NONLINEAR CLIMATE DYNAMICS

This book introduces stochastic dynamical systems theory to synthesise our current knowledge of climate variability. Nonlinear processes, such as advection, radiation and turbulent mixing, play a central role in climate variability. These processes can give rise to transition phenomena, associated with tipping or bifurcation points, once external conditions are changed. The theory of dynamical systems provides a systematic way to study these transition phenomena. Its stochastic extension also forms the basis of modern (nonlinear) data analysis techniques, predictability studies and data assimilation methods. Early chapters present the stochastic dynamical systems framework and a hierarchy of climate models to study climate variability. Later chapters analyse phenomena such as the North Atlantic Oscillation, the El Niño/Southern Oscillation, Atlantic Multidecadal Variability, Dansgaard-Oeschger Events, Pleistocene Ice Ages and climate predictability. This book will prove invaluable for graduate students and researchers in climate dynamics, physical oceanography, meteorology and paleoclimatology.

HENK A. DIJKSTRA is Professor of dynamical oceanography at the Institute for Marine and Atmospheric Research, Department of Physics and Astronomy, Utrecht University, The Netherlands. His main research interests are in the application of dynamical systems methods to problems in climate variability. He is the author of Nonlinear Physical Oceanography, Second Edition (2005) and Dynamical Oceanography (2008). He is a member of the Dutch Royal Academy of Arts and Sciences. In 2005, he received the Lewis Fry Richardson medal from the European Geosciences Union and in 2009 he was elected a Fellow of the Society for Industrial and Applied Mathematics.
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preface</td>
<td>ix</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>xi</td>
</tr>
<tr>
<td>1. Climate Variability</td>
<td>1</td>
</tr>
<tr>
<td>1.1 The observational database</td>
<td>2</td>
</tr>
<tr>
<td>1.2 Phenomena: temporal and spatial scales</td>
<td>4</td>
</tr>
<tr>
<td>1.3 The climate system</td>
<td>6</td>
</tr>
<tr>
<td>1.4 The stochastic dynamical systems framework</td>
<td>7</td>
</tr>
<tr>
<td>1.5 Overview of the book</td>
<td>9</td>
</tr>
<tr>
<td>2. Deterministic Dynamical Systems</td>
<td>11</td>
</tr>
<tr>
<td>2.1 Finite dimensional dynamical systems</td>
<td>11</td>
</tr>
<tr>
<td>2.2 Stability theory and transient behaviour</td>
<td>13</td>
</tr>
<tr>
<td>2.3 Bifurcation theory: fixed points</td>
<td>19</td>
</tr>
<tr>
<td>2.4 Bifurcation theory: periodic orbits</td>
<td>25</td>
</tr>
<tr>
<td>2.5 Beyond simple behaviour: attractors</td>
<td>28</td>
</tr>
<tr>
<td>3. Introduction to Stochastic Calculus</td>
<td>35</td>
</tr>
<tr>
<td>3.1 Random variables</td>
<td>35</td>
</tr>
<tr>
<td>3.2 Stochastic processes</td>
<td>38</td>
</tr>
<tr>
<td>3.3 Stochastic calculus</td>
<td>40</td>
</tr>
<tr>
<td>3.4 Stochastic differential equations</td>
<td>45</td>
</tr>
<tr>
<td>3.5 The Fokker-Planck equations</td>
<td>51</td>
</tr>
<tr>
<td>3.6 Numerical solutions of SDEs</td>
<td>58</td>
</tr>
<tr>
<td>4. Stochastic Dynamical Systems</td>
<td>64</td>
</tr>
<tr>
<td>4.1 Nonautonomous dynamical systems</td>
<td>64</td>
</tr>
<tr>
<td>4.2 Random dynamical systems</td>
<td>67</td>
</tr>
<tr>
<td>4.3 Stochastic bifurcations</td>
<td>77</td>
</tr>
</tbody>
</table>
Contents

5 Analysing Data from Stochastic Dynamical Systems 83
  5.1 Classical univariate methods 83
  5.2 Hypothesis testing and significance 89
  5.3 Bivariate and multivariate analysis 93
  5.4 Nonstationary methods: wavelets 102
  5.5 Nonlinear methods 103

6 The Climate Modelling Hierarchy 116
  6.1 Model traits: scales and processes 117
  6.2 Scientific questions and model choice 123
  6.3 Deterministic versus stochastic models 124
  6.4 Model error 128

7 The North Atlantic Oscillation 131
  7.1 Midlatitude atmospheric variability 131
  7.2 Minimal model 137
  7.3 Variability in conceptual models 139
  7.4 Beyond conceptual models 149
  7.5 Regime transitions and metastable states 160
  7.6 Synthesis 163

8 El Niño Variability 167
  8.1 Phenomena 167
  8.2 A minimal model 170
  8.3 The ENSO mode 177
  8.4 Excitation of the ENSO mode 185
  8.5 ENSO’s phase locking to the seasonal cycle 190
  8.6 The irregularity of El Niño 192
  8.7 Synthesis 200

9 Multidecadal Variability 202
  9.1 Introduction 202
  9.2 Stability of the thermally driven ocean flows 206
  9.3 Excitation of the AMO mode 213
  9.4 Dynamical mechanisms of excitation 217
  9.5 Context of the AMO: two time scales? 225
  9.6 Synthesis 227

10 Dansgaard-Oeschger Events 231
  10.1 Phenomena 231
  10.2 The meridional overturning circulation 233
  10.3 Sensitivity of the MOC: hysteresis 236
  10.4 Variability of the MOC: oscillations 240
  10.5 Effects of noise: stochastic resonance 248
  10.6 Climate models 263
  10.7 Synthesis 265
Contents

11 The Pleistocene Ice Ages 270
   11.1 Phenomena 270
   11.2 The ‘null hypothesis’: Milankovitch theory 272
   11.3 Potentially important feedbacks 274
   11.4 Basic theories of interglacial-glacial cycles 282
   11.5 The role of the carbon cycle 293
   11.6 Synthesis 299

12 Predictability 301
   12.1 The prediction problem 302
   12.2 Concepts of predictability 307
   12.3 Behaviour of nearby trajectories 314
   12.4 Data assimilation 324
   12.5 Outlook 334

References 337

Copyright Acknowledgements 353

Index 355

Colour plates follow page 180
Preface

Dynamical systems theory is an extremely powerful framework for understanding the behavior of complex systems. Its concepts apply to many scientific fields, and hence its language provides a multidisciplinary and unifying communication tool. The theory provides a systematic approach for assessing the sensitivity of a mathematical model of a particular phenomenon to changes in parameters and initial conditions. As such, it finds application in stability problems, transition behavior and predictability studies. In addition, techniques and concepts from dynamical systems theory have led to the development of a diverse set of nonlinear methods of time series analysis.

For many phenomena, existing models cannot resolve all relevant spatial and temporal scales, and hence small-scale features are often represented as ‘noise’. As a result of the increase in computational power, solutions of the resulting stochastic partial differential equations are now within reach. Although stochastic dynamical systems are difficult to deal with, in recent years, the theory of stochastic dynamical systems has matured and is ready to be applied to many scientific areas.

This book developed from a course on climate dynamics that I taught at Colorado State University in 2005 and a course on stochastic climate models that I taught at Utrecht University in 2008. My main motivation in writing this book was to provide both an introduction into stochastic dynamical systems theory and to show the application of these methods to problems in climate dynamics.

The book is, therefore, logically divided into two parts. In the first part, Chapters 2 through 5, introductions to dynamical systems (Chapter 2), stochastic calculus (Chapter 3) and random dynamical systems (Chapter 4) are given. With the level of mathematical detail provided, this material should be accessible to graduate students in climate physics. Chapters 7 through 11 provide a description of how stochastic dynamical systems theory has been used to understand particular phenomena in climate physics. Chapter 6 illustrates the hierarchy of climate models used in the description of climate variability and climate change. Chapter 12 shows the application of dynamical systems theory to predictability studies. The choice of all the material is
quite a personal one and I apologise for certainly having forgotten to cite and discuss results of additional very relevant publications.

In the present scientific world in which everybody can be a ‘Google’ specialist on any topic in fifteen minutes, and where the focus appears to be more and more on concepts that are appealing rather than precise, it is good to know that there are frameworks available that can be used to fit pieces of a complex puzzle. The material in this book forms the basic material for such a framework related to problems in climate variability. I hope it finds its way to the younger generation of climate scientists.
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The writing of this book has been a pleasure as a result of the interaction with many colleagues in the fields of climate dynamics and dynamical systems.

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Joint work with Michael Ghil (LMD, Paris, France) and Eric Simonnet (INLN, Nice, France) has been very important for the development of the material in this book. Chapter 2 is based on a joint paper that we wrote for the Handbook of Numerical Analysis (Vol XIV). Chapter 4 is based on Eric’s notes on random dynamical systems, which we discussed extensively during my many enjoyable visits to INLN (for which I thank the CNRS for support). I thank Eric Deleersnijder (Louvain-la-Neuve, Belgium) for inviting me to give several lectures on stochastic dynamical systems, which eventually led to Chapter 3. Most of the book was written during a four-month sabbatical at the Australian National University in 2011. I thank Andy Hogg and Ross Griffiths for a quiet and great time in Canberra and the ARC Centre of Excellence for Climate System Science for the generous support.

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