

VIBRATION OF AXIALLY LOADED STRUCTURES

This book concerns the vibration and the stability of slender structural components. The loss of stability of structures is an important aspect of structural mechanics and is presented here in terms of dynamic behavior. A variety of structural components are analyzed with a view to predict their response to various (primarily axial) loading conditions. A number of different techniques are presented, with experimental verification from the laboratory. Practical applications are widespread, ranging from cables to space structures. The book presents methods by which the combined effects of vibration and buckling on various structures can be assessed. Vibrations and buckling are usually treated separately, but in this book their influence on each other is examined together, with examples when a combined approach is necessary. The avoidance of instability is the primary goal of this material.

Dr. Lawrence N. Virgin completed his doctorate in structural mechanics in 1986 at University College London. Since 1988, he has been at Duke University, where he teaches and conducts research in engineering mechanics. His interests are centered on the instability behavior of non-linear dynamics systems in the context of experimental vibrations, with applications including aeroelasticity, systems with discontinuities (impact and friction), fluid–structure interaction, and buckling. He is currently Gardner Professor and Chair of the Department of Civil and Environmental Engineering and holds a secondary appointment in the Department of Mechanical Engineering and Materials Science.

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This book is dedicated to my wife Lianne, my children Elliot and Hayley,
and my parents Margaret and Alan

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Foreword

The concept of stability is intrinsically a dynamical one. This is recognized even by the simplistic classical definition, which ignores the random disturbances of the real world and just inquires what would happen if a system were displaced to an adjacent position in phase space. So we are lucky, indeed, to have this well-conceived book written by a leading researcher who has mastered both nonlinear dynamics and the static bifurcations of elastic stability theory. The latter theory works well for conservative systems, for which powerful energy theorems are available, but needs augmenting by dynamical methods in the presence of loading that is either nonconservative or time dependent.

Lawrence Virgin has of course just the right background, having chosen (in his usual thoughtful way) to work first at University College London, then with Earl Dowell at Duke University. He is currently the Chair of the Department of Civil and Environmental Engineering at Duke (which has an active interdisciplinary program in nonlinear dynamics) and has enjoyed productive collaborations with Raymond Plaut (Virginia Polytechnic Institute and State University). His previous book, *Introduction to Experimental Nonlinear Dynamics* (also published by Cambridge University Press), brought a welcome sense of realism into the often esoteric field of nonlinear dynamics by focusing on experimental investigations, and I am delighted to see a similar emphasis in this new book titled *Vibration of Axially Loaded Structures*.

Understanding the buckling and vibration of structures under axial compression is of very great importance to structural and aerospace engineers, to whom this book is primarily addressed. They, together with readers from many other areas of mechanics, will be well served by Lawrence's latest offering. The book covers a wide field, including buckling, dynamics (both linear and nonlinear), theory, and experiments, all explained in a clear and lucid style. Especially valuable are the comprehensive lists of references, which nicely complement the text.

I can heartily recommend this book to all who want to see a wide-ranging and scholarly treatment that brings new insights to an important long-standing but still emerging field.

Michael Thompson, FRS
Cambridge, England

Preface

General Comments

Rationale and Scope

- The material covered by this book spans the areas of *vibration* and *buckling*. Both of these areas can be considered as subsets of structural mechanics and play a central role in the disciplines of civil, mechanical, and aerospace engineering.
- Although vibration and buckling are key elements in the teaching of advanced engineering, they are typically taught separately. However, the *interplay* of dynamics and stability in structural mechanics and its coverage in a single text provide an opportunity to present material in an interesting way.
- The quest for stronger, stiffer, and more lightweight structural systems is making the material covered in this book *increasingly* important in practical applications.
- By using axially loaded structures as a consistent theme, the book covers a wide variety of types of structure, methods of analysis, and potential applications without trying to cover too much. Experimental verification appears throughout.
- The level of material is appropriate for upper-level, advanced undergraduate classes, and graduate students, but researchers and practicing engineers will find plenty of interest too.
- The text is liberally illustrated by figures, and close to 500 technical references are given.

Acknowledgments

The material presented in this book contains a synthesis of material from the general literature together with results from my own research program. In terms of the latter, this is by no means a solo endeavor, and there are a number of people I would like to thank.

First, and foremost, much of the work I have conducted in this area in the past 20 years or so has been done with Raymond Plaut from Virginia Polytechnic Institute and State University. I have learned a considerable amount from his deep

understanding of theoretical and applied mechanics as well as his attention to detail and meticulous approach to research. He also proofread this book, making useful suggestions and providing invaluable guidance.

My path along this road goes back to Terry Roberts in Cardiff, Michael Thompson in London, and Earl Dowell here in North Carolina. I have benefited immeasurably from their influence as mentors during my formative years (and beyond). In addition to my family, of course, I'd like to thank my friends and colleagues at Duke who have contributed to a supportive environment: Tod Laursen, John Dolbow, Henri Gavin, Ken Hall, Josiah Knight, and Bob Kielb.

I have had the privilege of working with many talented graduate students over a period of almost 20 years, and those whose research contributed directly or indirectly to material in this book include Phil Bayly, Kevin Murphy, Mike Todd, Kara Slade, Hui Chen, David Holland, Mike Hunter, Ilinca Stanciulescu, Sophia Santilan, and Ben Davis (who also diligently proofread the manuscript). Thanks to them all.

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