# DATA ANALYSIS FOR PHYSICAL SCIENTISTS: FEATURING EXCEL

The ability to summarise data, compare models and apply computer-based analysis tools are vital skills necessary for studying and working in the physical sciences. This textbook supports undergraduate students as they develop and enhance these skills.

Introducing data analysis techniques, this textbook pays particular attention to the internationally recognised guidelines for calculating and expressing measurement uncertainty. This new edition has been revised to incorporate Excel<sup>®</sup> 2010. It also provides a practical approach to fitting models to data using non-linear least squares, a powerful technique that can be applied to many types of model.

Worked examples using actual experimental data help students understand how the calculations apply to real situations. Over 200 in-text exercises and end of chapter problems give students the opportunity to use the techniques themselves and gain confidence in applying them. Answers to the exercises and problems are given at the end of the book.

LES KIRKUP is an Associate Professor in the School of Physics and Advanced Materials, University of Technology, Sydney. He is also an Australian Learning and Teaching Council National Teaching Fellow. A dedicated lecturer, many of his educational developments have focused on enhancing the laboratory experience of undergraduate students.

# Data Analysis for Physical Scientists

# Featuring Excel<sup>®</sup>

**Les Kirkup** University of Technology, Sydney



CAMBRIDGE UNIVERSITY PRESS Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi, Tokyo, Mexico City

Cambridge University Press The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org Information on this title: www.cambridge.org/9780521883726

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First published 2012

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data Kirkup, Les. Data analysis for physical scientists : featuring Excel / Les Kirkup. – 2nd ed. p. cm. Rev. ed. of: Data analysis with Excel. 2002. Includes bibliographical references and index. ISBN 978-0-521-88372-6 1. Research – Statistical methods – Data processing. 2. Electronic spreadsheets. 3. Microsoft Excel (Computer file) I. Kirkup, Les. Data analysis with Excel. II. Title. Q180.55.S7K57 2012 502.85'554-dc23 2011017662

ISBN 978-0-521-88372-6 Hardback

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> To Janet, Sarah and Amy nee more late neets!

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# Preface to the second edition

I thank Cambridge University Press, and in particular Simon Capelin, for the opportunity to revisit *Data Analysis with Excel*. I have revised sections of the book to include topics of contemporary relevance to undergraduate students, particularly in the area of uncertainty in measurement. I hope the book will continue to assist in developing the quantitative skills of students destined to graduate in the physical sciences. There is little doubt that the demand for such skills will continue to grow in society in general and particularly within industry, research, education and commerce.

This edition builds on the first with a new chapter added and others undergoing major or minor modifications (for example, to remedy mistakes, update references or include more end of chapter exercises).

I have taken the opportunity to include topics requested by several readers of the first edition. In particular, feedback indicated that the inclusion of a chapter on non-linear least squares and Excel's Solver would be valued and broaden the appeal of the book.

The treatment of error and uncertainty in the first edition paid insufficient attention to the international guidelines for calculating and expressing uncertainty. I hope a major rewrite of chapter 5 has gone a long way to remedying this. The international guidelines on uncertainty deserve to be better known and I trust this book can contribute something to raising awareness of the guidelines within universities and colleges. Terms not usually found in a data analysis textbook for undergraduates, such as coverage factor, standard uncertainty and expanded uncertainty, have been introduced and their relationship to more familiar terms explained as the book progresses.

Microsoft's Excel features regularly throughout the book. References to Excel and the descriptions of its functions have been updated to be consistent

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with Excel 2010. While there have been several important changes to the look and feel of Excel over earlier versions, my main aim as in the first edition has been to describe features of most value to data analysis. There have been modifications to some of Excel's built-in functions and several new functions added. Also some of the statistical algorithms (which came in for criticism in earlier versions of Excel) have been improved.

I believe that the title of the first edition of this book *Data Analysis with Excel* was somewhat misleading as it was possible to interpret that the book was dominated by Excel, when this wasn't (isn't) the case. I hope the new title betters reflects the role of Excel within the book.

Many of the problems and exercises in the book are based on real, though unpublished, data. For this I thank colleagues from my institution and beyond who have been so generous with their data. These same colleagues have been equally generous with their encouragement throughout the writing of this edition and I thank them whole-heartedly. For the contribution of extra data to this edition, I would particularly like to thank Fraser Torpy, Anna Wilson, Mike Cortie, Andy Leigh (who also supplied the image used on the front cover) Jonathan Edgar, Alison Beavis, Greg Skilbeck and Francois Malan. I would also like to thank the following people for stimulating conversations on data analysis methods: Bob Frenkel, Kendal McGuffie, Michael Bulmer, Kelly Matthews, Andy Buffler, Paul Francis, Manju Sharma, Darren Pearce, Jo McKenzie and Kate Wilson-Goosens.

# Preface to the first edition

Experiments and experimentation have central roles to play in the education of scientists. For many destined to participate in scientific enquiry through laboratory or field based studies, the ability to apply 'experimental methods' is a key skill that they rely upon throughout their professional careers. For others whose interests and circumstances take them into other fields upon completion of their studies, the experience of 'wrestling with nature' so often encountered in experimental work, offers enduring rewards: Skills developed in the process of planning, executing and deliberating upon experiments are of lasting value in a world in which some talents become rapidly redundant.

Laboratory and field based experimentation are core activities in the physical sciences. Good experimentation is a blend of insight, imagination, skill, perseverance and occasionally luck. Vital to experimentation is data analysis. This is rightly so, as careful analysis of data can tease out features and relationships not apparent at a first glance at the 'numbers' emerging from an experiment. This, in turn, may suggest a new direction for the experiment that might offer further insight into a phenomenon or effect being studied. Equally importantly, after details of an experiment are long forgotten, facility gained in applying data analysis methods remains as a highly valued and transferable skill.

My experience of teaching data analysis techniques at undergraduate level suggests that when the elements of content, relevance and access to contemporary analysis tools are sympathetically blended, students respond positively and enthusiastically. Believing that no existing text encourages or supports such a 'blend', I decided to write one. This text offers an introduction to data analysis techniques recognising the background and needs of students from the physical sciences. I have attempted to include those techniques most useful

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to students from the physical sciences and employ examples that have a physical sciences 'bias'.

It is natural to turn to the computer when the 'number crunching' phase of data analysis begins. Though many excellent computer based data analysis packages exist, I have chosen to exploit the facilities offered by spreadsheets throughout the text. In their own right, spreadsheets are powerful analysis tools which are likely to be familiar and readily accessible to students.

More specifically, my goals have been to,

- provide a readable text from which students can learn the basic principles of data analysis.
- ensure that problems and exercises are drawn from situations likely to be familiar and relevant to students from the physical sciences.
- remove much of the demand for manual data manipulation and presentation by incorporating the spreadsheet as a powerful and flexible utility.
- emphasise the analysis tools most often used in the physical sciences.
- focus on aspects often given little attention in other texts for scientists such as the treatment of systematic errors.
- encourage student confidence by incorporating 'worked' examples followed by exercises.
- provide access to extra topics not dealt with directly in the text through generally accessible Web pages.

Computers are so much a part of professional and academic life that I am keen to include their use, especially where this aids the learning and application of data analysis techniques. The Excel spreadsheet package by Microsoft has been chosen due to its flexibility, availability, longevity and the care that has been taken by its creators to provide a powerful yet 'user friendly' environment for the processing and presentation of data. This text does not, however, attempt a comprehensive coverage of the features of Excel. Anyone requiring a text focussing on Excel, and its many options, shortcuts and specialist applications must look elsewhere as only those features of most relevance to the analysis of experimental data are dealt with here.

While chapter 1 contains some material normally encountered at first year level, the text as a whole has been devised to be useful at intermediate and senior undergraduate levels. Derivations of formulae are mostly avoided in the body of the text. Instead, emphasis has been given to the assumptions underlying the formulae and range of applicability. Details of derivations may be found in the appendices. It is assumed that the reader is familiar with introductory calculus, graph plotting and the calculations of means and standard deviations. Experience of laboratory work at first year undergraduate level is also an advantage.

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I am fortunate that many people have given generously of their time to help me during the preparation of this book. Their ideas, feedback and not least their encouragement are greatly appreciated. I also acknowledge many intense Friday night discussions with students and colleagues on matters relating to data analysis and their frequent pleadings with me to 'get a life'.

I would like to express my appreciation and gratitude to the following people:

From the University of Technology, Sydney (UTS):

Geoff Anstis, Mark Berkahn, Graziella Caprarelli, Bob Cheary, Michael Dawson, Chris Deller, Sherri Hilario, Suzanne Hogg, Ann-Marie Maher, Kendal McGuffie, Mary Mulholland, Matthew Phillips, Andrew Searle, Brian Stephenson, Mike Stevens, Paul Swift.

Formerly of UTS:

Andreas Reuben, Tony Fisher-Cripps, Patsy Gallagher, Gary Norton

Finally, I thank Janet Sutherland for her encouragement and support during the preparation of this text.