1 Preliminaries

1.1 Introduction

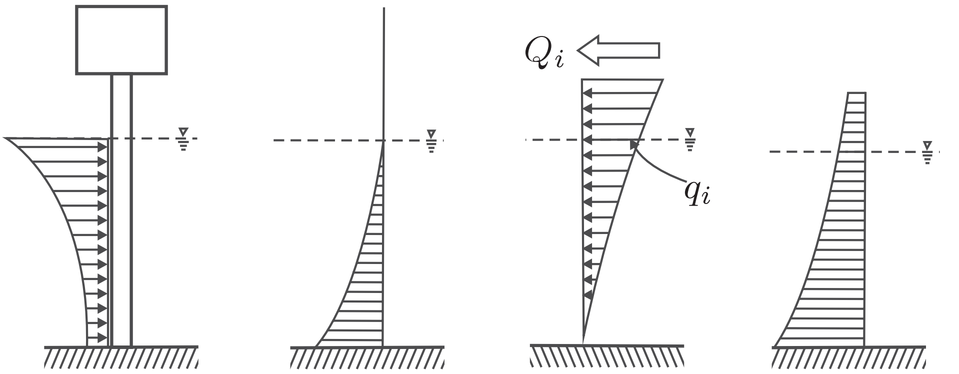
This book provides the reader with the basic procedures for performing a dynamic analysis of marine structures subjected to environmental stochastic load processes such as wind and ocean waves. The dynamics of rigid bodies and flexible structures are considered. As opposed to the static analysis of structures, the dynamic analysis of a structure concerns itself with time-variant external forces, inertia, damping, reaction forces, and the corresponding responses. Dynamic behavior differs from static behavior. Consider, for instance, the cantilevered tower shown in Fig. 1.1a.

Figures 1.1(b) and 1.1(d) display the bending moment in the tower with a deck (i.e., mass) on the top when the hydrodynamic forces from a long wave are considered as static and dynamic, respectively. Obviously, the dynamic case is relevant for a certain time variation and an instant in time. The difference is due to the inertia forces (on the deck Q_i and along the tower q_i).

Clearly, dynamic behavior is more complex than static, and its calculation is also more demanding. This becomes even more clear by considering a buoyant, rigid, articulated tower such as the one shown in Fig. 1.2. The static response is such that the tower rotates in such a way that the net effect of buoyancy and gravity balances the external forces (expressed by zero total moment with respect to the pin joint). If the external forces are considered as time variant, the external force at any time instant is balanced by the mentioned reaction forces as well as by inertia and damping forces. In practical situations (when resonance is not an issue), the inertia forces would typically dominate.

The loading on the structure is then described in terms of a load vector where one or more of the quantities, size, direction, or position, varies with time. As a consequence, the structural response to the dynamic loading that is, the resulting displacements, internal forces etc. will also be time varying or dynamic.

Broadly speaking, a dynamic analysis is performed in two different ways according to how the loading is specified. If the time-variant loading is given in such a way that we may consider it to be exactly known as a function of time, the same will apply to the response. In such a case, the dynamic analysis is called deterministic. This is in contrast to a stochastic analysis, where the loading is specified using probabilistic concepts. When this is done, the corresponding displacements and tensions can only

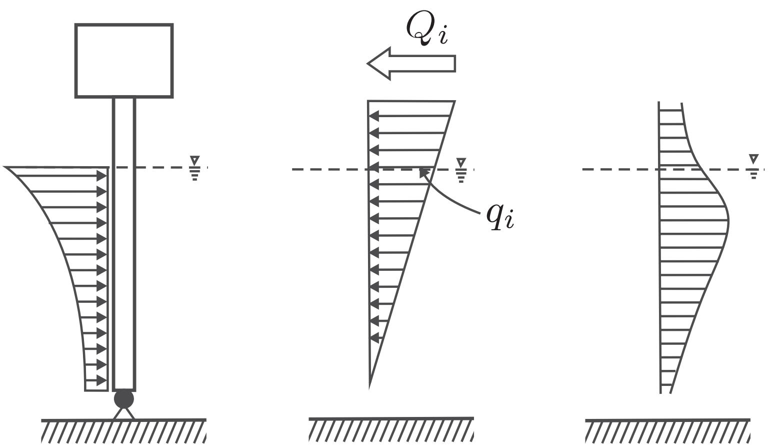


(a) Tower and external load due to a long wave (b) Static bending moment (c) Inertia reaction forces (d) Dynamic bending moment

Figure 1.1. Schematic of wave-induced static versus instantaneous dynamic forces and moments in a bottom-fixed cantilevered tower.

be described similarly. Even if, in principle, the naturally occurring loading that a structure is subjected to, such as wind and waves, can be claimed to be deterministic if the laws of physics and the initial conditions were known, their exact description and analysis within such a framework are believed to remain beyond reach for any foreseeable length of time. Fortunately, the complexity of these natural phenomena is such that they can be accurately modeled as random processes. This opens the way for practical analysis and prediction of the response of structures subjected to environmental loads.

The response analysis is performed as part of the design verification of structures, according to serviceability and safety requirements. Serviceability requirements relate to the function of the structure and the possible equipment it is carrying, while safety requirements refer to avoidance of ultimate or fatigue failure.



(a) Tower and external load due to a long wave (b) Inertia reaction forces (c) Dynamic bending moment

Figure 1.2. Schematic of instantaneous wave-induced forces and moments in an articulated tower.

The corresponding verification of ultimate or fatigue limit state criteria requires the extreme structural response and the whole response history, respectively.

1.2 Equations of Motion

The goal of a dynamic analysis of structures is to calculate the time history of the displacements, internal forces, or stresses at specific places in the structure. Before a dynamic analysis can be performed, most real structures would normally have to be represented by an idealized model.

Dynamic models fall into two basic categories, namely, continuous and discrete models. The number of variables – normally, displacement components – that need to be applied to describe the behavior is termed *the number of degrees of freedom* (nDOF). According to this definition, a continuous model represents an infinite DOF system, while a discrete model represents a finite DOF system.

Once the computational model of the structure is defined, the proper formulation of the equations of motion creates the mathematical model of the structure. A continuous model leads to partial differential equation(s), while a discrete model leads to ordinary differential equation(s).

When the mathematical model has been formulated, the next step deals with the solution of the differential equation(s) to predict the response of the real structure. Finally, the last step is the verification and confirmation of the results. This can be achieved by comparison of the computed results with dynamic tests or the responses of the models arrived at in different ways.

For plane motions of a rigid body in the reference (x, y)-plane, Newton's second law yields to the following equations:

$$m\ddot{u}_{cx} = \sum P_x(t), \tag{1.1}$$

$$m\ddot{u}_{cy} = \sum P_y(t), \tag{1.2}$$

$$J_c\ddot{\phi} = \sum M_c(t). \tag{1.3}$$

Herein, $\ddot{u}_{cx}$ and $\ddot{u}_{cy}$ denote the acceleration components of the center of mass C of the body along the x and the y axes; $\sum P_x(t)$ and $\sum P_y(t)$ are the corresponding resultant forces and $\ddot{\phi}$ is the angular acceleration; J_c represents the mass moment of inertia of the body with respect to an orthogonal axis through C, the center of mass; and $\sum M_c(t)$ is the sum of all moments acting on the body with respect to the axis through C.

Sometimes, especially when dealing with multiple degrees-of-freedom mass points with complex kinematic features, it is convenient to use Newton's second law recast in terms of Lagrange equations with reference to work and energy.

In analogy with the condition for static equilibrium, it is possible to introduce a corresponding condition for dynamic equilibrium by defining an auxiliary (vector) force, $\mathbf{f}_i(t) = -(m\ddot{u}_{cx}, m\ddot{u}_{cy}, J_c\ddot{\phi})^T$, which is often referred to as the inertia force. By rewriting Eqs. (1.1)–(1.3), letting $\mathbf{f}(t) = (\sum P_x(t), \sum P_y(t), \sum M_c(t))^T$, it is then obtained that

$$\mathbf{f}(t) + \mathbf{f}_i(t) = \mathbf{0}. \tag{1.4}$$

This rephrasing of Newton's second law of motion is usually called the *principle of d'Alembert*. Expressed in words, it may be stated as follows:

> *The condition for dynamic equilibrium is that the total force is in equilibrium with the inertia force.*

It is emphasized that the extension of the equilibrium concept we obtained by the introduction of the concept of dynamic equilibrium is synonymous with compliance with Newton's second law. This reformulation has resulted in a general condition of equilibrium, which is completely analogous to the condition of static equilibrium, and it contains this condition as a special case. When the acceleration is known (or is considered known), one can therefore recast the dynamic problem into the framework of a static problem by including the inertia force.

Although the treatment of a system of (rigid) mass points mainly requires the use of Newton's second law or derived energy formulations, the principles of continuum mechanics are needed to handle deformed bodies. They include equilibrium, kinematic compatibility and constitutive (stress–strain) relations, and may be used to formulate differential or energy expressions. In particular, the principle of virtual work provides a versatile tool for finite element discretizations of deformable bodies.

In this book, only the structural mechanics for bars, truss works, beams, and frames are described, while the continuum mechanics of plane stress, plate bending, shell, and solids are treated only in principle.

The principle of virtual work is briefly discussed. This principle is expressed here as an integrated form of d'Alembert's principle. Verbally expressed, it assumes the following form.

If the (generalized) forces in Eq. (1.4) are considered as functions of position, the principle of virtual work can be expressed as

$$\int_V \delta\mathbf{u}^T \mathbf{f}\, dV + \int_V \delta\mathbf{u}^T \mathbf{f}_i\, dV = 0, \tag{1.5}$$

where $\delta\mathbf{u}$ is a virtual displacement vector and V is the extension pr volume of the system. The force $\mathbf{f}$ is then understood as the external forces minus possible forces in the structure from damping and elastic resistance to displacement.

1.3 Stochastic Models

After the publication of the classical paper "On the motions of ships in confused seas," by St. Denis and Pierson (1953), where stochastic concepts were used for modeling the waves and the wave forces on a ship subjected to the ocean environment, it was generally recognized that this approach represented a rational and suitable way to account for the "unpredictability" of the hydrodynamic loading on a ship during an ocean voyage. During the more than 50 years since then, the stochastic modeling of environmental forces and the responses of the structures subjected to them has reached a fairly mature state. Today, stochastic modeling of the environmental loads and the induced responses is a routine procedure when designing ships and other ocean structures, even if the degree of sophistication varies depending on the type of structure and design provisions. In particular, taking account of the random character of wave and other environmental loads is important when the loads induce inertia and damping reaction forces. In this light, the main aspects of stochastic modeling of the environmental processes and loads, as well as the induced responses of structures, should be an important part of any textbook on the dynamics of marine structures. In this book, we therefore make an effort to discuss these aspects in some detail.

1.4 Organization of the Book

The first part of the book (Chapters 2–4) describes fundamental issues of a deterministic dynamic analysis with emphasis on simple, but important, vibration problems. The second part (Chapters 5–16) provides a fairly extensive introduction to stochastic dynamic analysis of marine structures, as well as applications of such analyses in the design process.

It is inevitable that a book of this type also to some extent reflects the authors' "world view." This is particularly apparent in the choice of a number of advanced topics that the authors have been particularly involved in.

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978-0-521-88155-5 - Stochastic Dynamics of Marine Structures
Arvid Naess and Torgeir Moan
Excerpt
More information

2 Dynamics of Single-Degree-of-Freedom Linear Systems

2.1 Introduction

This chapter deals with vibrations of structures that can be represented as a single-degree-of-freedom (SDOF) system. This means that the oscillatory response can be completely described by one displacement variable. This may seem like a gross oversimplification for structures of engineering interest that leads to a theory of little practical significance. However, the theory of vibrations for systems of an SDOF is crucial for understanding the vibration response of more complex structures. Frequently, it is also the case that one may investigate the vibration response characteristics of apparently complex structures by directly applying the theory of vibrations of SDOF systems. This is demonstrated in Chapter 3 on multi-degrees-of-freedom (MDOF) structures.

The word "vibration" used in this chapter should be interpreted as meaning oscillatory response in a fairly general sense, e.g., as applied to marine structures.

2.2 Harmonic Oscillator – Free Vibrations

Free vibrations or oscillations occur when there are no external forces imposed on the structure, e.g., after an initial displacement and release. Two different situations are discussed: translational oscillations and rotational oscillations.

2.2.1 Motions of Marine Structures

Because the main focus of this book is the motion response of marine structures, it is expedient to define the terms commonly used to describe the rigid-body motions of floating structures. This is most easily done by referring to Fig. 2.1. For a shiplike structure, it is common practice to place the x-axis along the beam of the ship (for the body-fixed coordinate system), and call the corresponding translational motion for surge. The y-axis is placed in the water plane (or parallel to it). Whether the z-axis points into the water (downward) or out of the water (upward) may vary. In this book, the z-axis is invariably positive upward. Thus, there are three translatory motions: surge (x-axis), sway (y-axis), and heave (z-axis). Similarly, there are three rotational motions: roll (x-axis), pitch (y-axis), and yaw (z-axis).

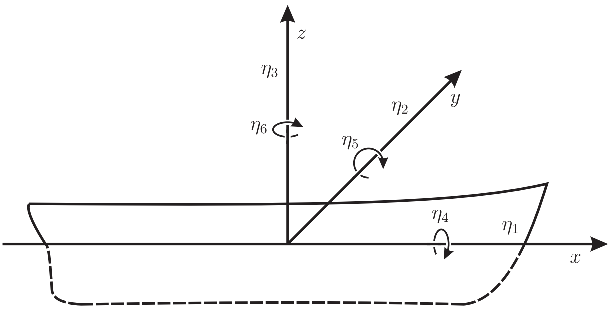

Figure 2.1. Definition of the motion response modes of a marine structure. η_1 = surge, η_2 = sway, η_3 = heave, η_4 = roll, η_5 = pitch, η_6 = yaw (SNAME, 1988).

For many offshore platform structures, there is no obvious "x-axis" in the same way as for a ship, and the placement of a local coordinate system is therefore to a larger extent arbitrary. In such cases, the x-axis is often placed along, or close to, the main wave direction.

2.2.2 Translational Oscillations

Figure 2.2 displays a schematic of an undamped (without friction) vibration system of an SDOF. The spring k is assumed to be without mass. It is also assumed to comply with Hooke's law; that is, it is assumed to be linearly elastic. The displacement u is considered positive to the right of the equilibrium point; that is, $u = 0$ at the equilibrium point. When the mass m is displaced a distance u, the spring k will impose a force $-ku$ on m, where the minus sign indicates that the force is directed against the displacement. Invoking Newton's second law then gives the relation

$$-ku = m\ddot{u} \tag{2.1}$$

or

$$m\ddot{u} + ku = 0 \tag{2.2}$$

To simplify language and notation, we let the letter m denote both the physical mass itself and its size in kilograms (kg). Similarly for the letter k, which denotes both a linearly elastic spring and the corresponding spring constant in Hooke's law ([k] = N/m).

Equation (2.2) can be rewritten as

$$\ddot{u} + \omega_e^2 u = 0 \tag{2.3}$$

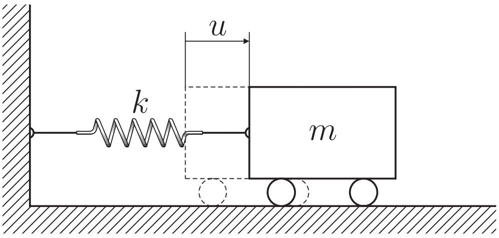

Figure 2.2. Principle sketch of the vibration system of an SDOF.

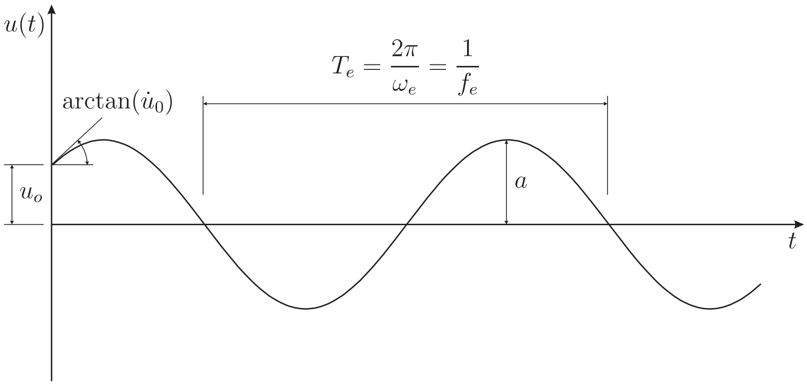

Figure 2.3. Undamped free oscillation response of amplitude a and period T_e.

by introducing the angular frequency ω_e defined by

$$\omega_e^2 = \frac{k}{m} \quad ([\omega_e] = \text{rad/s}) \tag{2.4}$$

ω_e is referred to as the natural frequency, or the eigenfrequency, of the oscillatory system.

One can easily verify that the harmonic functions $\cos \omega_e t$ and $\sin \omega_e t$ both satisfy Eq. (2.3). The general solution can therefore be written as

$$u(t) = a_1 \cos \omega_e t + a_2 \sin \omega_e t, \tag{2.5}$$

where a_1 and a_2 are two constants. The solution $u(t)$ represents an oscillation response with constant angular frequency ω_e, hence the name harmonic oscillator. Because the differential Eq. (2.3) is of second order, two constants are required to describe the general solution. If position and velocity at a particular point in time are given, for instance, $u(0) = u_0$ and $\dot{u}(0) = v_0$, then the solution will be uniquely determined as follows,

$$u(t) = u_0 \cos \omega_e t + \frac{v_0}{\omega_e} \sin \omega_e t. \tag{2.6}$$

This harmonic oscillation is illustrated in Fig. 2.3. It is seen from Eqs. (2.5) and (2.6) that $u(t) = u(t + 2\pi/\omega_e)$, while $u(t) \neq u(t + t')$ when $0 < t' < 2\pi/\omega_e$. This leads to the expression for the natural period of the system:

$$T_e = \frac{2\,\pi}{\omega_e} = 2\,\pi\sqrt{\frac{m}{k}}. \tag{2.7}$$

The natural frequency f_e ($[f_e] = \text{s}^{-1}$ or Hz) is therefore

$$f_e = \frac{1}{T_e} = \frac{\omega_e}{2\,\pi}. \tag{2.8}$$

Equation (2.5) may be rewritten as

$$u(t) = \sqrt{a_1^2 + a_2^2}\left(\frac{a_1}{\sqrt{a_1^2 + a_2^2}} \cos \omega_e t + \frac{a_2}{\sqrt{a_1^2 + a_2^2}} \sin \omega_e t\right). \tag{2.9}$$

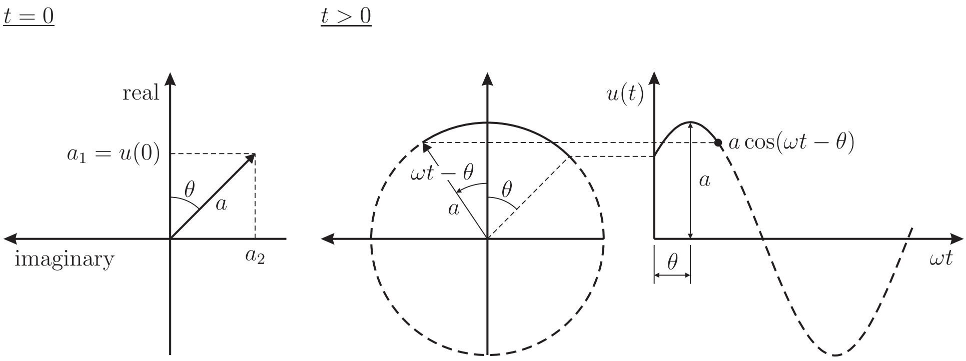

Figure 2.4. Argand diagram for the oscillation response shown in Fig. 2.3.

Because

$$\left(\frac{a_1}{\sqrt{a_1^2 + a_2^2}}\right)^2 + \left(\frac{a_2}{\sqrt{a_1^2 + a_2^2}}\right)^2 = 1, \tag{2.10}$$

there must exist an angle θ $(0 \le \theta < 2\pi)$ such that $\cos\theta = a_1/\sqrt{a_1^2 + a_2^2}$ and $\sin\theta = a_2/\sqrt{a_1^2 + a_2^2}$. This implies that

$$u(t) = \sqrt{a_1^2 + a_2^2}\,(\cos\omega_e t\,\cos\theta + \sin\omega_e t\,\sin\theta) = a\,\cos(\omega_e t - \theta), \tag{2.11}$$

where the amplitude $a = \sqrt{a_1^2 + a_2^2}$ and the phase angle θ are determined by the initial conditions at $t = 0$. Because $\tan\theta = a_2/a_1$, θ can also be calculated from the relation $\theta = \arctan(a_2/a_1)$. There are two solutions for θ in the interval $[0, 2\pi]$, and the sign of $u(0) = u_0$ determines the correct one. When $u(0)$ is positive, the phase angle will be in the first or fourth quadrant. If $u(0)$ is negative, the phase angle will be in the second or third quadrant.

The oscillation response can be represented in the complex plane by using an Argand diagram, where $u(t)$ is the real part of the complex number $\mathbf{a}$ ($|\mathbf{a}| = a$), which rotates with constant angular frequency ω_e, see Fig. 2.4.

2.2.3 Example – Amplitude and Phase of a Free Oscillation

Assume a vibration response determined by Eq. (2.2) with $m = 10$ kg and $k = 400$ N/m. What will be the resulting amplitude and phase angle for the free oscillation when $u_0 = 0.1$ m and $v_0 = -0.2$ m/s?

The natural frequency of the system is $\omega_e = \sqrt{k/m} = 2$ rad/s. From Eqs. (2.2) and (2.11), it follows that the amplitude $a = \sqrt{u_0^2 + (v_0/\omega_e)^2} = \sqrt{0.1^2 + 0.1^2} = 0.14$ m. The phase angle $\theta = \arctan(v_0/(\omega_e u_0)) = \arctan(-1) = 7\pi/4$, corresponding to a positive displacement and negative velocity.

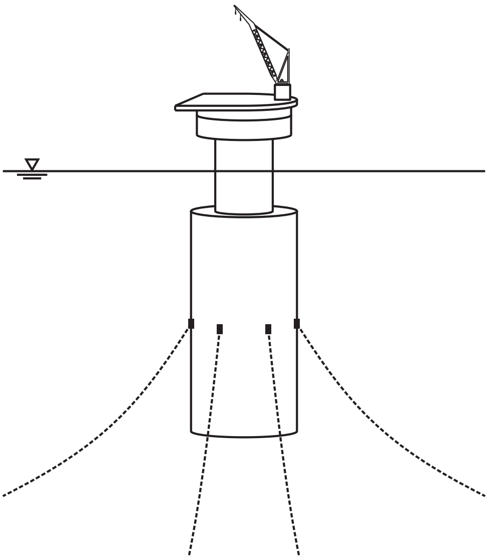

Figure 2.5. A coarse sketch of the Brent Spar.

2.2.4 Example – Heave Oscillations of a Spar Buoy

In the mid-1970s, the oil company Shell installed an oil storage and offloading spar buoy at the Brent offshore oil field in the North Sea at a water depth of 140 meters, see Fig. 2.5. The Brent Spar, or Brent E, was a conventional cylindrical buoy moored to the sea floor by six anchors. The cylindrical tank was 137 meters high with a diameter of 29 meters and a displacement of 66000 metric tons, which corresponds to a draft of 109 meters. The spar buoy was made up of oil storage tanks at the bottom, buoyancy tanks toward the middle and a topside containing the offloading and other equipment. Decommissioned in the mid-1990s, the Brent Spar was in operation for about 20 years. The original plans for decommissioning, which were basically to sink it in deep water, were fiercely attacked by Greenpeace and other environmental organizations, and these plans were eventually abandoned. Accounts of the "battle" of the Brent Spar can be found on the Internet.

The deep draft of the spar buoy makes it nonresponsive to normal sea states, and the heave response is hardly noticeable under ordinary operating conditions. This is connected with the long natural period of the heave oscillations of the spar, which can be determined by calculating the restoring force produced by a vertical displacement of the spar. The restoring force originates mainly from a change in the buoyancy effect. The mooring lines will contribute only marginally to the total restoring force in the heave direction. Hence, a vertical displacement of x meters will produce a restoring force $f_r(x)$ approximately equal to

$$f_r(x) = \rho g \pi R^2 x, \tag{a}$$

where $\rho = 1{,}025$ kg/m^3 = density of sea water, $g = 9.81$ m/s^2, and $R = 14.5$ m = radius of cylindrical tank. It follows that the spar buoy offers a restoring force that is a linear function of the displacement x; that is, $f_r(x) = Kx$. It is found that $K = 6.6 \cdot 10^6$ N/m for the Brent Spar.