A BREVIARY OF SEISMIC TOMOGRAPHY

Imaging the Interior of the Earth and Sun

This is the first textbook to cover all the major aspects of seismic tomography at a level accessible to students. While focusing on applications in solid earth geophysics, the book also includes numerous excursions into helioseismology in order to demonstrate the strong affinity between the two fields.

The book presents a comprehensive introduction to seismic tomography including the basic theory of wave propagation, the ray and Born approximations required for interpretation of amplitudes, travel times and phases, eigenvibrations and surface waves, observational methods, model parametrization, finite-frequency methods, inversion, error and resolution analysis, and seismic anisotropy. It presents in-depth consideration of observational aspects of the subject, as well as practical recommendations for implementing numerical models using publicly available software.

Written by one of the leaders in the field, and containing numerous student exercises, this textbook is appropriate for advanced undergraduate and graduate courses. It is also an invaluable guide for seismology research practitioners in geophysics and astronomy. Solutions to the exercises, and a link to the author's tomographic software and user manual are available online from www.cambridge.org/9780521882446.

Guust Nolet is the George J. Magee Professor of Geophysics Emeritus at Princeton University and currently teaches at the University of Nice/Sophia Antipolis in France. His research interests include seismic body wave tomography and investigations of the structure of the mantle and its role in shaping the Earth's surface. Professor Nolet is the winner of the 2006 Beno Gutenberg Medal of the European Geophysical Union, the 1983 Vening Meinesz prize, and the 1980 Prix Lagrange. He is a Fellow of the American Geophysical Union and is a member of the Academia Europea, the Royal Netherlands Academy of Sciences, and the American Academy of Arts and Sciences.

A BREVIARY OF SEISMIC TOMOGRAPHY

Imaging the Interior of the Earth and Sun

Guust Nolet
University of Nice/Sophia Antipolis, France

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521882446

First published 2008

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
Nolet, Guust, 1945–
A breviary of seismic tomography : imaging the interior of the earth and sun / Guust Nolet.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-521-88244-6 (hardback)
1. Seismic tomography. 2. Geodynamics. I. Title.
QE538.5.N65 2008
551.1′10284 – dc22 2008020508

ISBN 978-0-521-88244-6 hardback

To Tony Dahlen (1942–2007)

Bre"vi*a*ry, n. [F. bréviarie, L. breviarium summary, abridgment, neut. noun fr. breviarius abridged, fr. brevis short.]
1. An abridgment; a compend; an epitome; a brief account or summary. A book entitled the abridgment or breviary of those roots that are to be cut up or gathered.
2. A book containing the daily public or canonical prayers of the Roman Catholic or of the Greek Church for the seven canonical hours, namely, matins and lauds, the first, third, sixth, and ninth hours, vespers, and compline; – distinguished from the missal.

(Webster Dictionary, 1913, Page: 180)

Contents

Preface — *page* xi

1 Introduction — 1
- 1.1 Early efforts at seismic tomography — 2
- 1.2 Ocean acoustic tomography — 3
- 1.3 Global tomography — 3
- 1.4 Some major discoveries — 4
- 1.5 Helioseismology — 7
- 1.6 Finite-frequency tomography — 8

2 Ray theory for seismic waves — 11
- 2.1 The stress tensor — 11
- 2.2 Forces in continuous media — 12
- 2.3 Newton's law and the elastodynamic equations — 13
- 2.4 The acoustic wave equation — 16
- 2.5 The ray approximation — 20
- 2.6 Ray solutions in layered and spherical systems — 22
- 2.7 Geometrical spreading — 25
- 2.8 Rays in an isotropic, elastic Earth — 26
- 2.9 Fermat's Principle — 29
- 2.10 Huygens, Fresnel and Green — 30
- 2.11 Flow: solar p-waves or ocean acoustic waves — 34
- 2.12 Appendix A: Some elements of Fourier analysis — 36

3 Ray tracing — 40
- 3.1 The shooting method — 40
- 3.2 Ray bending — 42
- 3.3 Other raytracing algorithms for 3D media — 46
- 3.4 Ray-centred coordinates — 49
- 3.5 Dynamic ray tracing — 50
- 3.6 Ray tracing on the sphere — 53

3.7 Computational aspects 54
4 Wave scattering 58
4.1 The acoustic Green's function 60
4.2 An acoustic point scatterer 62
4.3 Green's functions for elastic waves 63
4.4 Green's functions in the ray approximation 68
4.5 The Born approximation 70
4.6 Scattering of a plane wave 73
4.7 The scattering matrix 77
4.8 Appendix B: The impulse response 79
5 Body wave amplitudes: theory 82
5.1 Geometrical spreading 82
5.2 The quality factor Q 84
5.3 The correspondence principle 86
5.4 Attenuating body waves 88
5.5 Scattering 91
6 Travel times: observations 93
6.1 Phase picks 96
6.2 Matched filters 100
6.3 Wavelet estimation 103
6.4 Differential times 107
6.5 Signal and noise 109
6.6 Time–distance analysis in helioseismology 110
7 Travel times: interpretation 116
7.1 The ray theoretical interpretation 116
7.2 Cross-correlation of seismic arrivals 121
7.3 Forward scattering 125
7.4 Finite frequency sensitivity: a simple example 126
7.5 Finite frequency kernels: general 129
7.6 Alternative arrival time measurements 135
7.7 Alternative methods for kernel computation 138
7.8 Computational aspects 139
8 Body wave amplitudes: observation and interpretation 145
8.1 Amplitude observations 146
8.2 t^* observations 149
8.3 Amplitude healing 151
8.4 Boundary topography 152
8.5 Finite-frequency Q tomography 154
9 Normal modes 158
9.1 The discrete spectrum 159

9.2 Rayleigh's Principle 166
9.3 Mode splitting 168
9.4 Observations of mode splits 175
10 Surface wave interpretation: ray theory 178
10.1 The theory of surface waves 180
10.2 Love and Rayleigh waves 182
10.3 Measuring fundamental mode dispersion 187
10.4 Measuring higher mode dispersion 190
10.5 Waveform fitting 192
10.6 Partitioned waveform inversion (PWI) 194
10.7 Appendix C: Asymptotic theory 197
11 Surface waves: finite-frequency theory 208
11.1 Phase and amplitude perturbations 209
11.2 Practical considerations 214
11.3 Phase velocity maps: an incompatibility 217
12 Model parametrization 219
12.1 Global parametrization 220
12.2 Local parametrization 222
12.3 Numerical considerations 228
12.4 Spectral analysis and model correlations 229
13 Common corrections 233
13.1 Ellipticity corrections 233
13.2 Topographic and bathymetric time corrections 238
13.3 Crustal time corrections 240
13.4 Surface wave corrections 243
13.5 Source corrections 245
13.6 Amplitude corrections for body waves 248
13.7 Dispersion corrections 250
13.8 Instrument response 251
13.9 Clock corrections 253
14 Linear inversion 255
14.1 Maximum likelihood estimation and least squares 256
14.2 Alternatives to least squares 260
14.3 Singular value decomposition 261
14.4 Tikhonov regularization 265
14.5 Bayesian inference 266
14.6 Information theory 270
14.7 Numerical considerations 272
14.8 Appendix D: Some concepts of probability theory and statistics 275

15 Resolution and error analysis 277
15.1 Resolution matrix 277
15.2 Backus–Gilbert theory 281
15.3 Sensitivity tests 285
16 Anisotropy 289
16.1 The elasticity tensor 290
16.2 Waves in homogeneous anisotropic media 295
16.3 S-wave splitting 296
16.4 Surface wave anisotropy 301
17 Future directions 306
17.1 Beyond Born 306
17.2 Adjoint methods 307
17.3 Global coverage of seismic sensors 310
17.4 Helioseismology and astroseismology 312
References 313
Author index 334
General index 339

Preface

Avec tout ce que je sais, on pourrait faire un livre. Il est vrai qu'avec tout ce que je ne sais pas, on pourrait faire une bibliothèque.

Sacha Guitry

After working on research topics related to seismic tomography for a quarter of a century, I decided it was time to write down all I know about the topic – but not in a grand unifying tome that covers everything from first principles to numerical applications. First of all, I have little patience for mathematical niceties; second, and more importantly, I wrote this book for the *practitioners* of the craft of seismic tomography. Those who go out into the field to collect data usually have no time for proofs of convergence or existence. The intended reader of this book is therefore an observational seismologist or helioseismologist who is not interested in lengthy derivations nor in the subtleties that fascinate the theoreticians, but who wants to understand the assumptions behind algorithms, even if these are mathematically intricate, and develop an understanding of the conditions for their validity, which forms the basis of that priceless commodity: scientific intuition. The level is such that it could be used for a one-semester course at upper undergraduate or beginning graduate level, perhaps following up on an introductory course based on Shearer [307] or Stein and Wysession [343]. Despite covering a wide range of topics, I have tried to keep it short (hence the title), while not economizing on references that may provide more detail if needed. As for references, choice is inevitable, and I have generally given preference to easily accessible papers in the English language. I realize that this gives short shrift to articles from Eastern Europe, Japan and China. This is unfortunate and I offer my apologies to colleagues in these countries.

The book is roughly divided into three parts. The basic theory of wave propagation and scattering needed to compute Fréchet (sensitivity) kernels for seismic tomography is expanded in the first half of the book. Although I made an attempt to write this as a self-contained part, even here I often steer away from lengthy derivations that the interested reader can find in the more general seismological literature, and strive instead to make results at least intuitively acceptable. I then discuss observations, paying attention to both the ambient noise and the capabilities of modern, digital, broadband instrumentation. The last part of the book is devoted to the tomographic inversion and imaging itself. I restrict the material to *transmission* tomography. The nonlinearities associated with reflection seismology form a topic apart, worthy of a monograph of equal or greater length than this book.

As a geophysicist, I have written this book from a 'terrestrial' viewpoint. Where the links with helioseismology are obvious, I have ventured onto the playing field of solar astronomy as well, mainly to demonstrate the large affinity between the two research fields and to help astronomers to recognize parallel developments more easily. However, geophysicists interested in the fascinating topics of helio- and astroseismology do well to consult other sources for an expert introduction into these fields. The lecture notes by Christensen-Dalsgaard [58] provide a general and very readable introduction to the theory of stellar oscillations. The 'living review' by Gizon and Birch [118] gives an up-to-date account of methods and results in local helioseismology. A special issue of Astronomische Nachrichten edited by Thierry Corbard, Laurent Gizon and Markus Roth [65] is an excellent source of information on current techniques and future plans and provides a wealth of further references.

If you are a student of the field, and undertake the journey of reading this book from first to last chapter, I hope that at the end you will not feel as though you have done the proverbial grand tour of six European cities in seven days. But I cannot deny that I try to cover a very large range of topics, each with its own jargon and notation. I have tried to stay close to notations one commonly finds in the literature. This implies that the same symbol is sometimes used with different meaning in different chapters. Usually, that meaning is clear from the context, but occasionally I have felt the need to explicitly comment on peculiarities in notation.

A special case is the notation of vectors, tensors and matrices. Throughout the book we deal with physical vectors such as the force $\boldsymbol{f} = (f_x, f_y, f_z)$, that have tensor-properties and are conceptually different from N-tuples such as the data 'vector' $\boldsymbol{d} = (d_1, d_2, ..., d_N)$ that we encounter in the last few chapters. I use the same bold font for both, and avoid the transpose notation (e.g. $\boldsymbol{d}^T$) that toggles between row and column vectors. A dot product $\boldsymbol{a} \cdot \boldsymbol{b}$ denotes $\sum_i a_i b_i$, without the complex conjugation (if it is needed, as in the definition of an inner product for normal modes in Chapter 9, it is explicitly used in the notation, e.g. $\boldsymbol{a} \cdot \boldsymbol{b}^*$). A matrix vector product such as $\sum_j A_{ij} b_j$ is written as $\boldsymbol{A}\boldsymbol{b}$, and quadratic products are therefore written as $\boldsymbol{a} \cdot \boldsymbol{A}\boldsymbol{b}$. The transpose of $\boldsymbol{A}$ is written as $\boldsymbol{A}^T$.

During the writing of this book, I have become painfully aware that the half life of a typo or sign convention or even error in math must be measured in weeks or months, not days. In view of the many that have been found before submitting the manuscript, some will undoubtedly remain. I shall be very grateful for readers to contact me about these.

Software for the computation of finite-frequency kernels is publicly available at the software website of CIG (Computational Infrastructure for Geodynamics).† In the text I refer to this as the *Software repository*.

Much of the research described in this book could never have been accomplished without the steady support of science foundations in a number of countries; in my own case the Dutch science foundation NWO and, over the last 15 years, the National Science Foundation in the US. Program directors like Robin Reichlin at NSF, who remain largely anonymous and rarely share in the glory, play a crucial role in allowing science to advance in the best way possible and making sure taxpayers' money is well spent. The ESA/NASA Solar and Heliospheric Observatory was instrumental in the acquisition of very high quality data for solar seismology as witnessed by some of the illustrations in this book.

Both as a researcher and as a teacher of the topic, I always felt a strong need for one text that covers all important aspects of the multidisciplinary science of seismic tomography. I started to write this book during a sabattical in 2005 with the intent to defy Richard's law (that one should multiply the expected time until submission – two years in my case – by π). That I succeeded is largely due to the help I received from many people. I wish to thank my colleagues at Geoazur of the Université de Nice/Sophia Antipolis, the Laboratoire de Geophysique Interne et Tectonophysique of the Université Joseph Fourier in Grenoble and the Institut de Physique du Globe in Paris, who all provided hospitable hiding space during the various stages of writing this book. A number of geoscientists and astronomers provided figures, valuable information, or commented on parts or all of earlier drafts of this book: Sebastien Chevrot, Jon Claerbout, Huub Douma, Adam Dziewonski, Bob Engdahl, Jim Fowler, Laurent Gizon, Brad Hindman, Shu-Huei Hung, Eystein Husebye, Alexander Kosovichev, Gabi Laske, Suzan van der Lee, Will Levandowski, Tolya Levshin, Guy Masters, Jean-Paul Montagner, Tarje Nissen-Meyer, Mark Panning, Jeroen Ritsema, Barbara Romanowicz, Génevieve Roult, Frederik Simons, Karin Sigloch, Roel Snieder, Toshiro Tanimoto, Albert Tarantola, Yue Tian, Jean Virieux, Cecily Wolfe, and Ying Zhou. I am very grateful to them and wish to make clear that the responsibility for any errors that survive is mine and mine alone. My beloved

† `http://geodynamics.org/cig/software/packages/seismo/`

Julia Frey corrected more than a few prepositions and other peculiarities in my use of the English language. But mostly I am indebted to my close friend Tony Dahlen with whom I collaborated intensively at Princeton and who died before he could see the final version of this book. Without his sharp theoretical insight and intellectual driving force the field of seismic tomography would never have evolved as rapidly and actively as it has.