Contents

Preface

1 Preliminary Concepts ................................. 1
  1.1 Introduction 1
  1.2 Historical Sketch 2
  1.3 The Nonlinear Programming Problem 4
  1.4 Optimization Problem Modeling 7
  1.5 Graphical Solution of One- and Two-Variable Problems 19
  1.6 Existence of a Minimum and a Maximum: Weierstrass Theorem 22
  1.7 Quadratic Forms and Positive Definite Matrices 25
  1.8 $C^n$ Continuity of a Function 26
  1.9 Gradient Vector, Hessian Matrix, and Their Numerical Evaluation Using Divided Differences 28
  1.10 Taylor’s Theorem, Linear, and Quadratic Approximations 33
  1.11 Miscellaneous Topics 36

2 One-Dimensional Unconstrained Minimization .................. 46
  2.1 Introduction 46
  2.2 Theory Related to Single Variable (Univariate) Minimization 46
  2.3 Unimodality and Bracketing the Minimum 54
  2.4 Fibonacci Method 55
  2.5 Golden Section Method 63
  2.6 Polynomial-Based Methods 67
  2.7 Shubert–Piyavskii Method for Optimization of Non-unimodal Functions 75
  2.8 Using MATLAB 77
  2.9 Zero of a Function 78
## Contents

### 3 Unconstrained Optimization

- **3.1 Introduction** 89
- **3.2 Necessary and Sufficient Conditions for Optimality** 90
- **3.3 Convexity** 94
- **3.4 Basic Concepts: Starting Design, Direction Vector, and Step Size** 96
- **3.5 The Steepest Descent Method** 99
- **3.6 The Conjugate Gradient Method** 106
- **3.7 Newton’s Method** 112
- **3.8 Quasi-Newton Methods** 116
- **3.9 Approximate Line Search** 121
- **3.10 Using MATLAB** 123

### 4 Linear Programming

- **4.1 Introduction** 131
- **4.2 Linear Programming Problem** 131
- **4.3 Problem Illustrating Modeling, Solution, Solution Interpretation, and Lagrange Multipliers** 132
- **4.4 Problem Modeling** 137
- **4.5 Geometric Concepts: Hyperplanes, Halfspaces, Polytopes, Extreme Points** 142
- **4.6 Standard form of an LP** 144
- **4.7 The Simplex Method – Starting with LE (≤) Constraints** 146
- **4.8 Treatment of GE and EQ Constraints** 152
- **4.9 Revised Simplex Method** 157
- **4.10 Duality in Linear Programming** 161
- **4.11 The Dual Simplex Method** 163
- **4.12 Sensitivity Analysis** 166
- **4.13 Interior Approach** 172
- **4.14 Quadratic Programming (QP) and the Linear Complementary Problem (LCP)** 176

### 5 Constrained Minimization

- **5.1 Introduction** 189
- **5.2 Graphical Solution of Two-Variable Problems** 192
- **5.3 Use of EXCEL SOLVER and MATLAB** 193
- **5.4 Formulation of Problems in Standard NLP Form** 195
- **5.5 Necessary Conditions for Optimality** 197
- **5.6 Sufficient Conditions for Optimality** 209
## Contents

5.7 Convexity 212  
5.8 Sensitivity of Optimum Solution to Problem Parameters 214  
5.9 Rosen’s Gradient Projection Method for Linear Constraints 216  
5.10 Zoutendijk’s Method of Feasible Directions (Nonlinear Constraints) 222  
5.11 The Generalized Reduced Gradient Method (Nonlinear Constraints) 232  
5.12 Sequential Quadratic Programming (SQP) 241  
5.13 Features and Capabilities of Methods Presented in this Chapter 247

6 Penalty Functions, Duality, and Geometric Programming 261  
6.1 Introduction 261  
6.2 Exterior Penalty Functions 261  
6.3 Interior Penalty Functions 267  
6.4 Duality 269  
6.5 The Augmented Lagrangian Method 276  
6.6 Geometric Programming 281

7 Direct Search Methods for Nonlinear Optimization 294  
7.1 Introduction 294  
7.2 Cyclic Coordinate Search 294  
7.3 Hooke and Jeeves Pattern Search Method 298  
7.4 Rosenbrock’s Method 301  
7.5 Powell’s Method of Conjugate Directions 304  
7.6 Nelder and Mead Simplex Method 307  
7.7 Simulated Annealing (SA) 314  
7.8 Genetic Algorithm (GA) 318  
7.9 Differential Evolution (DE) 324  
7.10 Box’s Complex Method for Constrained Problems 325

8 Multiobjective Optimization 338  
8.1 Introduction 338  
8.2 Concept of Pareto Optimality 339  
8.3 Generation of the Entire Pareto Curve 343  
8.4 Methods to Identify a Single Best Compromise Solution 345

9 Integer and Discrete Programming 359  
9.1 Introduction 359  
9.2 Zero–One Programming 361  
9.3 Branch and Bound Algorithm for Mixed Integers (LP-Based) 368
## Contents

9.4 Gomory Cut Method 372  
9.5 Farkas’ Method for Discrete Nonlinear Monotone Structural Problems 377  
9.6 Genetic Algorithm for Discrete Programming 380

10 **Dynamic Programming** ........................................ 385  
10.1 Introduction 385  
10.2 The Dynamic Programming Problem and Approach 387  
10.3 Problem Modeling and Computer Implementation 392

11 **Optimization Applications for Transportation, Assignment, and Network Problems** ........................................ 400  
11.1 Introduction 400  
11.2 Transportation Problem 400  
11.3 Assignment Problems 408  
11.4 Network Problems 413

12 **Finite Element-Based Optimization** ........................................ 424  
12.1 Introduction 424  
12.2 Derivative Calculations 427  
12.3 Sizing (i.e., Parameter) Optimization via Optimality Criteria and Nonlinear Programming Methods 432  
12.4 Topology Optimization of Continuum Structures 437  
12.5 Shape Optimization 441  
12.6 Optimization with Dynamic Response 449

*Index* 461