GRAVITATION
FOUNDATIONS AND FRONTIERS

Covering all aspects of gravitation in a contemporary style, this advanced textbook is ideal for graduate students and researchers in all areas of theoretical physics.

The 'Foundations' section develops the formalism in six chapters, and uses it in the next four chapters to discuss four key applications – spherical spacetimes, black holes, gravitational waves and cosmology. The six chapters in the 'Frontiers' section describe cosmological perturbation theory, quantum fields in curved spacetime, and the Hamiltonian structure of general relativity, among several other advanced topics, some of which are covered in-depth for the first time in a textbook.

The modular structure of the book allows different sections to be combined to suit a variety of courses. More than 225 exercises are included to test and develop the readers' understanding. There are also over 30 projects to help readers make the transition from the book to their own original research.

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GRAVITATION

Foundations and Frontiers

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521882231

First published 2010

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing-in-Publication Data

Padmanabhan, T. (Thanu), 1957-
Gravitation : foundations and frontiers / T. Padmanabhan.
p. cm.
ISBN 978-0-521-88223-1 (Hardback)
1. Gravitation–Textbooks. 2. Relativity (Physics)–Textbooks. 3. Gravitation–Problems, exercises, etc. I. Title.
QC178.P226 2010
531′.14–dc22

2009038009

ISBN 978-0-521-88223-1 Hardback

Dedicated to the fellow citizens of India

Contents

List of exercises

List of projects

Preface

There is a need for a *comprehensive, advanced level*, textbook dealing with all aspects of gravity, written *for the physicist* in a *contemporary* style. The italicized adjectives in the above sentence are the key: most of the existing books on the market are either outdated in emphasis, too mathematical for a physicist, not comprehensive or written at an elementary level. (For example, the two unique books – L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, and C. W. Misner, K. S. Thorne and J. A. Wheeler (MTW), *Gravitation* – which I consider to be masterpieces in this subject are more than three decades old and are out of date in their emphasis.) The current book is expected to fill this niche and I hope it becomes a standard reference in this field. Some of the features of this book, including the summary of chapters, are given below.

As the title implies, this book covers both Foundations (Chapters 1–10) and Frontiers (Chapters 11–16) of general relativity so as to cater for the needs of different segments of readership. The Foundations acquaint the readers with the basics of general relativity while the topics in Frontiers allow one to 'mix-and-match', depending on interest and inclination. This modular structure of the book will allow it to be adapted for different types of course work.

For a specialist researcher or a student of gravity, this book provides a comprehensive coverage of all the contemporary topics, some of which are discussed in a textbook for the first time, as far as I know. The cognoscenti will find that there is a fair amount of originality in the discussion (and in the Exercises) of even the conventional topics.

While the book is quite comprehensive, it also has a structure which will make it accessible to a wide target audience. Researchers and teachers interested in theoretical physics, general relativity, relativistic astrophysics and cosmology will find it useful for their research and adaptable for their course requirements. (The section *How to use this book*, just after this Preface, gives more details of this aspect.) The discussion is presented in a style suitable for physicists, ensuring that it caters for the current interest in gravity among physicists working in related areas. The large number (more than 225) of reasonably nontrivial Exercises makes it ideal for self-study.

Another unique feature of this book is a set of Projects at the end of selected chapters. The Projects are advanced level exercises presented with helpful hints to show the reader a direction of attack. Several of them are based on research literature dealing with key open issues in different areas. These will act as a bridge for students to cross over from textbook material to real life research. Graduate students and grad school teachers will find the Exercises and Projects extremely useful. Advanced undergraduate students with a flair for theoretical physics will also be able to use parts of this book, especially in combination with more elementary books.

Here is a brief description of the chapters of the book and their inter-relationship.

Chapters 1 and 2 of this book are somewhat unique and serve an important purpose, which I would like to explain. A student learning general relativity often finds that she simultaneously has to cope with (i) conceptual and mathematical issues which arise from the spacetime being curved and (ii) technical issues and concepts which are essentially special relativistic but were never emphasized adequately in a special relativity course! For example, manipulation of four-dimensional integrals or the concept and properties of the energy-momentum tensor have nothing to do with general relativity *a priori* – but are usually not covered in depth in conventional special relativity courses. The first two chapters give the student a rigorous training in four-dimensional techniques in flat spacetime so that she can concentrate on issues which are genuinely general relativistic later on. These chapters can also usefully serve as modular course material for a short course on advanced special relativity or classical field theory.

Chapter 1 introduces special relativity using four-vectors and the action principle right from the outset. Chapter 2 introduces the electromagnetic field through the four-vector formalism. I expect the student to have done a standard course in classical mechanics and electromagnetic theory but I do *not* assume familiarity with the relativistic (four-vector) notation. Several topics that are needed later in general relativity are introduced in these two chapters in order to familiarize the reader early on. Examples include the use of the relativistic Hamilton–Jacobi equation, precession of Coulomb orbits, dynamics of the electromagnetic field obtained from an action principle, derivation of the field of an arbitrarily moving charged particle, radiation reaction, etc. Chapter 2 also serves as a launch pad for discussing spin-0 and spin-2 interactions, using electromagnetism as a familiar example.

Chapter 3 attempts to put together special relativity and gravity and explains, in clear and precise terms, why it does not lead to a consistent picture. Most textbooks I know (except MTW) do not explain the issues involved clearly and with adequate detail. For example, this chapter contains a detailed discussion of the spin-2 tensor field which is not available in textbooks. It is important for a student to realize that the description of gravity in terms of curvature of spacetime is inevitable and natural. This chapter will also lay the foundation for the description of the spin-2 tensor field h_{ab}, which will play an important role in the study of gravitational waves and cosmological perturbation theory later on.

Having convinced the reader that gravity is related to spacetime geometry, Chapter 4 begins with the description of general relativity by introducing the metric tensor and extending the ideas of four-vectors, tensors, etc., to a nontrivial background. There are two points that I would like to highlight about this chapter. First, I have introduced every concept with a physical principle rather than in the abstract language of differential geometry. For example, direct variation of the line interval leads to the geodesic equation *through* which one can motivate the notion of Christoffel symbols, covariant derivative, etc., in a simple manner. During the courses I have taught over years, students have found this approach attractive and simpler to grasp. Second, I find that students sometimes confuse issues which arise when curvilinear coordinates are used in flat spacetime with those related to genuine curvature. This chapter clarifies such issues.

Chapter 5 introduces the concept of the curvature tensor from three different perspectives and describes its properties. It then moves on to provide a complete description of electrodynamics, statistical mechanics, thermodynamics and wave propagation in curved spacetime, including the Raychaudhuri equation and the focusing theorem.

Chapter 6 starts with a clear and coherent derivation of Einstein's field equations from an action principle. I have provided a careful discussion of the surface term in the Einstein–Hilbert action (again not usually found in textbooks) in a manner which is quite general and turns out to be useful in the discussion of Lanczos–Lovelock models in Chapter 15. I then proceed to discuss the general structure of the field equations, the energy-momentum pseudo-tensor for gravity and the weak field limit of gravity.

After developing the formalism in the first six chapters, I apply it to discuss four key applications of general relativity – spherically symmetric spacetimes, black hole physics, gravitational waves and cosmology – in the next four chapters. (The only other key topic I have omitted, due to lack of space, is the physics of compact stellar remnants.)

Chapter 7 deals with the simplest class of exact solutions to Einstein's equations, which are those with spherical symmetry. The chapter also discusses the orbits of particles and photons in these spacetimes and the tests of general relativity. These are used in Chapter 8, which covers several aspects of black hole physics, concentrating mostly on the Schwarzschild and Kerr black holes. It also introduces important concepts like the maximal extension of a manifold, Penrose–Carter diagrams and the geometrical description of horizons as null surfaces. A derivation of the zeroth law of black hole mechanics and illustrations of the first and second laws are also provided. The material developed here forms the backdrop for the discussions in Chapters 13, 15 and 16.

Chapter 9 takes up one of the key new phenomena that arise in general relativity, viz. the existence of solutions to Einstein's equations which represent disturbances in the spacetime that propagate at the speed of light. A careful discussion of gauge invariance and coordinate conditions in the description of gravitational waves is provided. I also make explicit contact with similar phenomena in the case of electromagnetic radiation in order to help the reader to understand the concepts better. A detailed discussion of the binary pulsar is included and a Project at the end of the chapter explores the nuances of the post-Newtonian approximation.

Chapter 10 applies general relativity to study cosmology and the evolution of the universe. Given the prominence cosmology enjoys in current research and the fact that this interest will persist in future, it is important that all general relativists are acquainted with cosmology at the same level of detail as, for example, with the Schwarzschild metric. This is the motivation for Chapter 10 as well as Chapter 13 (which deals with general relativistic perturbation theory). The emphasis here will be mostly on the geometrical aspects of the universe rather than on physical cosmology, for which several other excellent textbooks (e.g. mine!) exist. However, in order to provide a complete picture and to appreciate the interplay between theory and observation, it is necessary to discuss certain aspects of the evolutionary history of the universe – which is done to the extent needed.

The second part of the book (Frontiers, Chapters 11–16) discusses six separate topics which are reasonably independent of each other (though not completely). While a student or researcher specializing in gravitation should study all of them, others could choose the topics based on their interest after covering the first part of the book.

Chapter 11 introduces the language of differential forms and exterior calculus and translates many of the results of the previous chapters into the language of forms. It also describes briefly the structure of gauge theories to illustrate the generality of the formalism. The emphasis is in developing the key concepts rapidly and connecting them up with the more familiar language used in the earlier chapters, rather than in maintaining mathematical rigour.

Chapter 12 describes the $(1 + 3)$-decomposition of general relativity and its Hamiltonian structure. I provide a derivation of Gauss–Codazzi equations and Einstein's equations in the $(1 + 3)$-form. The connection between the surface term in the Einstein–Hilbert action and the extrinsic curvature of the boundary is also spelt out in detail. Other topics include the derivation of junction conditions which are used later in Chapter 15 while discussing the brane world cosmologies.

Chapter 13 describes general relativistic linear perturbation theory in the context of cosmology. This subject has acquired major significance, thanks to the observational connection it makes with cosmic microwave background radiation. In view of this, I have also included a brief discussion of the application of perturbation theory in deriving the temperature anisotropies of the background radiation.

Chapter 14 describes some interesting results which arise when one studies standard quantum field theory in a background spacetime with a nontrivial metric. Though the discussion is reasonably self-contained, some familiarity with simple ideas of quantum theory of free fields will be helpful. The key result which I focus on is the intriguing connection between thermodynamics and horizons. This connection can be viewed from very different perspectives not all of which can rigorously be proved to be equivalent to one another. In view of the importance of this result, most of this chapter concentrates on obtaining this result using different techniques and interpreting it physically. In the latter part of the chapter, I have added a discussion of quantum field theory in the Friedmann universe and the generation of perturbations during the inflationary phase of the universe.

Chapter 15 discusses a few *selected* topics in the study of gravity in dimensions other than $D = 4$. I have kept the discussion of models in $D < 4$ quite brief and have spent more time on the $D > 4$ case. In this context – after providing a brief, but adequate, discussion of brane world models which are enjoying some popularity currently – I describe the structure of Lanczos–Lovelock models in detail. These models share several intriguing features with Einstein's theory and constitute a natural generalization of Einstein's theory to higher dimensions. I hope this chapter will fill the need, often felt by students working in this area, for a textbook discussion of Lanczos–Lovelock models.

The final chapter provides a perspective on gravity as an emergent phenomenon. (Obviously, this chapter shows my personal bias but I am sure that is acceptable in the *last* chapter!) I have tried to put together several peculiar features in the standard description of gravity and emphasize certain ideas which the reader might find fascinating and intriguing.

Because of the highly pedagogical nature of the material covered in this textbook, I have not given detailed references to original literature except on rare occasions when a particular derivation is not available in the standard textbooks. The annotated list of Notes given at the end of the book cites several other text books which I found useful. Some of these books contain extensive bibliographies and references to original literature. The selection of books and references cited here clearly reflects the bias of the author and I apologize to anyone who feels their work or contribution has been overlooked.

Discussions with several people, far too numerous to name individually, have helped me in writing this book. Here I shall confine myself to those who provided detailed comments on earlier drafts of the manuscript. Donald Lynden-Bell and Aseem Paranjape provided extensive and very detailed comments on most of the chapters and I am very thankful to them. I also thank A. Das, S. Dhurandar, P. P. Divakaran, J. Ehlers, G. F. R. Ellis, Rajesh Gopakumar, N. Kumar, N. Mukunda, J. V. Narlikar, Maulik Parikh, T. R. Seshadri and L. Sriramkumar for detailed comments on selected chapters.

Vince Higgs (CUP) took up my proposal to write this book with enthusiasm. The processing of this volume was handled by Laura Clark (CUP) and I thank her for the effort she has put in.

This project would not have been possible without the dedicated support from Vasanthi Padmanabhan, who not only did the entire TEXing and formatting but also produced most of the figures. I thank her for her help. It is a pleasure to acknowledge the library and other research facilities available at IUCAA, which were useful in this task.

How to use this book

This book can be adapted by readers with varying backgrounds and requirements as well as by teachers handling different courses. The material is presented in a fairly modular fashion and I describe below different sub-units that can be combined for possible courses or for self-study.

1 Advanced special relativity

Chapter 1 along with parts of Chapter 2 (especially Sections 2.2, 2.5, 2.6, 2.10) can form a course in advanced special relativity. No previous familiarity with four-vector notation (in the description of relativistic mechanics or electrodynamics) is required.

2 Classical field theory

Parts of Chapter 1 along with Chapter 2 and Sections 3.2, 3.3 will give a comprehensive exposure to classical field theory. This will require familiarity with special relativity using four-vector notation which can be acquired from specific sections of Chapter 1.

3 Introductory general relativity

Assuming familiarity with special relativity, a basic course in general relativity (GR) can be structured using the following material: Sections 3.5, Chapter 4 (except Sections 4.8, 4.9), Chapter 5 (except Sections 5.2.3, 5.3.3, 5.4.4, 5.5, 5.6), Sections 6.2.5, 6.4.1, 7.2.1, 7.4.1, 7.4.2, 7.5. This can be supplemented with selected topics in Chapters 8 and 9.

4 Relativistic cosmology

Chapter 10 (except Sections 10.6, 10.7) along with Chapter 13 and parts of Sections 14.7 and 14.8 will constitute a course in relativistic cosmology and perturbation theory from a contemporary point of view.

5 Quantum field theory in curved spacetime

Parts of Chapter 8 (especially Sections 8.2, 8.3, 8.7) and Chapter 14 will constitute a first course in this subject. It will assume familiarity with GR but not with detailed properties of black holes or quantum field theory. Parts of Chapter 2 can supplement this course.

6 Applied general relativity
For students who have already done a first course in GR, Chapters 6, 8, 9 and 12 (with parts of Chapter 7 not covered in the first course) will provide a description of *advanced* topics in GR.

Exercises and Projects

None of the Exercises in this book is trivial or of simple 'plug-in' type. Some of them involve extending the concepts developed in the text or understanding them from a different perspective; others require detailed application of the material introduced in the chapter. There are more than 225 exercises and it is strongly recommended that the reader attempts as many as possible. Some of the nontrivial exercises contain hints and short answers.

The Projects are more advanced exercises linking to original literature. It will often be necessary to study additional references in order to comprehensively grasp or answer the questions raised in the projects. Many of them are open-ended (and could even lead to publishable results) but all of them are presented in a graded manner so that a serious student will be able to complete most parts of any project. They are included so as to provide a bridge for students to cross over from the textbook material to original research and should be approached in this light.

Notation and conventions

Throughout the book, the Latin indices $a, b, \ldots i, j \ldots$, etc., run over $0, 1, 2, 3$ with the 0-index denoting the time dimension and $(1, 2, 3)$ denoting the standard space dimensions. The Greek indices, $\alpha, \beta, \ldots$, etc., will run over 1, 2, 3. Except when indicated otherwise, the units are chosen with $c = 1$.

We will use the vector notation for both three-vectors and four-vectors by using different fonts. The four-momentum, for example, will be denoted by $\mathsf{\boldsymbol{p}}$ while the three-momentum will be denoted by $\boldsymbol{p}$.

The signature is $(-, +, +, +)$ and curvature tensor is defined by the convention $R^a{}_{bcd} \equiv \partial_c \Gamma^a_{bd} - \cdots$ with $R_{bd} = R^a{}_{bad}$.

The symbol $\equiv$ is used to indicate that the equation defines a new variable or notation.