SUPERSYMMETRY IN PARTICLE PHYSICS

Supersymmetry has been a central topic in particle physics since the early 1980s, and represents the culmination of the search for fundamental symmetries that has dominated particle physics for the last 50 years. Traditionally, the constituents of matter (fermions) have been regarded as essentially different from the particles (bosons) that transmit the forces between them. In supersymmetry, however, fermions and bosons are unified.

This is the first textbook to provide a simple pedagogical introduction to what has been a formidably technical field. The elementary and practical treatment brings readers to the frontier of contemporary research, in particular, to the confrontation with experiments at the Large Hadron Collider. Intended primarily for first-year graduate students in particle physics, both experimental and theoretical, this volume will also be of value to researchers in experimental and phenomenological supersymmetry. Supersymmetric theories are constructed through an intuitive 'trial and error' approach, rather than being formal and deductive. The basic elements of spinor formalism and superfields are introduced, allowing readers to access more advanced treatments. Emphasis is placed on physical understanding, and on detailed, explicit derivations of all important steps. Many short exercises are included making for a valuable and accessible self-study tool.

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SUPERSYMMETRY IN PARTICLE PHYSICS

An Elementary Introduction

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521880237

First published 2007

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-88023-7 hardback

For Danny

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Preface

This book is intended to be an elementary and practical introduction to supersymmetry in particle physics. More precisely, I aim to provide an accessible, self-contained account of the basic theory required for a working understanding of the 'Minimal Supersymmetric Standard Model' (MSSM), including 'soft' symmetry breaking. Some simple phenomenological applications of the model are also developed in the later chapters.

The study of supersymmetry (SUSY) began in the early 1970s, and there is now a very large, and still growing, research literature on the subject, as well as many books and review articles. However, in my experience the existing sources are generally suitable only for professional (or intending) theorists. Yet searches for SUSY have been pursued in experimental programmes for some time, and are prominent in experiments planned for the Large Hadron Collider at CERN. No direct evidence for SUSY has yet been found. Nevertheless, for the reasons outlined in Chapter 1, supersymmetry at the TeV scale has become the most highly developed framework for guiding and informing the exploration of physics beyond the Standard Model. This dominant role of supersymmetry, both conceptual and phenomenological, suggests a need for an entry-level introduction to supersymmetry, which is accessible to the wider community of particle physicists.

The first difficulty presented by conventional texts on supersymmetry – and it deters many students – is one of notation. Right from the start, discussions tend to be couched in terms of a spinor notation that is generally not familiar from standard courses on the Dirac equation – namely, that of either 'dotted and undotted 2-component Weyl spinors', or '4-component Majorana spinors'. This creates something of a conceptual discontinuity between what most students already know, and what they are trying to learn; it becomes a pedagogical barrier. By contrast, my approach builds directly on knowledge of Dirac spinors in a conventional representation, using 2-component ('half-Dirac') spinors, without necessarily requiring the more sophisticated dotted and undotted formalism. The latter is, however, introduced early on (in Section 2.3), but it can be treated as an optional extra; the essential elements of SUSY and the MSSM (contained in Sections 3.1, 3.2, 4.2, 4.4, 5.1 and Chapters 7 and 8) can be understood quite reasonably without it.

Apart from its simple connection to standard Dirac theory, a second advantage of the 2-component formalism is, I think, that it is simpler to use than the Majorana one for motivating and establishing the forms of simple SUSY-invariant Lagrangians. Again, a more powerful route is available via the superfield formalism, to which I provide access in Chapter 6, but the essentials do not depend on it.

On the other hand, I don't think it is wise to eschew the Majorana formalism altogether. For one thing, there are some important sources which adopt it exclusively, and which students might profitably consult. Furthermore, the Majorana formalism appears to be the one generally used in SUSY calculations, since, with some modifications, it allows the use of short-cuts familiar from the Dirac case. So I provide an early introduction to Majorana spinors as well, in Section 2.5; and at various places subsequently I point out the Majorana equivalents for what is going on. I make use of Majorana forms in Section 8.2, where I recover the Standard Model interactions in the MSSM, and also in the calculations of Section 5.2 and of Chapter 12. I believe that the indicated arguments justify the added burden, to the interested reader, of having to acquire some familiarity with a second language.

Moving on from notation, my approach is generally intuitive and constructive, rather than formal and deductive. It is very much a do-it-yourself treatment. Thus in Sections 2.1 and 2.2 I provide a gentle and detailed introduction to the use of Weyl spinors in the 'half-Dirac' notation. Care is taken to introduce a simple (free) SUSY theory very slowly and intuitively in Section 3.1, and this is followed by an appetite-whetting preview of the MSSM, as a relief from the diet of formalism. The simple SUSY transformations learned in Section 3.1 are used to motivate the SUSY algebra in Section 4.2 (rather than just postulating it), and simple consequences for supermultiplet structure are explained in Section 4.4. The more technical matter of the necessity for auxiliary fields (even in such a simple case) is discussed at the end of Chapter 4.

The introduction of interactions in a chiral multiplet follows reasonably straightforwardly in Section 5.1 (the Wess–Zumino model). The more technical – but theoretically crucial – property of cancellation of quadratic divergences is illustrated for some simple cases in Section 5.2.

After the optional detour into chiral superfields, the main thread is taken up again in Chapter 7, where supersymmetric gauge theories are introduced via vector supermultiplets, which are then combined with chiral supermultiplets. Here the superfield formalism has been avoided in favour of a more direct try-it-and-see approach similar to that of Section 3.1.

At roughly the half-way stage in the book, all the elements necessary for understanding the construction of the MSSM (or variants thereof) are now in place. The model is defined in Chapter 8, and immediately applied to exhibit gauge-coupling unification. Elementary ideas of SUSY breaking are introduced in Chapter 9, together with the phenomenolgically important notion of 'soft' supersymmetry-breaking parameters. The remainder of the book is devoted to simple applications: Higgs physics (Chapter 10), sparticle masses (Chapter 11) and sparticle production processes (Chapter 12).

Throughout, emphasis is placed on providing elementary, explicit and detailed derivations of important formal steps wherever possible. Many short exercises are included, which are designed to help the reader to engage actively with the text, and to keep abreast of the formal development through practice at every stage.

In keeping with the stated aim, the scope of this book is strictly limited. A list of omitted topics would be long indeed. It includes, for example: the superfield formalism for vector supermultiplets; Feynman rules in super-space; wider phenomenological implications of the MSSM; local supersymmetry (supergravity); more detail on SUSY searches; SUSY and cosmology; non-perturbative aspects of SUSY; SUSY in dimensions other than 4, and for values of N other than $N = 1$. Fortunately, a number of excellent and comprehensive monographs are now available; readers interested in pursuing matters beyond where I leave them, or in learning about topics I omit, can confidently turn to these professional treatments.

I am very conscious that the list of references is neither definitive nor comprehensive. In a few instances (for example, in reviewing the beginnings of SUSY and the MSSM) I have tried to identify the relevant original contributions, although I have probably missed some. Usually, I have not attempted to trace priorities carefully, but have referred to more comprehensive reviews, or have simply quoted such references as came to hand as I worked my own way into the subject. I apologize to the many researchers whose work, as a consequence, has not been referenced here.

The book has grown out of lectures to graduate students at Oxford working in both experimental and theoretical particle physics. In this, its genesis is very similar to my book with Tony Hey, *Gauge Theories in Particle Physics*, first published in 1982 and now in its third (two-volume) edition. The present book aims to reach a similar readership: in particular, I have tried to design the level so that it follows smoothly on from the earlier one. Indeed, as the title suggests, this book may be seen as 'volume 3' in the series.

However, I would expect theorists and experimentalists to use the book differently. For theorists, it should be a relatively easy read, setting them up for immediate access to the professional literature and more advanced monographs. On the other hand, many experimentalists are likely to find some of the formal parts indigestible, even with the support provided. They should be able to find a reasonably friendly route to the physics they want to learn via the 'essential elements' mentioned earlier (that is, Sections 2.1, 2.2, 3.1, 3.2, 4.2, 4.4, 5.1 and Chapters 7 and 8), to be followed by whatever applications they are most interested in. Much of this material should not be beyond final year maths or physics undergraduates who have taken courses in relativistic quantum mechanics, introductory quantum field theory, and gauge theories. By the same token, the book may also be useful to a wide range of physicists in other areas, who wish to gain a first-hand appreciation of the excitement and anticipation which surround the possible discovery of supersymmetry at the TeV scale.

Ian J. R. Aitchison
February 2007

Acknowledgements

Questions raised by three successive generations of Oxford students have led to many improvements in my original notes. I am grateful to John March-Russell for many clear and patient explanations when I was trying (not for the first time) to learn supersymmetry while on leave at CERN in 2001–2, and subsequently at Oxford. I have also benefited from being able to consult Graham Ross as occasion demanded. I thank Michael Peskin and Stan Brodsky for welcoming me as a visitor to the Stanford Linear Accelerator Center Theory Group, where work on the book continued, supported by the Department of Energy under contract DE-AC02-76SF00515. As regards written sources, I owe most to Stephen Martin's '*A Supersymmetry Primer*' (reference 46), and to '*Weak Scale Supersymmetry*' by Howard Baer and Xerxes Tata (reference 49); I hope I have owned up to all my borrowings. In developing the material for Chapter 6 I was greatly helped by unpublished notes of lectures on superfields by the late Caroline Fraser, given at Annecy in 1981–2.

Above all, and once again, I owe a special debt to my good friend George Emmons, who has been an essential part of this project from the beginning: his comments and queries revealed misconceptions on my part, as well as obscurities in the presentation, and led to many improvements in the developing text; he read carefully through several successive LaTex drafts, spotting many errors, and he corrected the final proofs. His encouragement and support have been invaluable.