Computability and Logic, Fifth Edition

Computability and Logic has become a classic because of its accessibility to students without a mathematical background and because it covers not simply the staple topics of an intermediate logic course, such as Gödel's incompleteness theorems, but also a large number of optional topics, from Turing's theory of computability to Ramsey's theorem. This fifth edition has been thoroughly revised by John P. Burgess. Including a selection of exercises, adjusted for this edition, at the end of each chapter, it offers a new and simpler treatment of the representability of recursive functions, a traditional stumbling block for students on the way to the Gödel incompleteness theorems. This new edition is also accompanied by a Web site as well as an instructor's manual.

"[This book] gives an excellent coverage of the fundamental theoretical results about logic involving computability, undecidability, axiomatization, definability, incompleteness, and so on."

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"A valuable asset to those who want to enhance their knowledge and strengthen their ideas in the areas of artificial intelligence, philosophy, theory of computing, discrete structures, and mathematical logic. It is also useful to teachers for improving their teaching style in these subjects."

– *Computer Engineering*

Computability and Logic

Fifth Edition

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FURTHER TOPICS

Preface to the Fifth Edition

The original authors of this work, the late George Boolos and Richard Jeffrey, stated in the preface to the first edition that the work was intended for students of philosophy, mathematics, or other fields who desired a more advanced knowledge of logic than is supplied by an introductory course or textbook on the subject, and added the following:

> The aim has been to present the principal fundamental theoretical results *about* logic, and to cover certain other meta-logical results whose proofs are not easily obtainable elsewhere. We have tried to make the exposition as readable as was compatible with the presentation of complete proofs, to use the most elegant proofs we knew of, to employ standard notation, and to reduce *hair* (as it is technically known).

Such have remained the aims of all subsequent editions.

The "principal fundamental theoretical results *about* logic" are primarily the theorems of Gödel, the completeness theorem, and especially the incompleteness theorems, with their attendant lemmas and corollaries. The "other meta-logical results" included have been of two kinds. On the one hand, filling roughly the first third of the book, there is an extended exposition by Richard Jeffrey of the theory of Turing machines, a topic frequently alluded to in the literature of philosophy, computer science, and cognitive studies but often omitted in textbooks on the level of this one. On the other hand, there is a varied selection of theorems on (in-)definability, (un-)decidability, (in-)completeness, and related topics, to which George Boolos added a few more items with each successive edition, until by the third, the last to which he directly contributed, it came to fill about the last third of the book.

When I undertook a revised edition, my special aim was to increase the pedagogical usefulness of the book by adding a selection of problems at the end of each chapter and by making more chapters independent of one another, so as to increase the range of options available to the instructor or reader as to what to cover and what to defer. Pursuit of the latter aim involved substantial rewriting, especially in the middle third of the book. A number of the new problems and one new section on undecidability were taken from Boolos's *Nachlass*, while the rewriting of the précis of first-order logic – summarizing the material typically covered in a more leisurely way in an introductory text or course and introducing the more abstract modes of reasoning that distinguish intermediate- from introductory-level logic – was undertaken in consultation with Jeffrey. Otherwise, the changes have been my responsibility alone.

The book runs now in outline as follows. The basic course in intermediate logic culminating in the first incompleteness theorem is contained in Chapters 1, 2, 6, 7, 9, 10, 12, 15, 16, and 17, minus any sections of these chapters starred as optional. Necessary background on enumerable and nonenumerable sets is supplied in Chapters 1 and 2. All the material on computability (recursion theory) that is strictly needed for the incompletness theorems has now been collected in Chapters 6 and 7, which may, if desired, be postponed until after the needed background material in logic. That material is presented in Chapters 9, 10, and 12 (for readers who have not had an introductory course in logic including a proof of the completeness theorem, Chapters 13 and 14 will also be needed). The machinery needed for the proof of the incompleteness theorems is contained in Chapter 15 on the arithmetization of syntax (though the instructor or reader willing to rely on Church's thesis may omit all but the first section of this chapter) and in Chapter 16 on the representability of recursive functions. The first completeness theorem itself is proved in Chapter 17. (The second incompleteness theorem is discussed in Chapter 18.)

A semester course should allow time to take up several supplementary topics in addition to this core material. The topic given the fullest exposition is the theory of Turing machines and their relation to recursive functions, which is treated in Chapters 3 through 5 and 8 (with an application to logic in Chapter 11). This now includes an account of Turing's theorem on the existence of a universal Turing machine, one of the intellectual landmarks of the last century. If this material is to be included, Chapters 3 through 8 would best be taken in that order, either after Chapter 2 or after Chapter 12 (or 14).

Chapters 19 through 21 deal with topics in general logic, and any or all of them might be taken up as early as immediately after Chapter 12 (or 14). Chapter 19 is presupposed by Chapters 20 and 21, but the latter are independent of each other. Chapters 22 through 26, all independent of one another, deal with topics related to formal arithmetic, and any of them could most naturally be taken up after Chapter 17. Only Chapter 27 presupposes Chapter 18. Users of the previous edition of this work will find essentially all the material in it still here, though not always in the same place, apart from some material in the former version of Chapter 27 that has, since the last edition of this book, gone into *The Logic of Provablity*.

All these changes were made in the fourth edition. In the present fifth edition, the main change to the body of the text (apart from correction of errata) is a further revision and simplification of the treatment of the representability of recursive functions, traditionally one of the greatest difficulties for students. The version now to be found in section 16.2 represents the distillation of more than twenty years' teaching experience trying to find ever easier ways over this hump. Section 16.4 on Robinson arithmetic has also been rewritten. In response to a suggestion from Warren Goldfarb, an explicit discussion of the distinction between two different kinds of appeal to Church's thesis, avoidable and unavoidable, has been inserted at the end of section 7.2. The avoidable appeals are those that consist of omitting the verification that certain obviously effectively computable functions are recursive; the unavoidable appeals are those involved whenever a theorem about recursiveness is converted into a conclusion about effective computability in the intuitive sense.

On the one hand, it should go without saying that in a textbook on a classical subject, only a small number of the results presented will be original to the authors. On the other hand, a textbook is perhaps not the best place to go into the minutiæ of the history of a field. Apart from a section of remarks at the end of Chapter 18, we have indicated the history of the field for the student or reader mainly by the names attached to various theorems. See also the annotated bibliography at the end of the book.

There remains the pleasant task of expressing gratitude to those (beyond the dedicatees) to whom the authors have owed personal debts. By the third edition of this work the original authors already cited Paul Benacerraf, Burton Dreben, Hartry Field, Clark Glymour, Warren Goldfarb, Simon Kochen, Paul Kripke, David Lewis, Paul Mellema, Hilary Putnam, W. V. Quine, T. M. Scanlon, James Thomson, and Peter Tovey, with special thanks to Michael J. Pendlebury for drawing the "mop-up" diagram in what is now section 5.2.

In connection with the fourth edition, my thanks were due collectively to the students who served as a trial audience for intermediate drafts, and especially to my very able assistants in instruction, Mike Fara, Nick Smith, and Caspar Hare, with special thanks to the last-named for the "scoring function" example in section 4.2. In connection with the present fifth edition, Curtis Brown, Mark Budolfson, John Corcoran, Sinan Dogramaci, Hannes Eder, Warren Goldfarb, Hannes Hutzelmeyer, David Keyt, Brad Monton, Jacob Rosen, Jada Strabbing, Dustin Tucker, Joel Velasco, Evan Williams, and Richard Zach are to be thanked for errata to the fourth edition, as well as for other helpful suggestions.

Perhaps the most important change connected with this fifth edition is one not visible in the book itself: It now comes supported by an instructor's manual. The manual contains (besides any errata that may come to light) suggested hints to students for odd-numbered problems and solutions to all problems. Resources are available to students and instructors at www.cambridge.org/us/9780521877527.

January 2007 JOHN P. BURGESS