This introduction to general equilibrium modeling takes an integrated approach to the analysis of macroeconomics and finance. It provides students, practitioners, and policymakers with an easily accessible set of tools that can be used to analyze a wide range of economic phenomena.

Key features:
- Provides a consistent framework for understanding dynamic economic models.
- Introduces key concepts in finance in a discrete time setting.
- Develops a simple recursive approach for analyzing a variety of problems in a dynamic, stochastic environment.
- Sequentially builds up the analysis of consumption, production, and investment models to study their implications for allocations and asset prices.
- Reviews business cycle analysis and the business cycle implications of monetary and international models.
- Covers latest research on asset pricing in overlapping generations models and on models with borrowing constraints and transaction costs.
- Includes end-of-chapter exercises allowing readers to monitor their understanding of each topic.

Online resources available at www.cambridge.org/altug_labadie

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Preface

The starting point for any analysis in finance involves assigning a current price to a future stream of uncertain payoffs. This is the basic notion behind any asset-pricing model. Take, for example, the price of a share to a competitive firm. Since the share entitles the owner to claims for the future profits of the firm, a central problem is to assign a value to these future profits. Take another asset – a house. This provides housing services in all states of nature and at all dates. Consequently, the value of the house today must reflect the value of these future services. Other examples include the pricing of durable goods or investment projects based on their future expected marginal products. One approach to monetary economics also follows this basic principle – if money as an asset has value in equilibrium (in the absence of any legal restrictions), then this value must reflect the stream of services provided by this asset.

Our approach is to derive pricing relationships for different assets by specifying the economic environment at the outset. One of the earliest examples of this approach is Merton [342]. However, Merton does not relate the technological sources of uncertainty to the equilibrium prices of the risky assets. Alternatively, he assumes a given stochastic process for the returns of different types of assets and then prices them given assumptions about consumer preferences. Consequently, the supply side is not explicitly considered by Merton. The asset-pricing model of Lucas [317] is fully general equilibrium but it is an endowment economy, so that consumption and investment decisions are trivial. Brock [76] develops an asset-pricing model with both the demand and supply side fully specified and links it up to Ross's [369] arbitrage pricing model.

In this book, we will start from an explicit economic environment and deduce the implications for asset prices, and the form of the asset-pricing function from the equilibrium in these environments. To study the problem of asset pricing, we could also follow another approach: we could take a very general and abstract approach, viewing asset pricing as the valuation of a future stream of uncertain payoffs from the asset according to a general pricing function. Given a minimal set of assumptions about the set of payoffs, we could try to characterize the properties of this abstract pricing function. This is the approach taken by Ross [371],
Harrison and Kreps [240], Chamberlain and Rothschild [100], amongst others. One general point to note about the relationship between the two approaches to asset pricing is that the former abstract approach acquires economic content when interpreted in terms of the equilibrium approach. In fact, the benchmark payoff in the pricing function used to price streams of uncertain payoffs turns out to be the intertemporal marginal rate of substitution function for consumption. Depending on the nature of heterogeneity among consumers, the existence of complete contingent claims markets, and the role of money for acquiring consumption goods, the form of this intertemporal marginal rate of substitution function changes.

The purpose of this book is to provide an integrated treatment of a variety of dynamic equilibrium frameworks and to examine their empirical implications. The book is organized in four main parts. In Part I, we present material that constitutes the basis for much thinking in dynamic macroeconomics and finance. We begin by describing a simple version of the Arrow-Debreu contingent claims model, which is one of the building blocks of asset pricing. We also present the basics of arbitrage and asset valuation, expected utility analysis, CAPM and APT, and consumption/savings decisions under uncertainty. In Part II, we present a more fully developed set of results for dynamic economies under uncertainty using a recursive approach. In this part, we describe a pure exchange, representative consumer economy as well as economies with production. This framework allows us to derive the form of the asset-valuation function and to examine such issues as the effects of taxation on asset returns, the optimal financial structure of a firm, and the role of uncertainty in determining asset pricing and equilibrium allocations. Part III is devoted to cash-in-advance models, which allow us to examine the effects of inflation and exchange rate risk. Part IV presents material at a slightly more advanced level. In this part, we examine questions related to market incompleteness and the effects of frictions such as transactions costs. We consider the effects of borrowing constraints on equilibrium allocations and prices in a model with consumer heterogeneity and idiosyncratic risk. The stochastic overlapping generations model has been suggested as a convenient framework for analyzing issues related to “bubbles” in asset prices and the determinants of savings decisions with intergenerational heterogeneity among consumers. We examine a variety of issues using the stochastic overlapping generations framework. In many recent empirical applications of dynamic models, numerical solution methods have been combined with simulation or estimation procedures to assess quantitatively the importance of alternative model features. In this book, we also describe how numerical dynamic programming methods and other numerical methods can be used for solving and simulating a variety of dynamic economic problems.

There are many excellent texts in macroeconomics and finance that also cover material that is presented in this text. Cochrane’s [109] text is an
excellent reference that covers all of the standard issues in finance, updated using the modern approach to asset pricing. The texts by Darell Duffie [159, 161] also present the modern general equilibrium approach to finance but they are more technical in nature and help to serve as useful references, especially for advanced graduate students. The texts by Ingersoll [261], Huang and Litzenberger [256], Jarrow [263], Copeland and Weston [127], Hull [259], amongst others, present many of the standard issues of finance at differing levels of abstraction. They are recommended for students who (i) either lack a more traditional knowledge of finance and financial markets, or (ii) wish to obtain more detailed knowledge of some of the issues that we cover in this book. There is also some overlap between the topics we consider in this book and other texts dealing with dynamic general equilibrium modeling or macroeconomics such as Sargent [384] or Ljungqvist and Sargent [325].

A unifying feature of our discussion is that many of the dynamic equilibrium models that we consider can be formulated as dynamic programming problems and solved using a contraction mapping approach. Rather than introduce explicit measure-theoretic considerations for analyzing dynamic stochastic models, we describe uncertainty in terms of Markov uncertainty in a discrete-time setting. In Chapter 6, we provide a review of some results from functional analysis that we use in later chapters. For a review of basic results from functional analysis, we refer the reader to Kreyszig [290] and Naylor and Sell [351], and to Papoulis [356] for a review of probability theory and stochastic processes.

We have provided a set of detailed exercises at the end of each chapter and their solutions as a separate file. These exercises are intended to introduce some new topics at the same time that they allow the student to apply the methods described earlier. We developed this book from our teaching of finance, graduate financial economics and macroeconomics at the University of York, Duke University, the University of Minnesota, the University of Wisconsin, and Columbia University. It reflects our desire to provide a unified treatment of material that we could not find in one place. For teaching purposes, this text can be used as the basis for a graduate macroeconomics or financial economics course. We hope that this text will also prove useful to students and practitioners in the fields of macroeconomics, finance, applied general equilibrium modeling, and structural econometrics.

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