1

Introduction and motivation

Supersymmetry (SUSY) – a symmetry relating bosonic and fermionic degrees of freedom – is a remarkable and exciting idea, but its implementation is technically rather complicated. It can be discouraging to find that after standard courses on, say, the Dirac equation and quantum field theory, one has almost to start afresh and master a new formalism, and moreover one that is not fully standardized. On the other hand, 30 years have passed since the first explorations of SUSY in the early 1970s, without any direct evidence of its relevance to physics having been discovered. The Standard Model (SM) of particle physics (suitably extended to include an adequate neutrino phenomenology) works extremely well. So the hard-nosed seeker after truth may well wonder: why spend the time learning all this intricate SUSY formalism? Indeed, why speculate at all about how to go 'beyond' the SM, unless or until experiment forces us to? If it's not broken, why try and fix it?

As regards the formalism, most standard sources on SUSY use either the 'dotted and undotted' 2-component (Weyl) spinor notation found in the theory of representations of the Lorentz group, or 4-component Majorana spinors. Neither of these is commonly included in introductory courses on the Dirac equation (although perhaps they should be), but it is perfectly possible to present simple aspects of SUSY using a notation which joins smoothly on to standard 4-component Dirac equation courses, and a brute force, 'try-it-and-see' approach to constructing SUSY-invariant theories. That is the approach to be followed in this book, at least to start with. However, as we go along the more compact Weyl spinor formalism will be introduced, and also (more briefly) the Majorana formalism. Later, we shall include an introduction to the powerful superfield formalism. All this formal concentration is partly because the simple-minded approach becomes too cumbersome after a while, but mainly because discussions of the phenomenology of the Minimal Supersymmetric Standard Model (MSSM) generally make use of one or other of these more sophisticated notations.

What of the need to go beyond the Standard Model? Within the SM itself, there is a plausible historical answer to that question. The V–A current–current (four-fermion) theory of weak interactions worked very well for many years, when used at lowest order in perturbation theory. Yet Heisenberg [1] had noted as early as 1939 that problems arose if one tried to compute higher-order effects, perturbation theory apparently breaking down completely at the then unimaginably high energy of some 300 GeV (the scale of $G_{\rm F}^{-1/2}$). Later, this became linked to the non-renormalizability of the four-fermion theory, a purely theoretical problem in the years before experiments attained the precision required for sensitivity to electroweak radiative corrections. This perceived disease was alleviated but not cured in the 'Intermediate Vector Boson' model, which envisaged the weak force between two fermions as being mediated by massive vector bosons. The non-renormalizability of such a theory was recognized, but not addressed, by Glashow [2] in his 1961 paper proposing the SU(2) × U(1) structure. Weinberg [3] and Salam [4], in their gauge-theory models, employed the hypothesis of spontaneous symmetry breaking to generate masses for the gauge bosons and the fermions, conjecturing that this form of symmetry breaking would not spoil the renormalizability possessed by the massless (unbroken) theory. When 't Hooft [5] demonstrated this in 1971, the Glashow–Salam–Weinberg theory achieved a theoretical status comparable to that of quantum electrodynamics (QED). In due course the precision electroweak experiments spectacularly confirmed the calculated radiative corrections, even yielding a remarkably accurate prediction of the top quark mass, based on its effect as a virtual particle . . . but note that even this part of the story is not yet over, since we have still not obtained experimental access to the proposed symmetry-breaking (Higgs [6]) sector. If and when we do, it will surely be a remarkable vindication of theoretical preoccupations dating back to the early 1960s.

It seems fair to conclude that worrying about perceived imperfections of a theory, even a phenomenologically very successful one, can pay off. In the case of the SM, a quite serious imperfection (for many theorists) is the 'SM fine-tuning problem', which we shall discuss in a moment. SUSY can suggest a solution to this perceived problem, provided that supersymmetric partners to known particles have masses no larger than a few TeV (roughly).

In addition to the 'fine-tuning' motivation for SUSY – to which, as we shall see, there are other possible responses – there are some quantitative results (Section 1.2), and theoretical considerations (Section 1.3) , which have inclined many physicists to take SUSY and the MSSM (or something like it) very seriously. As always, experiment will decide whether these intuitions were correct or not. A lot of work has been done on the phenomenology of such theories, which has influenced the Large Hadron Collider (LHC) detector design. Once again, it will surely be extraordinary if, in fact, the world turns out to be this way.

Cambridge University Press
978-0-521-88023-7 - Supersymmetry in Particle Physics: An Elementary Introduction
Ian J. R. Aitchison
Excerpt
More information

1.1 The SM fine-tuning problem

The electroweak sector of the SM contains within it a parameter with the dimensions of energy (i.e. a 'weak scale'), namely

$$v \approx 246 \text{ GeV}, \tag{1.1}$$

where $v/\sqrt{2}$ is the vacuum expectation value (or 'vev') of the neutral Higgs field, $\langle 0|\phi^0|0\rangle = v/\sqrt{2}$. The occurrence of the vev signals the 'spontaneous' breaking of electroweak gauge symmetry (see, for example [7], Chapter 19), and the associated parameter v sets the scale, in principle, of all masses in the theory. For example, the mass of the $\mathrm{W}^{\pm}$ (neglecting radiative corrections) is given by

$$M_{\mathrm{W}} = gv/2 \sim 80\,\text{GeV}, \tag{1.2}$$

and the mass of the Higgs boson is

$$M_{\mathrm{H}} = v\sqrt{\frac{\lambda}{2}}, \tag{1.3}$$

where g is the SU(2) gauge coupling constant, and λ is the strength of the Higgs self-interaction in the Higgs potential

$$V = -\mu^2\phi^\dagger\phi + \frac{\lambda}{4}(\phi^\dagger\phi)^2, \tag{1.4}$$

where $\lambda > 0$ and $\mu^2 > 0$. Here ϕ is the SU(2) doublet field

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \tag{1.5}$$

and all fields are understood to be quantum, no 'hat' being used.

Recall now that the *negative* sign of the 'mass^2' term $-\mu^2$ in (1.4) is essential for the spontaneous symmetry-breaking mechanism to work. With the sign as in (1.4), the minimum of V interpreted as a classical potential is at the non-zero value

$$|\phi| = \sqrt{2}\mu/\sqrt{\lambda} \equiv v/\sqrt{2}, \tag{1.6}$$

where $\mu \equiv \sqrt{\mu^2}$. This classical minimum (equilibrium value) is conventionally interpreted as the expectation value of the quantum field in the quantum vacuum (i.e. the vev), at least at tree level. If '$-\mu^2$' in (1.4) is replaced by the positive quantity 'μ^2', the classical equilibrium value is at the origin in field space, which would imply $v = 0$, in which case all particles would be massless. Hence it is vital to preserve the sign, and indeed magnitude, of the coefficient of $\phi^\dagger\phi$ in (1.4).

The discussion so far has been at tree level (no loops). What happens when we include loops? The SM is renormalizable, which means that finite results are obtained for all higher-order (loop) corrections even if we extend the virtual momenta

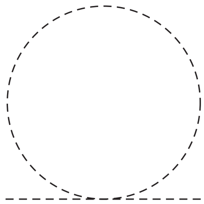

Figure 1.1 One-loop self-energy graph in ϕ^4 theory.

in the loop integrals all the way to infinity; but although this certainly implies that the theory is well defined and calculable up to infinite energies, in practice no one seriously believes that the SM is really all there is, however high we go in energy. That is to say, in loop integrals of the form

$$\int^{\Lambda} \mathrm{d}^4 k \ f(k, \text{external momenta}) \tag{1.7}$$

we do not think that the cut-off Λ *should* go to infinity, physically, even though the reormalizability of the theory assures us that no inconsistency will arise if it does. More reasonably, we regard the SM as part of a larger theory which includes as yet unknown 'new physics' at high energy, Λ representing the scale at which this new physics appears, and where the SM must be modified. At the very least, for instance, there surely must be some kind of new physics at the scale when quantum gravity becomes important, which is believed to be indicated by the Planck mass

$$M_{\mathrm{P}} = (G_{\mathrm{N}})^{-1/2} \simeq 1.2 \times 10^{19} \ \mathrm{GeV}. \tag{1.8}$$

If this is indeed the scale of the new physics beyond the SM or, in fact, if there is *any* scale of 'new physics' even several orders of magnitude different from the scale set by v, then we shall see that we meet a problem with the SM, once we go beyond tree level.

The 4-boson self-interaction in (1.4) generates, at one-loop order, a contribution to the $\phi^\dagger\phi$ term, corresponding to the self-energy diagram of Figure 1.1, which is proportional to

$$\lambda \int^{\Lambda} \mathrm{d}^4 k \, \frac{1}{k^2 - M_{\mathrm{H}}^2}. \tag{1.9}$$

This integral clearly diverges quadratically (there are four powers of k in the numerator, and two in the denominator), and it turns out to be *positive*, producing a correction

$$\sim \lambda \Lambda^2 \phi^\dagger \phi \tag{1.10}$$

to the 'bare' $-\mu^2\phi^\dagger\phi$ term in V. (The '$\sim$' represents a numerical factor, such as $1/4\pi^2$, which is unimportant for the argument here: we shall include such factors explicitly in a later calculation, in Section 5.2.) The coefficient $-\mu^2$ of $\phi^\dagger\phi$ is then replaced by the one-loop corrected 'physical' value $-\mu^2_{\rm phys}$, where (ignoring the numerical factor) $-\mu^2_{\rm phys} = -\mu^2 + \lambda\Lambda^2$, or equivalently

$$\mu^2_{\rm phys} = \mu^2 - \lambda\Lambda^2. \tag{1.11}$$

Re-minimizing V, we obtain (1.6) but with μ replaced by $\mu_{\rm phys} \equiv \sqrt{\mu^2_{\rm phys}}$. Consider now what is the likely value of $\mu_{\rm phys}$. With v fixed phenomenologically by (1.1), equation (1.6), as corrected to involve $\mu_{\rm phys}$, provides a relation between the two unknown parameters $\mu_{\rm phys}$ and λ: $\mu_{\rm phys} \approx \sqrt{\lambda}\,123\,\text{GeV}$. It follows that if we want to be able to treat the Higgs coupling λ perturbatively, $\mu_{\rm phys}$ can hardly be much greater than a few hundred GeV at most. (A value considerably greater than this would imply that λ is very much greater than unity, and the Higgs sector would be 'strongly interacting'; while not logically excluded, this possibility is generally not favoured, because of the practical difficulty of making reliable non-perturbative calculations.) On the other hand, if $\Lambda \sim M_{\rm P} \sim 10^{19}$ GeV, the one-loop correction in (1.11) is then vastly greater than $\sim(100\text{ GeV})^2$, so that to arrive at a value $\sim(100\text{ Gev})^2$ *after* inclusion of this loop correction would seem to require that we start with an equally huge value of the Lagrangian parameter μ^2, relying on a remarkable cancellation, or *fine-tuning*, to get us from $\sim(10^{19}\text{ GeV})^2$ down to $\sim(10^2\text{ GeV})^2$.

In the SM, this fine-tuning problem involving the parameter $\mu_{\rm phys}$ affects not only the mass of the Higgs particle, which is given in terms of $\mu_{\rm phys}$ (combining (1.3) and (1.6)) by

$$M_{\rm H} = \sqrt{2}\mu_{\rm phys}, \tag{1.12}$$

but also the mass of the W,

$$M_{\rm W} = g\mu_{\rm phys}/\sqrt{\lambda}, \tag{1.13}$$

and ultimately all masses in the SM, which derive from v and hence $\mu_{\rm phys}$. The serious problem posed for the SM by this 'unnatural' situation, which is caused by quadratic mass divergences in the scalar sector, was pointed out by K. G. Wilson in a private communication to L. Susskind [8].[1]

[1] From a slightly different perspective, 't Hooft [9] also drew attention to difficulties posed by theories with 'unnaturally' light scalars. In the context of Grand Unified gauge theories, Weinberg [10] emphasized the difficulty of finding a natural theory (i.e. one that is not fine-tuned) in which scalar fields associated with symmetry breaking are elementary, and some symmetries are broken at the GUT scale $\sim 10^{16}$ GeV whereas others are broken at the very much lower weak scale; this is usually referred to as the 'gauge hierarchy problem'.

This fine-tuning problem would, of course, be much less severe if, in fact, 'new physics' appeared at a scale Λ which was much smaller than M_P. How much tuning is acceptable is partly a subjective matter, but for many physicists the only completely 'natural' situation is that in which the scale of new physics is within an order of magnitude of the weak scale, as defined by the quantity v of equation (1.1), i.e. no higher than a few TeV. The question then is: what might this new physics be?

Within the framework of the discussion so far, the aim of an improved theory must be somehow to eliminate the quadratic dependence on the (assumed high) cut-off scale, present in theories with fundamental (or 'elementary') scalar fields. In the SM, such fields were introduced to provide a simple model of spontaneous electroweak symmetry breaking. Hence one response – the first, historically – to the fine-tuning problem is to propose [8] (see also [11]) that symmetry breaking occurs 'dynamically'; that is, as the result of a new strongly interacting sector with a mass scale in the TeV region. In such theories, generically called 'technicolour', the scalar states are not elementary, but rather fermion–antifermion bound states. The dynamical picture is analogous to that in the BCS theory of superconductivity (see, for example, Chapters 17, 18 and 19 of [7]). In this case, the Lagrangian for the Higgs sector is only an effective theory, valid for energies significantly below the scale at which the bound state structure would be revealed, say 1–10 TeV. The integral in (1.9) can then only properly be extended to this scale, certainly not to a hierarchically different scale such as M_P, or the GUT scale. This scheme works very nicely as far as generating masses for the weak bosons is concerned. However, in the SM the fermion masses also are due to the coupling of fermions to the Higgs field, and hence, if the Higgs field is to be completely banished from the 'fundamental' Lagrangian, the proposed new dynamics must also be capable of generating the fermion mass spectrum. This has turned out to require increasingly complicated forms of dynamics, to meet the various experimental constraints. Still, technicolour theories are not conclusively ruled out. Reviews are provided by Fahri and Susskind [12], and more recently by Lane [13]; see also the somewhat broader review by Hill and Simmons [14].

If, on the other hand, fundamental scalars are to be included in the theory, how might the quadratic divergences be controlled? A clue is provided by considering why such divergences only seem to affect the scalar sector. In QED the photon self-energy diagram of Figure 1.2 is apparently quadratically divergent (there are two fermion propagators, each of which depends linearly on the integrated 4-momentum). As in the scalar case, such a quadratic divergence would imply an enormous quantum correction to the photon mass. In fact this divergence is absent, provided the theory is regularized in a gauge-invariant way (see, for example [15], Section 11.3). In other words, the symmetry of gauge invariance guarantees that no

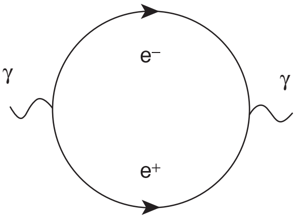

Figure 1.2 One-loop photon self-energy diagram in QED.

term of the form

$$m_\gamma^2 A^\mu A_\mu \tag{1.14}$$

can be radiatively generated in an unbroken gauge theory: the photon is massless. The diagram of Figure 1.2 is divergent, but only logarithmically; the divergence is absorbed in a field strength renormalization constant, and is ultimately associated with the running of the fine structure constant (see [7], Section 15.2).

We may also consider the electron self-energy in QED, generated by a one-loop process in which an electron emits and then re-absorbs a photon. This produces a correction δm to the fermion mass m in the Lagrangian, which seems to vary linearly with the cut-off:

$$\delta m \sim \alpha \int^\Lambda \frac{\mathrm{d}^4 k}{\not{k} k^2} \sim \alpha\Lambda. \tag{1.15}$$

(Here we have neglected both the external momentum and the fermion mass, in the fermion propagator, since we are interested in the large k behaviour.) Although perhaps not so bad as a quadratic divergence, such a linear one would still lead to unacceptable fine-tuning in order to arrive at the physical electron mass. In fact, however, when the calculation is done in detail one finds

$$\delta m \sim \alpha m \ln \Lambda, \tag{1.16}$$

so that even if $\Lambda \sim 10^{19}$ GeV, we have $\delta m \sim m$ and no unpleasant fine-tuning is necessary after all.

Why does it happen in this case that $\delta m \sim m$? It is because the Lagrangian for QED (and the SM for that matter) has a special symmetry as the fermion masses go to zero, namely chiral symmetry. This is the symmetry under transformations (on fermion fields) of the form

$$\psi \rightarrow \mathrm{e}^{\mathrm{i}\alpha\gamma_5}\psi \tag{1.17}$$

in the U(1) case, or

$$\psi \rightarrow \mathrm{e}^{\mathrm{i}\boldsymbol{\alpha}\cdot\boldsymbol{\tau}/2\gamma_5}\psi \tag{1.18}$$

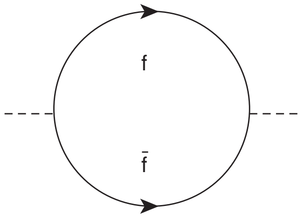

Figure 1.3 Fermion loop contribution to the Higgs self-energy.

in the SU(2) case. This symmetry guarantees that all radiative corrections to m, computed in perturbation theory, will vanish as $m \to 0$. Hence δm must be proportional to m, and the dependence on Λ is therefore (from dimensional analysis) only logarithmic.

In these two examples from QED, we have seen how unbroken gauge and chiral symmetries keep vector mesons and fermions massless, and remove 'dangerous' quadratic and linear divergences from the theory. If we could find a symmetry which grouped scalar particles with either massless fermions or massless vector bosons, then the scalars would enjoy the same 'protection' from dangerous divergences as their symmetry partners. Supersymmetry is precisely such a symmetry: as we shall see, it groups scalars together with fermions (and vector bosons with fermions also). The idea that supersymmetry might provide a solution to the SM fine-tuning problem was proposed by Witten [16], Veltman [17] and Kaul [18].

We can understand qualitatively how supersymmetry might get rid of the quadratic divergences in the scalar self-energy by considering a possible fermion loop correction to the $-\mu^2\phi^\dagger\phi$ term, as shown in Figure 1.3. At zero external momentum, such a contribution behaves as

$$\left(-g_{\rm f}^2\int^\Lambda \mathrm{d}^4k\,\mathrm{Tr}\left[\frac{1}{(\not k - m_{\rm f})^2}\right]\right)\phi^\dagger\phi = \left(-4g_{\rm f}^4\int^\Lambda \mathrm{d}^4k\,\frac{k^2+m_{\rm f}^2}{\left(k^2-m_{\rm f}^2\right)^2}\right)\phi^\dagger\phi. \tag{1.19}$$

The sign here is crucial, and comes from the closed fermion loop. The term with the k^2 in the numerator in (1.19) is quadratically divergent, and of opposite sign to the quadratic divergence (1.10) due to the Higgs loop. Ignoring numerical factors, these two contributions together have the form

$$\left(\lambda - g_{\rm f}^2\right)\Lambda^2\phi^\dagger\phi. \tag{1.20}$$

The possibility now arises that *if* for some reason there existed a boson–fermion coupling $g_{\rm f}$ related to the Higgs coupling by

$$g_{\rm f}^2 = \lambda \tag{1.21}$$

then this quadratic sensitivity to Λ would not occur.

A relation between coupling constants, such as (1.21), is characteristic of a symmetry, but in this case it must evidently be a symmetry which relates a purely bosonic vertex to a boson–fermion (Yukawa) one. Relations of the form (1.21) are indeed just what occur in a SUSY theory, as we shall see in Chapter 5. In addition, the masses of bosons and fermions belonging to the same SUSY multiplet are equal, if SUSY is unbroken; in this simplified model, then, we would have $m_{\rm f} = M_{\rm H}$. Note, however, that the cancellation of the quadratic divergence occurs whatever the values of $m_{\rm f}$ and $M_{\rm H}$, since these masses do not enter the expression (1.20). We shall show this explicitly for the Wess–Zumino model [19] in Chapter 5. It is a general result in any SUSY theory, and has the important consequence that SUSY-breaking mass terms (as are certainly required phenomenologically) can be introduced 'by hand' without spoiling the cancellation of quadratic divergences. As we shall see in Chapter 9, other SUSY-breaking terms which do not compromise this cancellation are also possible; they are referred to generically as 'soft SUSY-breaking terms'.

To implement this idea in the context of the (MS)SM, it will be necessary to postulate the existence of new fermionic 'superpartners' of the Higgs field – 'Higgsinos' – as discussed in Chapters 3 and 8. But this will by no means deal with all the quadratic divergences present in the $-\mu^2\phi^\dagger\phi$ term. In principle, every SM fermion can play the role of 'f' in (1.19), since they all have a Yukawa coupling to the Higgs field. To cancel all these quadratic divergences will require the introduction of scalar superpartners for all the SM fermions, that is, an appropriate set of squarks and sleptons. There are also quadratic divergences associated with the contribution of gauge boson loops to the '$-\mu^2$' term, and these too will have to be cancelled by fermionic superpartners, 'gauginos'. In this way, the outlines of a supersymmetrized version of the SM are beginning to emerge.

After cancellation of the Λ^2 terms via (1.21), the next most divergent contributions to the '$-\mu^2$' term grow logarithmically with Λ, but even terms logarithmic in the cut-off can be unacceptably large. Consider a simple 'one Higgs – one new fermion' model. The $\ln\Lambda$ contribution to the '$-\mu^2$' term has the form

$$\sim \lambda\left(aM_{\rm H}^2 - bm_{\rm f}^2\right)\ln\Lambda, \tag{1.22}$$

where a and b are numerical factors. Even though the dependence on Λ is now tamed, a fine-tuning problem will arise in the case of any fermion (coupling to the Higgs field) whose mass $m_{\rm f}$ is very much larger than the weak scale. In general, if the Higgs sector has any coupling, even indirect via loops, to very massive states (as happens in Grand Unified Theories for example), the masses of these states will dominate radiative corrections to the '$-\mu^2$' term, requiring large cancellations once again.

This situation is dramatically improved by SUSY. Roughly speaking, in a supersymmetric version of our 'one Higgs – one new fermion' model, the boson and fermion masses would be equal ($M_{\rm H} = m_{\rm f}$), and so would the coefficients a and b in (1.22), with the result that the correction (1.22) would vanish! Similarly, other contributions to the self-energy from SM particles and their superpartners would all cancel out, if SUSY were exact. More generally, in supersymmetric theories only wavefunction renormalizations are infinite as $\Lambda \to \infty$, as we shall discuss further in the context of the Wess–Zumino model in Section 5.2; these will induce corresponding logarithmic divergences in the values of physical (renormalized) masses (see, for example, Section 10.4.2 of [15]). However, no superpartners for the SM particles have yet been discovered, so SUSY – to be realistic in this context – must be a (softly) broken symmetry (see Chapter 9), with the masses of the superpartners presumably lying at too high values to have been detected yet. In our simple model, this means that $M_{\rm H}^2 \neq m_{\rm f}^2$. In this case, the quadratic divergences still cancel, as previously noted, and the remaining correction to the physical '$-\mu^2$' term will be of order $\lambda(M_{\rm H}^2 - m_{\rm f}^2)\ln\Lambda$. We conclude that (softly) broken SUSY may solve the SM fine-tuning problem, provided that the new SUSY superpartners are not too much heavier than the scale of v (or $M_{\rm H}$), or else we are back to some form of fine-tuning.[2] Of course, how much fine-tuning we are prepared to tolerate is a matter of taste, but the argument strongly suggests that the discovery of SUSY should be within the reach of the LHC – if not, as it now seems, of either LEP or the Tevatron. Hence the vast amount of work that has gone into constructing viable theories, and analysing their expected phenomenologies.

In summary, SUSY can *stabilize* the hierarchy $M_{\rm H,W} \ll M_{\rm P}$, in the sense that radiative corrections will not drag $M_{\rm H,W}$ up to the high scale Λ; and the argument implies that, for the desired stabilization to occur, SUSY should be visible at a scale not much greater than a few TeV. The origin of this latter scale (that of SUSY-breaking – see Chapter 9) is a separate problem. It is worth emphasizing that a theory of the MSSM type, with superpartner masses no larger than a few TeV, is a consistent effective field theory which is perturbatively calculable for all energies up to, say, the Planck, or a Grand Unification, scale without requiring fine-tuning (but see Section 10.3 for further discussion of this issue, within the MSSM specifically). Whether such a post-SUSY 'desert' exists or not is, of course, for experiment to decide.

Notwithstanding the foregoing motivation for seeking a supersymmetric version of the SM (a view that became widely accepted from the early 1980s), the reader should be aware that, historically, supersymmetry was not invented as a response to

[2] The application of the argument to motivate a supersymmetric SU(5) grand unified theory (in which Λ is now the unification scale), which is softly broken at the TeV mass scale, was made by Dimopoulos and Georgi [20] and Sakai [21]. Well below the unification scale, the effective field content of these models is that of the MSSM.