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978-0-521-88042-8 - Vibration of Axially Loaded Structures
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Excerpt
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1 Context: The Point of Departure

In the engineered world (and in a good deal of the natural world), stable equilibrium, or some kind of stationary or steady-state behavior, is the order of the day. Systems are designed to operate in a predictable fashion to fulfill their intended functions despite disturbances and changing conditions. Control systems have been spectacularly successful in maintaining a desirable (stable[1]) response given inevitable uncertainty in modeling system physics. However, there are plenty of examples of systems becoming unstable – and often the consequences of instability are severe. This book looks at the interplay between vibrations and stability in elastic structures.

A brief view of an ecological system provides an effective analogy. The competition between certain species can be viewed as a coupled dynamic system in a slowly changing environment. External influences are provided by various factors including the climate, disease, and human influence. The delicate interaction is played out as conditions evolve and populations respond accordingly – usually in a correspondingly slow way also. However, an instability may occur leading to extinction on a relatively short time scale, perhaps when a disease (or massive meteorite) wipes out an entire population. This situation is not that dissimilar to the fluctuations of the stock markets (in which prediction of sudden changes is of concern to individuals and governments).

In an engineering context, we typically have considerable knowledge about the underlying physics and governing equations of our systems, are able to test a system both analytically and in the laboratory, and thus have a much better chance of assessing the robustness of a system, especially its propensity to failure. However, unforeseen circumstances do occur, and it would, of course, be remiss in a book concerning stability in engineering mechanics not to mention the Tacoma Narrows suspension bridge disaster. But many other bridges and buildings have collapsed, aircraft wings and rotorblades have a tendency to flutter, ships sometimes capsize, the tracks of a railroad will buckle from time to time, electric circuits sporadically exhibit unintended feedback, machine parts are prone to fatigue, and once in a while satellites disappear into deep space. What most of these systems have in common is that they were either subject to external influences with which they could not cope

[1] Some aeronautics control systems take advantage of a brief loss of stability for enhanced maneuverability.

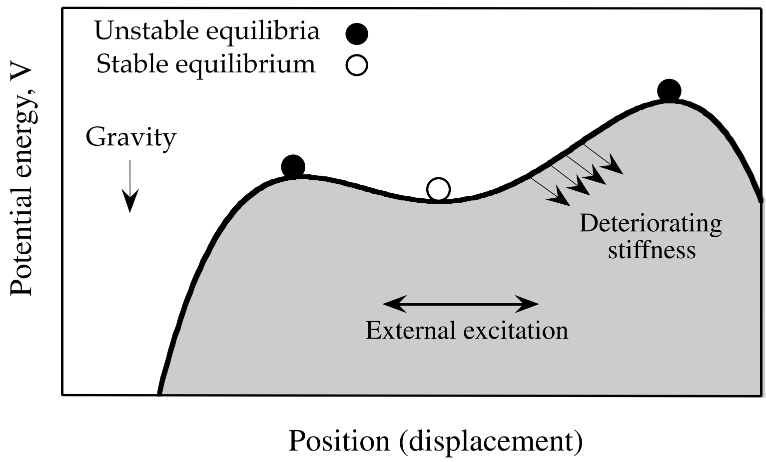

Figure 1.1. A deteriorating scenario.

or they changed. Perhaps an encounter with a rogue wave in the case of a ship, or collision with space debris in the case of the satellite. In this last instance an error in the units used in trajectory calculations may cause disaster but in the sense that the system was designed correctly but for the wrong conditions. Of course, there are always practical limits to how much safety or redundancy can be built into a system; the World Trade Center provided a sobering example. But it is also likely that a system is subject to slowly changing conditions, which may, of course, lead to catastrophe, but in a gradual deteriorating sense. It is with these systems that we have scope for monitoring and prediction, as their (dynamic) response may give clues about future performance.

Hence, given a (structural) system in some state of rest (equilibrium) or steady-state motion (an oscillation), we seek to understand those conditions that cause a change in the nominal response, and especially where such a change is *large* (and instability falls squarely into this category). The theoretical framework underlying this statement is of course based on Newton's laws and subsequent developments especially concerning concepts of energy. To crystallize this approach, consider the schematic diagram shown in Fig. 1.1.

Here we might consider the behavior of a small ball allowed to roll (under the influence of gravity) on a curved surface to represent a generic structural or mechanical system. The analogy is really brought into focus if we further assume that the curve is actually associated with the underlying potential energy of the system and that the surface causes a little energy dissipation as the ball rolls. Hence the bottom of the energy "well" is identified as a position of stable equilibrium, with linear theory based on a locally quadratic minimum. Linear stability theory will also tell us that the "hilltops" are points of *unstable* equilibrium. In both cases, the ball will remain at rest at these extremum values of the potential energy surface. However, the important behavior is observed if the system is subject to a *disturbance*. In the stable case, the ball might begin to oscillate but typically return to rest at the bottom of the well. In the unstable case, the ball picks up speed and

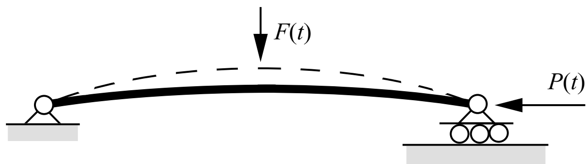

Figure 1.2. A slender axially loaded structure and its dynamic response.

departs the local neighborhood of the hilltop. These situations are well covered by linear stability theory providing the size of the perturbation is *small*.

Extending this concept further, it is natural to ask what happens

- if the morphology of the potential energy surface changes (typically slowly) such that the potential energy at a stable equilibrium position ceases to be a minimum,
- or if the ball is subject to a relatively *large* perturbation or disturbance that may push the ball well beyond the local neighborhood of the minimum.

These are the two situations depicted in Fig. 1.1. The former case is the basis of most studies in classical buckling. The application of an external axial load is assumed to take place quasi-statically, and buckling occurs (typically leading to large deflections) as the ball can no longer maintain its position. Many practical examples like this can be handled very effectively by use of statics. Most interest is naturally focused on the behavior of the system prior to buckling when the system is changing sufficiently slowly that kinetic energy can safely be ignored in the Lagrangian description (although it may still be useful to gain information based on dynamics). However, in the latter case, the application of a large (say, sudden or periodic) perturbation inevitably leads to a dynamic, perhaps unbounded, response. In fact, even in those cases in which a static approach works well, if we want to track the postcritical behavior, we may still need to use a dynamic approach, for example, one in which a system subject to a slowly increasing load results in a fast dynamic jump at buckling.

Figure 1.2 adds some specificity to the scope of the material covered in this book using the behavior exhibited by a vibrating thin beam:

- Figure 1.2 illustrates a beam undergoing small-amplitude free vibrations, that is, with $P(t) = F(t) = 0$. This is a thoroughly linear situation, with the straight configuration the only equilibrium and damping causing dynamic behavior to decay. Exact solutions are available; natural frequencies are constant and scale with the stiffness of the beam. For example, a longer beam is less stiff and thus natural frequencies are lower. Clamped boundary conditions lead to higher natural frequencies than simply supported, and so on.
- The presence of a constant axial load [but with $F(t) = 0$] tends to reduce the natural frequencies if the load is compressive and below its critical value. If the axial load is sufficiently large (i.e., greater than critical), postbuckled (nontrivial)

equilibria exist, and natural frequencies can be computed about these nontrivial equilibria.

- For laterally excited systems ($F(t) \not\simeq 0$ but with $P = 0$), we can have *resonance*. This may also occur about postbuckled equilibria when $P > P_{\mathrm{cr}}$.
- If the axial load is a function of time (say, periodic), then the system may also lose stability (depending on the frequency of excitation) through *parametric resonance*.
- If the ends of the beam are both constrained against moving (in-plane) then membrane, or *stretching*, forces arise.
- In each of the preceding scenarios the vibration may have *large* amplitude.
- Many of these scenarios might occur simultaneously. For example, a postbuckled beam might *snap through* if excited laterally.

Thus this range of behavior encompasses both small-amplitude and large-amplitude motion about both trivial and nontrivial equilibria. Access to analytic solutions becomes restricted as the complexity (and nonlinearity) of the system increases. Damping oftens needs to be considered also. Although the example of the prismatic beam has been used here, extensions to other types of axially loaded structures, like plates and shells is easy to envision. Furthermore, some of these situations may lead to instability (both static and dynamic), which is of particular concern to engineers. It is worth mentioning that aerospace structures provide a natural context for much of this material; the continual quest for lighter vehicles naturally brings with it issues of vibration and stability.

Some practical examples of slender structures in aerospace engineering in which axial loads and dynamics may need to be considered are shown in Fig. 1.3. These images all portray aerospace systems. Spacecraft applications tend to be very lightweight: Thin-film solar sails designed for deep-space propulsion; high-altitude unmanned surveillence craft like the Predator; lightweight solar-powered high-endurance aircraft like the Pathfinder; the shuttle; international space station; rotorcraft; and military aircraft all possess slender structural components subject to a variety of loading conditions including vibration and axial-load effects.

Figure 1.4 shows some other examples of slender structures. They range from bridges to pipelines, telescopes to submarines, oil tankers to high-rise buildings. The vibrations of axially loaded structures also occur at very small scales, including the increasingly important range of applications in nanotechnology. The guitar string is an obvious case. The axial load in this case can only be tensile, but it is interesting to note the slightly angled bridge of the guitar – this accounts for the slight amount of bending stiffness in the thicker strings.

Hence this book is broadly divided into two main parts to cover these rather wide-ranging scenarios. A conventional division in the presentation of vibration problems is between free and forced vibration. That convention is somewhat followed here in the development of the material. However, there are occasions for which this division is not clear (e.g., an impulsive force can also be viewed as an initial velocity), but in terms of organizing the material, this seemed to be a natural

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Figure 1.3. Examples of slender structures in an aerospace context. Courtesy of NASA. See color plates I–IV following page xvi.

Figure 1.4. More examples of slender structures. See color plates V–VIII following page xvi.

Figure 1.4. (continued) More examples of slender structures. See color plates V–VIII following page xvi.

choice. The next chapter will provide a brief overview of basic mechanics (which can be omitted by the more advanced reader), followed by a treatment of the interplay of dynamics and stability, without introducing too much in the way of mathematics, but still providing a flavor of the types of more practical structural systems considered later in the book.

2 Elements of Classical Mechanics

2.1 Introduction

This chapter develops the theoretical basis for the derivation of governing equations of motion. It starts with Newton's second law and then uses Hamilton's principle to derive Lagrange's equations. A number of conservation laws are introduced. The theory is developed initially for a single particle and extended to systems of particles where appropriate. The emphasis is placed on building the theory relevant to the types of physical system of interest in structural dynamics. Other than the usual limitations regarding relativistic and quantum effects, we also restrict ourselves to translational (rather than rotational) systems, which is largely a matter of coordinates. The majority of problems in this book involve systems in which the forces developed during elastic deformation play a crucial role. Certain standard problems in classical mechanics, for example the central force motion leading to the two-body problem or particle scattering, are not relevant here and are not considered. We shall see the important role played by energy methods in studying the dynamics of structures. Classical mechanics has a long history and in-depth treatment of the subject can be found in Goldstein [1], Whittaker [2], and Synge and Griffith [3] and, of course, going back to the early developments of Newton [4], Euler [5], and Lagrange [6].

2.2 Newton's Second Law

The natural starting point in any text covering an aspect of classical mechanics are Newton's laws of motion. They date back to 1686, with the second being the most important:

> *A body acted upon by a force moves in such a manner that the time rate of change of momentum equals the force.*

Mathematically we introduce the concept of a linear momentum vector $\mathbf{p}$ defined as the product of mass and velocity:

$$\mathbf{p} = m\mathbf{v}, \tag{2.1}$$

where m is the mass and $\mathbf{v}$ is the velocity vector. We can thus write Newton's second law as

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(m\mathbf{v}), \tag{2.2}$$

in which $\mathbf{F}$ is the force vector.

To apply this law we need to specify motion relative to a reference frame. If we define an absolute position vector, $\mathbf{r}$, in an inertial frame (i.e., a frame at rest or moving with a constant velocity relative to the "fixed" stars), then the corresponding absolute velocity vector is given by

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}}, \tag{2.3}$$

where an overdot signifies a time derivative. Thus we can further express Newton's second law in its more familiar form as

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} = m\ddot{\mathbf{r}} = m\mathbf{a}, \tag{2.4}$$

where $\mathbf{a}$ is an absolute acceleration vector and we have assumed m does not vary with time.

Equation (2.4) is a (set of) second-order ordinary differential equation fundamental to the study of mechanics. In general,

$$\mathbf{F} = \mathbf{F}(\mathbf{r}, \dot{\mathbf{r}}, t), \tag{2.5}$$

and a solution $\mathbf{r}(t)$ that satisfies this equation can be obtained given appropriate initial conditions $\mathbf{r}(t_0)$ and $\dot{\mathbf{r}}(t_0)$. For the types of systems of relevance to the material covered in this book, these solutions are unique. The forces entering Eq. (2.5) arise from a number of different sources in structural dynamics: stiffness, inertia, excitation and damping being the most important. The SI units of force are newtons (N), where $1\,\text{N} = 1\,\text{kg m/s}^2$.

Clearly, if $\mathbf{F} = \mathbf{F}(t)$, then it would be a straightforward task to integrate Eq. (2.4) directly to obtain $\mathbf{v}(t)$ and then $\mathbf{r}(t)$. However, this will not typically be the case (as elastic forces tend to depend on the change in position), and a variety of techniques can be called on to solve differential equations. We observe at this point that solutions to equations of the type (2.4) will often involve *oscillations*, and also that there may not be analytic solutions available, especially in those situations in which *nonlinear* terms are present. Further discussion of nonlinearity and other aspects of differential equations are left to later chapters. However, the concept of *stability* (which will be developed continuously throughout this book) involves considering the manner in which closely adjacent solutions of Eq. (2.4) behave as a function of time, and specifically, when one of those solutions represents some kind of steady or equilibrium solution.

2.3 Energy and Work

Now suppose $\mathbf{F} = \mathbf{F}(\mathbf{r})$. We can obtain information about the solution to Eq. (2.4) by performing a path integral with respect to $\mathbf{r}$ along the trajectory:

$$\int_{\mathbf{r}(t_0)}^{\mathbf{r}(t)} \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r} = \int_{t_0}^{t} \mathbf{F}(\mathbf{r}) \cdot \dot{\mathbf{r}} dt = m \int_{t_0}^{t} \frac{d^2\mathbf{r}}{dt^2} \cdot \frac{d\mathbf{r}}{dt} dt \tag{2.6}$$

$$= \frac{1}{2} m \int_{t_0}^{t} \frac{d}{dt}(\dot{r}^2) dt = \frac{1}{2} m v^2(t) - \frac{1}{2} m v^2(t_0), \tag{2.7}$$

which gives the magnitude of the velocity [rather than $\mathbf{r}(t)$] provided the integral on the left-hand side of Eq. (2.6) can be performed. This is not a straightforward matter because $\mathbf{r}(t)$ (which is unknown) appears in the upper limit and a path integral depends on the path of integration.

However, if we let the path of this integral [in Eq. (2.6)] be called C, then we can introduce the *work done* by the force $\mathbf{F}$ moving along this path as

$$W_C = \int_C \mathbf{F} \cdot d\mathbf{r}, \tag{2.8}$$

and, defining the kinetic energy as

$$T = \frac{1}{2} m v^2, \tag{2.9}$$

we can rewrite Eq. (2.7) as

$$W_C = T_2 - T_1, \tag{2.10}$$

which is a statement of the work – energy theorem. It turns out that there is a relatively large class of problems for which the work done for any admissible path between points 1 and 2 depends on only the end points of the path. In these cases forces are called *conservative*, and they play a dominant role in the static analysis of buckling, for example.

For a conservative force $\mathbf{F}(\mathbf{r})$, consider two paths C_1 and C_2 connecting two points $\mathbf{r}_1$ and $\mathbf{r}_2$. In this case we can write

$$\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}, \tag{2.11}$$

which implies that

$$\oint \mathbf{F} \cdot d\mathbf{r} = 0, \tag{2.12}$$

where the closed integral is performed from $\mathbf{r}_1$ to $\mathbf{r}_2$ and back again. We define the work done by a conservative force in moving a particle from a reference point, $\mathbf{r}_0$, to an arbitrary position $\mathbf{r}$ as the potential energy,

$$V(\mathbf{r}) = \int_{\mathbf{r}}^{\mathbf{r}_0} \mathbf{F}_c \cdot d\mathbf{r}, \tag{2.13}$$