Introduction

Plates are structural elements given by a flat surface with a given thickness h. The flat surface is the middle surface of the plate; the upper and lower surfaces delimiting the plate are at distance $h/2$ from the middle surface. The thickness is small compared with the in-plane dimensions and can be either constant or variable. Thin plates are very stiff for in-plane loads, but they are quite flexible in bending. Many applications of plates, made of extremely different materials, can be found in engineering. For example, very thin circular plates are used in computer hard-disk drives; rectangular and trapezoidal plates can be found in the wing skin, horizontal tail surfaces, flaps and vertical fins of aircraft; cantilever rectangular plates are used as nano-resonators for drug detection; and flat rectangular panels are largely used in civil buildings.

If the middle surface describing the structural elements is folded, shells are obtained. Such structures are abundantly present in nature. In fact, because of the curvature of the middle surface, shells are very stiff for both in-plane and bending loads; therefore, they can span over large areas by using a minimum amount of material.

Shells are largely used in engineering; some shell structures are impressive and beautiful. In automotive engineering, the bodies of cars are shells; in biomechanics, arteries are shells conveying flow. Shell structural elements are largely present in the NASA space shuttle in Figure 1, where the solid rocket boosters and the big orange tank for liquid fuels are large shell bodies. Actually, the super lightweight orange tank is composed of a smaller top tank of ogival shape containing liquid oxygen and a bigger bottom tank of circular cylindrical shape containing liquid hydrogen joined by an intertank element. Shells are: the hull of the *Queen Mary 2* transatlantic boat shown in Figure 2 and of the Los Angeles-class fast attack submarine in Figure 3; the roof of *L'Hemisféric* designed by Santiago Calatrava in the Ciutat del les Arts i les Ciéncies, Valencia, Spain (Figure 4); and the fuselage and wing panels of the huge Airbus A380 civil aircraft in Figure 5.

One of the main targets in the design of shell structural elements is to make the thickness as small as possible to spare material and to make the structure light. The analysis of shells has difficulty related to the curvature, which is also the reason for the carrying load capacity of these structures. In fact, a change of the curvature can give a totally different strength (Chapelle and Bathe 2003). Moreover, because of the optimal distribution of material, shells collapse for buckling much before the failure strength of the material is reached. For their thin nature, they can present large displacements, with respect to the shell thickness, associated to small strains

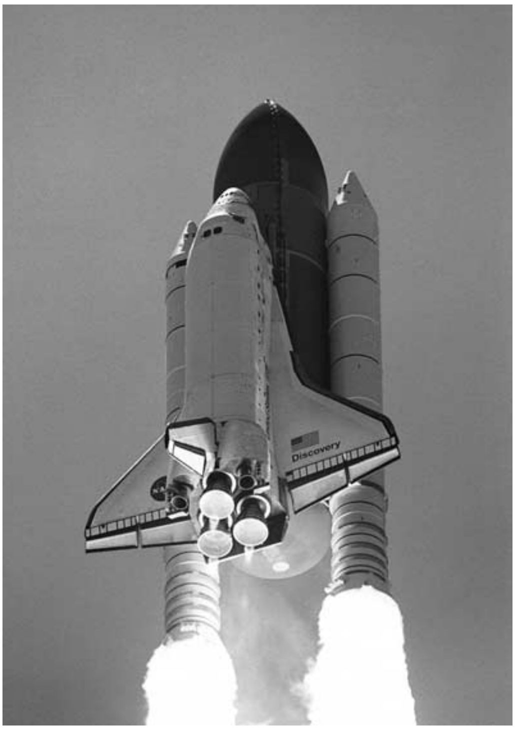

Figure 1. NASA space shuttle: orbiter *Discovery* with the two solid rocket boosters and the large orange external tank. Courtesy of NASA.

before collapse. This is the rationale for using a nonlinear shell and plate theory for studying shell stability.

Shells are often subjected to dynamic loads that cause vibrations; vibration amplitudes of the order of the shell thickness can be easily reached in many applications. Therefore, a nonlinear shell theory should be applied.

Figure 2. The *Queen Mary 2* transatlantic boat. Courtesy of Carnival Corporation & PLC.

Figure 3. The Los Angeles-class fast attack submarine USS *Asheville*. Courtesy of the U.S. Navy.

The book is organized into 15 chapters. Chapter 1 obtains classical nonlinear theories for rectangular and circular plates, circular cylindrical and spherical shells. Classical shell theories for doubly curved shallow shells and for shells of arbitrary shape are obtained in Chapter 2. Composite and innovative functionally graded materials

Figure 4. *L'Hemisféric* designed by Santiago Calatrava in the Ciutat del les Arts i les Ciéncies, Valencia, Spain. Alan Copson/age fotostock.

Figure 5. Airbus A380 aircraft. Plath/age fotostock.

are introduced and nonclassical nonlinear theories, including shear deformation and rotary inertia, for laminated and functionally graded shells are developed; thermal stresses are also introduced. The first two chapters are self-contained with the full development of the theories under clear hypotheses and limitations. They present material that is usually spread in several articles and books with different approaches and symbols. The shell theories are expressed in lines-of-curvature coordinates, which is the form suitable for applications and computer implementation. In some cases, slightly improved formulations, suitable for moderately thick shells and large rotations, have been developed.

The nonlinear dynamics, stability, bifurcation analysis and modern computational tools are introduced in Chapter 3. The Galerkin method and the energy approach that leads to the Lagrange equations of motion are introduced here.

Linear and nonlinear vibrations of rectangular plates and simply supported circular cylindrical shells (closed around the circumference) are throughly studied in Chapters 4 and 5, respectively. In particular, the Lagrange equations are used for discretizing plates and the Galerkin method is used for discretizing circular shells to show both methods. Numerical and experimental results are presented and compared. The effect of geometric imperfections is also addressed in both chapters. The problem of inertial coupling in the equations of motion is analyzed in Chapter 4. Fluid-filled circular shells are investigated with great care in Chapter 5 for their important applications. Chaotic vibrations are analyzed for large harmonic excitation.

Modern numerical techniques, specifically proper orthogonal decomposition (POD) and nonlinear normal modes (NNM) methods, that reduce the number of degrees of freedom in nonlinear shell models are presented in Chapter 6. The comparison of different classical nonlinear shell theories to study large-amplitude vibrations of simply supported circular cylindrical shells is performed in Chapter 7, where the Lagrange equations are used; the nonlinear stability of pressurized shells is also investigated. Nonlinear vibrations of circular cylindrical shells with different boundary conditions are addressed in Chapter 8. Linear and nonlinear vibrations of circular cylindrical panels (open shells) with different boundary conditions are studied in Chapter 9. Also in this case numerical and experimental results are presented and successfully compared. Chaotic vibrations are detected for large harmonic excitations. Nonlinear vibrations of doubly curved shallow (i.e. with small rise) shells with rectangular base, including spherical and hyperbolic paraboloidal shells, are investigated in Chapter 10. Both classical and first-order shear deformation theories are used to study nonlinear vibrations of laminated composite shells. Static buckling, including the example of the external tank of the NASA space shuttle, is also addressed.

The R-function method for meshless discretization of shells and plates of complex shape is introduced in Chapter 11. This is a method with great potential to develop commercial software, but very little is still known about it outside of the Ukraine and Russia. Linear and nonlinear vibrations of circular plates and rotating disks are investigated in Chapter 12. They have an important application in engineering; for example, in hard-disk, CD and DVD drives of computers.

Nonlinear stability of circular cylindrical shells under static and periodic axial loads are studied in Chapter 13. The fundamental effects of geometric imperfections on the buckling load are investigated, and the period-doubling bifurcation giving dynamic instability in periodic loads is deeply analyzed. The problem of stability of circular cylindrical shells conveying a subsonic flow is addressed in Chapter 14, where a strongly subcritical divergence is detected. The fluid-structure interaction problem for inviscid and incompressible flow is fully studied and the numerical and experimental results obtained in a water tunnel are compared. Nonlinear forced vibrations of circular cylindrical shells conveying water flow are studied for both moderate and large excitations, giving periodic response and complex dynamics, respectively. Flutter instability of circular cylindrical shells inserted in axial supersonic airflow is finally investigated in Chapter 15 by using either linear or nonlinear piston theory to model the aerodynamic loads, and a nonlinear shell model taking into account geometric imperfections. Numerical results are compared with the results of experiments performed by the National Aeronautics and Space Administration (NASA).

REFERENCE

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1 Nonlinear Theories of Elasticity of Plates and Shells

1.1 Introduction

It is well known that certain elastic bodies may undergo large displacements while the strain at each point remains small. The classical theory of elasticity treats only problems in which displacements and their derivatives are small. Therefore, to treat such cases, it is necessary to introduce a theory of nonlinear elasticity with small strains. If the strains are small, the deformation in the neighborhood of each point can be identified with a deformation to which the linear theory is applicable. This gives a rationale for adopting Hooke's stress-strain relations, and in the resulting nonlinear theory large parts of the classical theory are preserved (Stoker 1968). However, the original and deformed configuration of a solid now cannot be assumed to be coincident, and the strains and stresses can be evaluated in the original undeformed configuration by using Lagrangian description, or in the deformed configuration by using Eulerian description (Fung 1965).

In this chapter, the classical geometrically nonlinear theories for rectangular plates, circular cylindrical shells, circular plates and spherical shells are derived, classical theories being those that neglect the shear deformation. Results are obtained in Lagrangian description, the effect of geometric imperfections is considered and the formulation of the elastic strain energy is also given. Classical theories for shells of any shape, as well as theories including shear deformation, are addressed in Chapter 2.

1.1.1 Literature Review

A short overview of some theories for geometrically nonlinear shells and plates will now be given. Some information is taken from the review by Amabili and Païdoussis (2003).

In the classical linear theory of plates, there are two fundamental methods for the solution of the problem. The first method was proposed by Cauchy (1828) and Poisson (1829) and the second by Kirchhoff (1850). The method of Cauchy and Poisson is based on the expansion of displacements and stresses in the plate in power series of the distance z from the middle surface. Disputes concerning the convergence of these series and about the necessary boundary conditions made this method unpopular. Moreover, the method proposed by Kirchhoff has the advantage of introducing physical meaning into the theory of plates. Von Kármán (1910) extended this method to study finite deformation of plates, taking into account nonlinear terms. The nonlinear dynamic case was studied by Chu and Herrmann (1956), who were the pioneers in studying nonlinear vibrations of rectangular plates. In order to deal with thicker and laminated composite plates, the Reissner-Mindlin theory of plates (first-order shear deformation theory) was introduced to take into account transverse shear strains. Five variables are used in this theory to describe the deformation: three displacements of the middle surface and two rotations. The Reissner-Mindlin approach does not satisfy the transverse shear boundary conditions at the top and bottom surfaces of the plate, because a constant shear angle through the thickness is assumed, and plane sections remain plane after deformation. As a consequence of this approximation, the Reissner-Mindlin theory of plates requires shear correction factors for equilibrium considerations. For this reason, Reddy (1990) has developed a nonlinear plate theory that includes cubic terms (in the distance from the middle surface of the plate) in the in-plane displacement kinematics. This higher-order shear deformation theory satisfies zero transverse shear stresses at the top and bottom surfaces of the plate; up to cubic terms are retained in the expression of the shear, giving a parabolic shear strain distribution through the thickness, resembling with good approximation the results of three-dimensional elasticity. The same five variables of the Reissner-Mindlin theory are used to describe the kinematics in this higher-order shear deformation theory, but shear correction factors are not required.

Donnell (1934) established the nonlinear theory of circular cylindrical shells under the simplifying shallow-shell hypothesis. Because of its relative simplicity and practical accuracy, this theory has been widely used. The most frequently used form of Donnell's nonlinear shallow-shell theory (also referred to as Donnell-Mushtari-Vlasov theory) introduces a stress function in order to combine the three equations of equilibrium involving the shell displacements in the radial, circumferential and axial directions into two equations involving only the radial displacement w and the stress function F. This theory is accurate only for modes with circumferential wavenumber n that are not small: specifically, $1/n^2 \ll 1$ must be satisfied, so that $n \geq 4$ or 5 is required in order to have fairly good accuracy. Donnell's nonlinear shallow-shell equations are obtained by neglecting the in-plane inertia, transverse shear deformation and rotary inertia, giving accurate results only for very thin shells. The predominant nonlinear terms are retained, but other secondary effects, such as the nonlinearities in curvature strains, are neglected; specifically, the curvature changes are expressed by linear functions of w only.

Von Kármán and Tsien (1941) performed a seminal study on the stability of axially loaded circular cylindrical shells, based on Donnell's nonlinear shallow-shell theory. In their book, Mushtari and Galimov (1957) presented nonlinear theories for moderate and large deformations of thin elastic shells. The nonlinear theory of shallow-shells is also discussed in the book by Vorovich (1999), where the classical Russian studies, for example, due to Mushtari and Vlasov, are presented.

Sanders (1963) developed a more refined nonlinear theory of shells, expressed in tensorial form. The same equations were obtained by Koiter (1966) around the same period, leading to the designation of these equations as the Sanders-Koiter equations. Later, this theory was reformulated in lines-of-curvature coordinates, that is, in a form that can be more suitable for applications; see, for example, Budiansky (1968), where only linear terms are given. According to the Sanders-Koiter theory, all three displacements are used in the equations of motion. Changes in curvature and torsion are linear according to both the Donnell and the Sanders-Koiter nonlinear theories (Yamaki 1984). The Sanders-Koiter theory gives accurate results for vibration amplitudes significantly larger than the shell thickness for thin shells (Amabili 2003).

Details on the above-mentioned nonlinear shell theories may be found in Yamaki (1984) and Amabili (2003), with an introduction to another accurate theory called the modified Flügge nonlinear theory of shells, also referred to as the Flügge-Lur'e-Byrne nonlinear shell theory (Ginsberg 1973). The Flügge-Lur'e-Byrne theory is close to the general large deflection theory of thin shells developed by Novozhilov (1953) and differs only in terms for change in curvature and torsion.

Additional nonlinear shell theories were formulated by Naghdi and Nordgren (1963), using the Kirchhoff hypotheses, and by Libai and Simmonds (1988).

In order to treat moderately thick laminated shells, the nonlinear first-order shear deformation theory of shells was introduced by Reddy and Chandrashekhara (1985), which is based on the linear first-order shear deformation theory introduced by Reddy (1984). Five independent variables, three displacements and two rotations, are used to describe the shell deformation. This theory may be regarded as the thick-shell version of the Sanders theory for linear terms and of the Donnell nonlinear shell theory for nonlinear terms. A linear higher-order shear deformation theory of shells has been introduced by Reddy and Liu (1985); see also Reddy (2003). Dennis and Palazotto have extended this theory to nonlinear deformations (1990); see also Soldatos (1992).

The nonlinear mechanics of composite laminated shells has also been investigated by many authors. Librescu (1987) developed refined nonlinear theories for anisotropic laminated shells. Other theories applied to the dynamics of laminated shells have been developed, for example, by Tsai and Palazotto (1991), Kobayashi and Leissa (1995), Sansour et al. (1997), Gummadi and Palazotto (1999) and Pai and Nayfeh (1994). Nonlinear electromechanics of piezoelectric laminated shallow spherical shells was developed by Zhou and Tzou (2000).

1.2 Large Deflection of Rectangular Plates

1.2.1 Green's and Almansi Strain Tensors for Finite Deformation

It is assumed that a continuous body changes its configuration under physical actions and the change is continuous (no fractures are considered). A system of coordinates x_1, x_2, x_3 is chosen so that a point P of a body at a certain instant of time is described by the coordinates x_i ($i = 1, 2, 3$). At a later instant of time, the body has moved and deformed to a new configuration; the point P has moved to Q with coordinates $a_i (i = 1, 2, 3)$ with respect to a new coordinate system a_1, a_2, a_3 (see Figure 1.1). Both coordinate systems are assumed to be the same rectangular Cartesian (rectilinear and orthogonal) coordinates for simplicity. The point transformation from P to Q is considered to be one-to-one, so that there is a unique inverse of the transformation. The functions x_i and a_i, describing the coordinates, are assumed to be continuous and differentiable, and the Jacobian determinant of the transformation is positive (i.e. a right-hand set of coordinates is transformed into another right-hand set) and does not vanish at any point. The displacement vector $\mathbf{u}$ is introduced having the following components:

$$u_i = a_i - x_i \quad \text{for } i = 1, \ldots, 3. \tag{1.1}$$

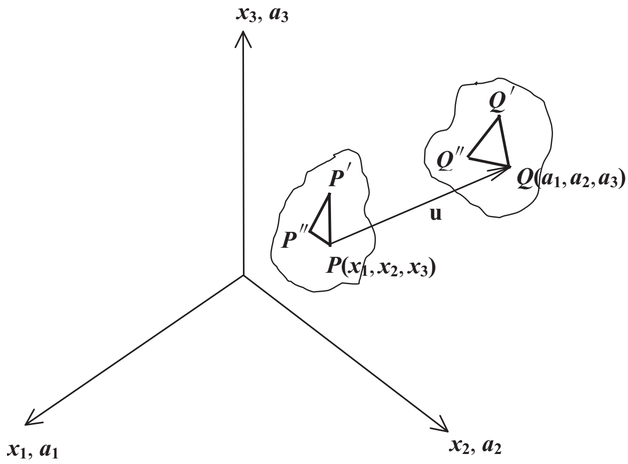

Figure 1.1. Body in the original configuration and after displacement **u**, which moves point P to Q.

In the present book, the Lagrangian description of the continuous systems is used for convenience; therefore u_i are considered to be functions of x_i in order to evaluate the Lagrangian (or Green's) strain tensor, that is, the strains are referred to the original undeformed configuration.

If P, P', P'' are three neighboring points forming a triangle in the original configuration, and if they are transformed to points Q, Q', Q'' in the deformed configuration, the change in the area and angles of the triangle is completely determined if the change in the length of the sides is known. However, the location of the triangle in space is not determined by the change of the sides. Similarly, if the change in length between any two arbitrary points of the body is known, the new configuration of the body is completely defined except for the location of the body in the space. Because interest here is on strains, and these are related to stresses, attention is now focused on the change in distance between any two points of the body.

An infinitesimal line connecting point P (x_1, x_2, x_3) to a neighborhood point P' $(x_1 + \mathrm{d}x_1, x_2 + \mathrm{d}x_2, x_3 + \mathrm{d}x_3)$ is considered; the square of its length in the original configuration is given by

$$\overline{PP'}^2 = \mathrm{d}s_0^2 = \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2. \tag{1.2}$$

When, due to deformation, P and P' become $Q(a_1, a_2, a_3)$ and $Q'(a_1 + \mathrm{d}a_1, a_2 + \mathrm{d}a_2, a_3 + \mathrm{d}a_3)$, respectively, the square of the distance is

$$\overline{QQ'}^2 = \mathrm{d}s^2 = \mathrm{d}a_1^2 + \mathrm{d}a_2^2 + \mathrm{d}a_3^2, \tag{1.3}$$

in the coordinate system a_i. The differentials $\mathrm{d}a_i$ can be transformed in the original coordinate system x_i:

$$\mathrm{d}a_i = \frac{\partial a_i}{\partial x_1}\mathrm{d}x_1 + \frac{\partial a_i}{\partial x_2}\mathrm{d}x_2 + \frac{\partial a_i}{\partial x_3}\mathrm{d}x_3. \tag{1.4}$$

Therefore, by using equation (1.4), equation (1.3) is transformed into

$$\mathrm{d}s^2 = \sum_{k=1}^{3}\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{\partial a_k}{\partial x_i}\frac{\partial a_k}{\partial x_j}\,\mathrm{d}x_i\,\mathrm{d}x_j. \tag{1.5}$$

The difference between the squares of the length of the elements may be written in the following form by using equations (1.2) and (1.5):

$$\mathrm{d}s^2 - \mathrm{d}s_0^2 = \sum_{k=1}^{3}\sum_{i=1}^{3}\sum_{j=1}^{3}\left(\frac{\partial a_k}{\partial x_i}\frac{\partial a_k}{\partial x_j} - \delta_{ij}\right)\mathrm{d}x_i\,\mathrm{d}x_j, \tag{1.6}$$

where δ_{ij} is the Kronecker delta, equal to 1 if $i = j$ and otherwise equal to zero. By definition, Green's strain tensor ε_{ij} is obtained as

$$\mathrm{d}s^2 - \mathrm{d}s_0^2 = 2\sum_{i=1}^{3}\sum_{j=1}^{3}\varepsilon_{ij}\,\mathrm{d}x_i\,\mathrm{d}x_j, \tag{1.7}$$

and therefore is given by

$$\varepsilon_{ij} = \frac{1}{2}\left(\sum_{k=1}^{3}\frac{\partial a_k}{\partial x_i}\frac{\partial a_k}{\partial x_j} - \delta_{ij}\right). \tag{1.8}$$

By using equation (1.1), the following expression is obtained:

$$\frac{\partial a_k}{\partial x_i} = \frac{\partial u_k}{\partial x_i} + \delta_{ki}. \tag{1.9}$$

Finally, by substituting equation (1.9) into equation (1.8), Green's strain tensor is expressed as

$$\begin{aligned}\varepsilon_{ij} &= \frac{1}{2}\left[\left(\sum_{k=1}^{3}\frac{\partial u_k}{\partial x_i} + \delta_{ki}\right)\left(\sum_{k=1}^{3}\frac{\partial u_k}{\partial x_j} + \delta_{kj}\right) - \delta_{ij}\right]\\ &= \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \sum_{k=1}^{3}\frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right).\end{aligned} \tag{1.10}$$

In unabridged notation (x, y, z for x_1, x_2, x_3), the typical formulations are obtained:

$$\varepsilon_{xx} = \frac{\partial u_1}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u_1}{\partial x}\right)^2 + \left(\frac{\partial u_2}{\partial x}\right)^2 + \left(\frac{\partial u_3}{\partial x}\right)^2\right], \tag{1.11a}$$

$$\gamma_{xy} = \frac{1}{2}\left[\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} + \left(\frac{\partial u_1}{\partial x}\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial x}\frac{\partial u_3}{\partial y}\right)\right], \tag{1.11b}$$

where the usual symbol γ_{xy} has been used instead of ε_{xy} in equation (1.11b). Equation (1.10) shows that Green's strain tensor is symmetric.

If the Eulerian description of the continuous systems is used, u_i are considered functions of a_i in order to evaluate the Eulerian (usually referred to as the Almansi) strain tensor (i.e. the strains are referred to the deformed configuration). With analogous mathematical development, the Almansi strain tensor is given by

$$\begin{aligned}\varepsilon_{ij}^{(E)} &= \frac{1}{2}\left[\delta_{ij} - \left(-\sum_{k=1}^{3}\frac{\partial u_k}{\partial a_i} + \delta_{ki}\right)\left(-\sum_{k=1}^{3}\frac{\partial u_k}{\partial a_j} + \delta_{kj}\right)\right]\\ &= \frac{1}{2}\left(\frac{\partial u_i}{\partial a_j} + \frac{\partial u_j}{\partial a_i} - \sum_{k=1}^{3}\frac{\partial u_k}{\partial a_i}\frac{\partial u_k}{\partial a_j}\right),\end{aligned} \tag{1.12}$$

which is also symmetric; the superscript denotes "Eulerian."