exploration based on the theory of gravity-driven flow systems will be shown in Section 5.5). However, sites where such conditions might occur cannot be identified in practice from Munn's concept because the relations between flow directions and the factors controlling them are not specified.

Perhaps the earliest published conceptualization of hierarchically distributed groundwater flow systems is reproduced by Fourmarier in his 'Hydrogéologie' (1939, Fig. 43, p. 87, from D'Andrimont, 1906). Figure 1.2 shows a major water divide with a sub-basin to the left from its crest. From both sides, the sub-basin attracts two, what we call today *local*, groundwater flow-systems. The local systems are superimposed on a larger system that originates on the principal divide and moves towards the main valley of the watershed. A similar notion seems to be reflected by two tiny flow systems leading to a saline 'Discharge area in sidehill valley' on the right-hand side flank of Meyboom's (1962, Fig. 2, not reproduced here) 'Prairie Profile'. But the minuteness of the feature suggests a lack of

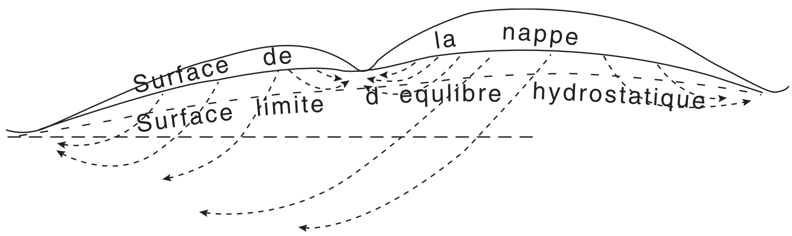

Fig. 1.2 'Allure complex des filets liquids dans une nappe libre': Complex pattern of fluid flow in an unconfined aquifer (Fourmarier, 1939, Figure 43, p. 87: after D'Andrimont, 1906).

conviction by the author concerning its existence or importance in nature. Both of these diagrams are perceptive generalizations of possible patterns of gravity-flow in complex drainage basins. However, they are only the mental images of individuals without the accompanying mathematical descriptions that would enable others to reproduce and further develop them.

The door to the transition from the age of speculative and qualitative conceptualizations of regional groundwater flow to rigorous mathematical analyses was opened by Hubbert's (1940) paper, *'The Theory of Ground-Water Motion'*. In this classic treatise, Hubbert derived the concept of fluid potential, Φ, from first principles. He also showed that, for subsurface liquids in general, Φ comprises two terms: one related to pore pressure, p, the other to topographic elevation, z, (because of low velocities the inertia-dependent kinetic energy associated with the fluid's motion can be neglected) and that the impelling force acting upon a unit mass of fluid is the negative first derivative of the fluid potential. Consequently, the force field can be calculated, or modelled, for any given flow domain along the boundaries for which Φ or its first derivatives can be stated. In turn, the flow field can be determined by combining the force field with the rock's hydraulic properties (porosity, permeability, storativity). Basinal-scale groundwater flow patterns could thus now be produced mathematically as solutions to formal boundary value problems. However, another twenty years went by before this gift to hydrogeology was exploited.

In the course of my regular duties as a hydrogeologist in Central Alberta, Canada, I noticed a discrepancy between what I expected on the basis of Hubbert's (1940) Figure 45, on the one hand (Fig. 1.3), and what I saw in the field, on the other.

Hubbert's figure showed all infiltrating water resurfacing in the thalweg of the valley, as though the watercourse were a drainage ditch. In reality, the beds of the numerous creeks in my area were dry in many places and whatever water they had was frozen to the bottom in the winter. Based on 'Figure 45', and considering the steep topographic slopes, shallow water tables (<3 m deep) and permeable rock, together providing sufficient supplies of water to the area's farms and towns, I

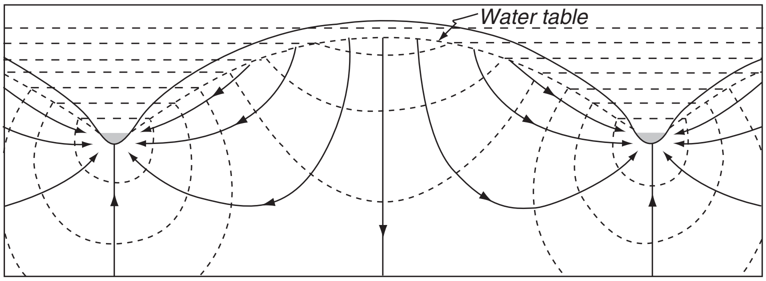

Fig. 1.3 'Approximate flow pattern in uniformly permeable material between the sources distributed over the air–water interface and the valley sinks' (Hubbert, 1940, Fig. 45, p. 930).

would have expected healthy runoffs in the creeks. A possible solution to the riddle occurred to me one day when I realized that the convergence of the flow lines in the figure was *a postulate, not a result;* Hubbert *made* the flow lines converge on the thalweg! I decided to find out where the water wants to go by itself, and solved the Laplace equation for a drainage basin of simple geometry (Fig. 1.4, App. A; Tóth, 1962a, Fig. 3, p. 4380).

The results were revealing: they showed that instead of the 'sinks' being 'limited to the bottoms of valleys containing streams' (Hubbert, 1940, p. 928), 'groundwater discharge is not concentrated in the valley bottom' (Tóth, 1962a, p. 4386). Thus the entire lower half of the basin was revealed to be a 'discharge area'. This simple discovery has triggered a number of follow-up studies in rapid succession.

During the preparation of the above, my first, paper (Tóth, 1962a), I already knew that assuming a linearly sloping valley flank was an oversimplification. I solved the Laplace equation again, now for a drainage basin with a sinusoidal surface superimposed on a linear regional slope (Fig. 1.4c; App. B; Tóth, 1962b, 1963 Fig. 3, p. 4807, reprinted in 1983). The analysis resulted in the groundwater flow-pattern for composite basins with homogeneous and isotropic rock framework. It was aptly called the ' hierarchically nested flow systems' by Engelen (Engelen and Jones, 1986, p. 9).

By fortunate coincidence, at the time when numerical methods just started to gain popularity R. Allan Freeze was looking for a Ph.D. thesis topic at the University of California, Berkeley. Advised by P. Witherspoon, he intended to show the value of the method to groundwater-related problems. Freeze took off from my solution to the composite-basin problem and produced a trail-blazing series of three papers from his thesis showing that quantitative flow-nets can be calculated by numerical, as opposed to analytical, methods for gravity-driven groundwater flow in drainage basins of arbitrary topography and heterogeneous and anisotropic rock framework (Freeze and Witherspoon, 1966, 1967, 1968).

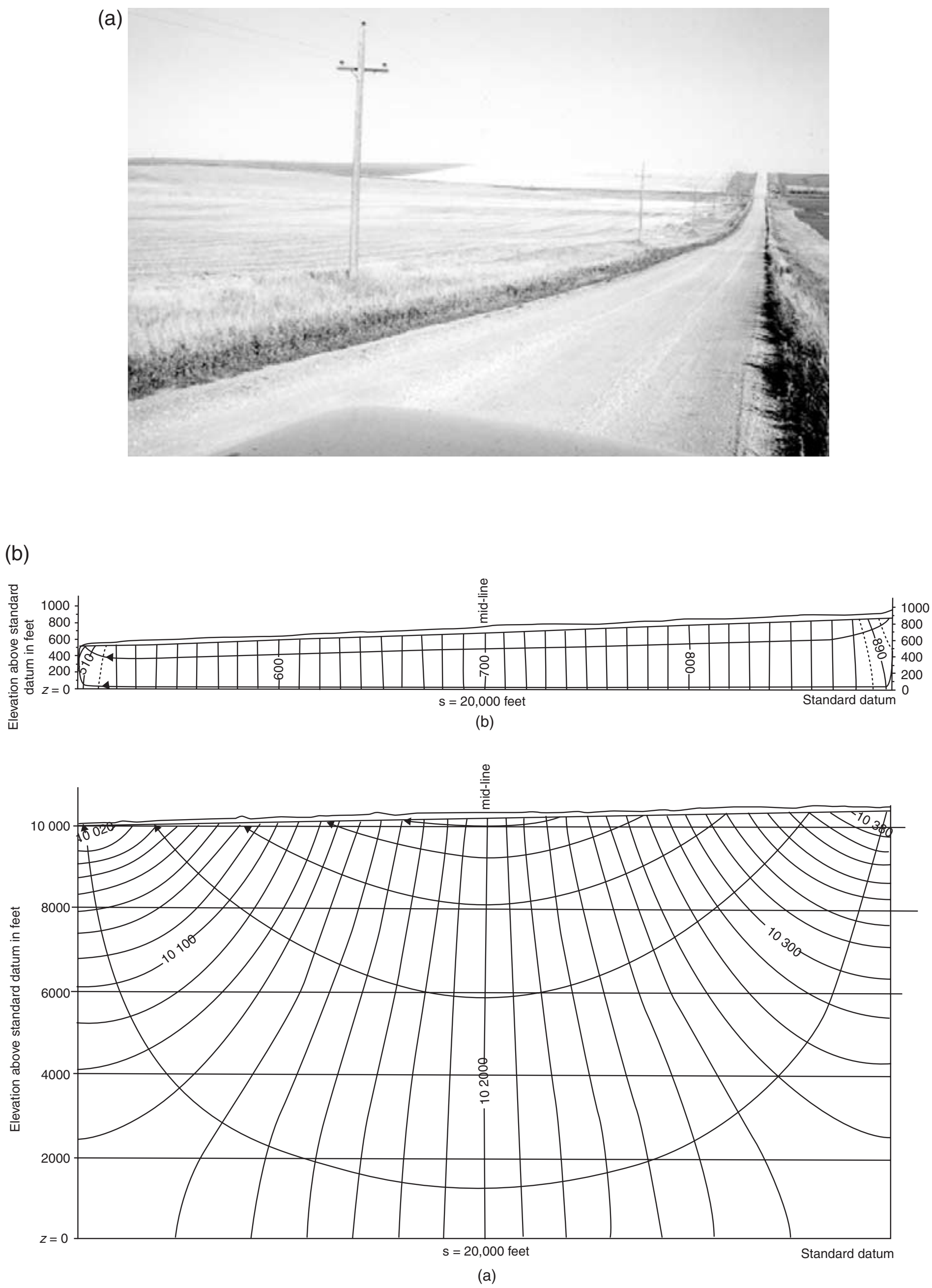

Fig. 1.4 Groundwater flow in a simple drainage basin: (a) the concept-inspiring topography, Central Alberta, Canada (photo by J. Tóth); (b) two-dimensional theoretical fluid-potential distributions and flow patterns for different depths to the horizontal impermeable boundary in a drainage basin with linearly sloping water table (Tóth, 1962a, Fig. 3, p. 4380). (c) Hierarchically nestled gravity-flow systems of groundwater in drainage basin with complex topography (Tóth, 1963, Fig. 3, p. 4807).

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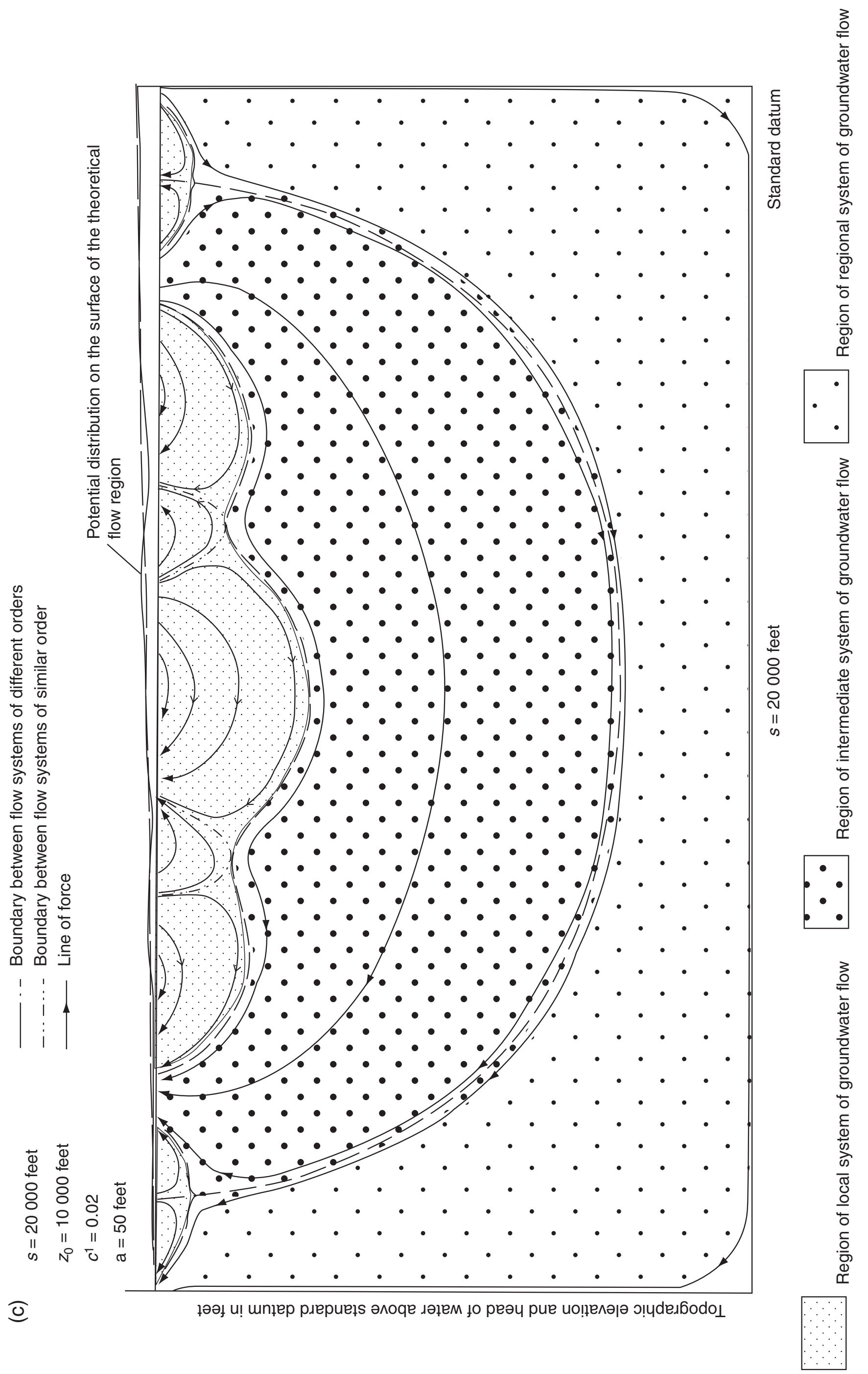

Fig. 1.4 (cont.)

Collectively, the above papers, published between 1962 and 1968, have fundamentally affected the direction, the scope and the rate of the subsequent evolution of hydrogeology. In the search for and the development of groundwater resources, basin-scale considerations have been added to aquifer- and well-hydraulics. The scope of the field was broadened by dedicated field studies, recognizing moving groundwater as a geologic agent of diverse consequences and introducing the flow-system concept into a wide variety of other disciplines such as soil science, petroleum exploration, economic geology, geothermics, hydrogeochemistry, soil mechanics, sedimentology, diagenesis and ecology. Hydrogeology has thus been turned into both a basic and a specialty discipline of the earth and hydrological sciences. In addition, the broadened scope of hydrogeologically-related activities attracted talented new researchers and practitioners who accelerated the rates of theoretical progress and methodological innovations. Some of the many studies inspired by the flow-system concept and prompting further developments include: Meyboom *et al*. (1966), Tóth (1966a, 1978, 1980), Fritz (1968), Mifflin (1968), Williams (1968, 1970), Freeze (1969), Freeze and Harlan (1969), Kiraly (1970), Deere and Patton (1971), Domenico and Palciauskas (1973), Schwartz and Domenico (1973), Galloway (1978), Winter (1978), Garven and Freeze (1984), Garven (1989) and so on.

Regional, or basinal, groundwater flow can be studied, characterized, and evaluated by three different methods: (i) mathematical modelling; (ii) field measurements of fluid-dynamic parameters; and (iii) mapping of flow-generated natural field-phenomena.

(i) Mathematical models produce spatially and, in the case of Equation (1.1) also temporally, distributed patterns of flow-related fluid-dynamic parameters. The patterns can be obtained as solutions to the *Diffusion Equation* for transient, or non-steady-state, flow:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \nabla^2 h = \text{div grad}\, h = \frac{S_0}{K}\frac{\partial h}{\partial t} \quad (1.1)$$

or as solutions to its particular case, the *Laplace Equation,* for steady-state flow (1.2):

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \nabla^2 h = \text{div grad}\, h = 0. \quad (1.2)$$

In these equations h is hydraulic head, t is time, K is hydraulic conductivity and S_0 is specific storage. The equations can be solved analytically, numerically or by analogue modelling, using appropriate initial and boundary conditions.

(ii) Fluid-dynamic parameters [hydraulic heads, h; vertical gradients of pore pressure or pressure vs. depth profiles, $p(d)$; and dynamic pressure-increments, Δp] can be obtained or derived from field measurements of pore pressures and/or groundwater levels, and interpreted as patterns of fluid potential and flow.

(iii) Many different processes and phenomena are in cause-and-effect relation to gravity-driven groundwater in the realm of hydrology, ground- and/or surface-water chemistry, plants and plant ecology, mineralogy, pedology, soil- and rock-mechanics, subsurface transport of heat and mass, and so on. The manifestations of these natural conditions can thus be interpreted in terms of direction and intensity of flow.

1.2 Portrayal of groundwater flow-systems

1.2.1 Darcy's experiment and Law

The quantitative relation between the strength and sense of the fluid-driving force and the rate and direction of the flow that it induces through permeable materials was first stated, based on laboratory experiments, by the French engineer Henry Darcy (1856). This empirical relation is the fluid-flow equivalent of Ohm's Law for electrical current or Fourier's Law for heat flow. Because it encapsulates the main aspects of the physics of flow, is stated in terms of basic fluid-dynamic parameters and requires the use of essential terminology, Darcy's Law is the natural introduction to any discussion of subsurface fluid flow.

In Darcy's experiments, water was passed through a vertical iron pipe filled with sand and equipped with manometers along its side (Fig. 1.5).

The size of the sand grains and the rate of water flow were varied in the different experiments. The changes resulted in variation of the water levels in the manometer

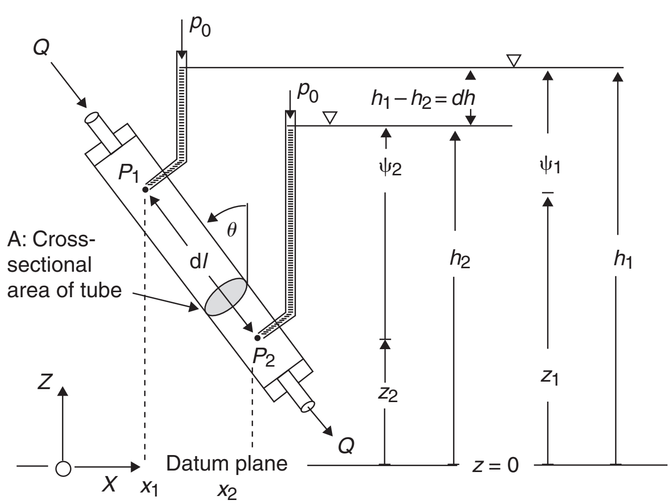

Fig. 1.5 Apparatus (with tilted pipe) to demonstrate Darcy's Law and the meaning of hydraulic (potentiometric) head.

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tubes as measured vertically above a horizontal datum plane. The summary conclusion that Darcy made from his observations, which is called 'Darcy's Law' in his honour, can be expressed mathematically in various forms, for instance:

$$Q = -KA\frac{\mathrm{d}h}{\mathrm{d}l}, \tag{1.3a}$$

$$q = -K\frac{\mathrm{d}h}{\mathrm{d}l}. \tag{1.3b}$$

The terms in Equations (1.3a) and (1.3b) are defined and named below (dimensions are shown in brackets) and illustrated in Figure 1.5.

$A[L^2]$: *cross-sectional area* of flow field normal to the direction of flow; $Q[L^3/T]$ total volume of fluid passing through A during a unit length of time t, or *volume discharge;* $q = Q/A[L^3/T]/[L^2] = [L/T]$: volume of fluid passing through a unit cross-sectional area of A during a unit length of time, *specific volume discharge, flux, Darcy velocity* or *flow strength;* $h[L]$: height to which the fluid rises above the datum plane of elevation $z = 0$, from an observation point P in the flow field, i.e. from a manometer's intake, *hydraulic head;* $\mathrm{d}l[L]$: distance measured along the flow path between points in which hydraulic head is determined, *flow length;* $\mathrm{d}h$: difference in hydraulic heads determined in different points separated by the distance $\mathrm{d}l$ along the flow path; $\mathrm{d}h/\mathrm{d}l$ $[L/]/[L] = [L_0]$, change in hydraulic head over a unit length of flow path, *hydraulic gradient,* taken positive in the direction of increasing hydraulic head; $K[L^3/TL^2][L/L] = [L/T]$: a constant of proportionality found by Darcy to depend on the grain size of the sand, it represents the volume discharge during a unit length of time through a unit cross-sectional area normal to flow, under a unit change in hydraulic head over a unit length of flow path, *hydraulic conductivity;* the negative sign in the equation is used by convention, in order to obtain a positive value for the volume discharge in the direction of decreasing hydraulic head, i.e. in the direction opposite to the hydraulic gradient.

Darcy conducted his experiments with descending flow in vertically positioned columns. Nevertheless, his conclusions as expressed by Equations (1.3a) and (1.3b) are valid for any flow direction relative to vertical, including horizontal and uphill flow. Some of the more important observations that can be made immediately from these equations are:

(i) flow is always in the direction of decreasing hydraulic head;
(ii) the volume discharge Q is directly proportional to the flow-field's cross-sectional area A, the hydraulic conductivity K and the hydraulic gradient $\mathrm{d}h/\mathrm{d}l$;
(iii) the flux q is directly and linearly proportional to both the hydraulic conductivity K and the hydraulic gradient $\mathrm{d}h/\mathrm{d}l$;

(iv) although the flux q has dimensions of velocity (hence the term: Darcy velocity) it represents a volumetric discharge rate;
(v) similarly, the hydraulic conductivity K has dimensions of velocity but in reality it is the specific flux that occurs under a unit hydraulic gradient;
(vi) since K is the flux normalized to a unit hydraulic gradient $\mathrm{d}h/\mathrm{d}l = 1$, its magnitude expresses the ease with which the fluid passes through the permeable medium.

1.2.2 Fluid-dynamic parameters

The fluid-dynamic parameters are variables with magnitudes and distribution in space and time that define and characterize the fields of flow and driving force. Consequently, they can be used graphically and/or numerically to map and analyse the intensity and direction of groundwater flow.

1.2.2.1 Fluid potential, Φ, and hydraulic head, h

The *fluid potential,* Φ, and the *hydraulic head, h*, are fluid-dynamic parameters whose physical meaning can be understood from Darcy's Law. One of the fundamental conclusions drawn from Darcy's experiments is that flow is not controlled either by elevation or by pressure exclusively. Figure 1.6 shows fluid pressures p to be proportional to the height of the water column ψ above measurement points P of elevation z in manometers that are open at the top. Accordingly, flow takes place from higher to lower elevation (from z_1 to z_2) but from lower to higher pressure

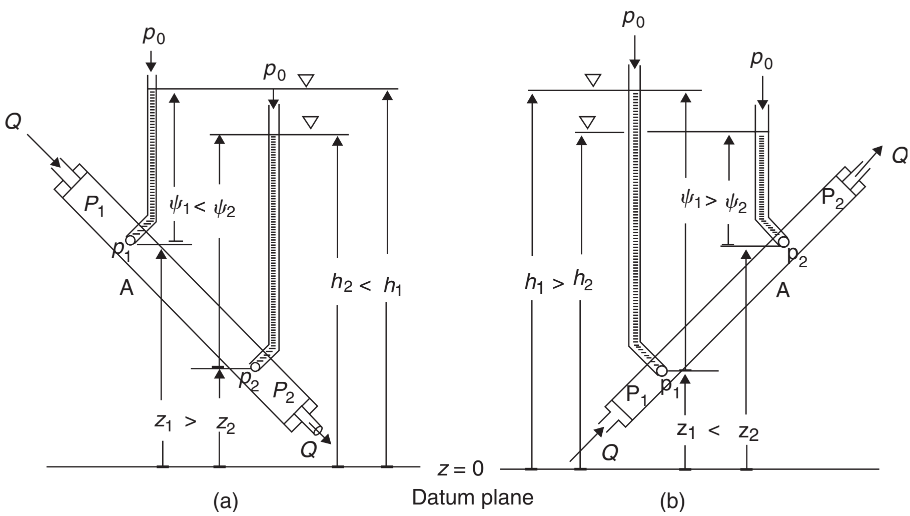

Fig. 1.6 Illustration of hydraulic-head change as the unique control on the direction of water flow: (a) flow from low to high pressure; (b) flow from low to high elevation.