

Cambridge University Press  
978-0-521-87585-1 - Asset Pricing for Dynamic Economies  
Sumru Altug and Pamela Labadie  
Frontmatter  
[More information](#)

---

## ASSET PRICING FOR DYNAMIC ECONOMIES

This introduction to general equilibrium modeling takes an integrated approach to the analysis of macroeconomics and finance. It provides students, practitioners, and policymakers with an easily accessible set of tools that can be used to analyze a wide range of economic phenomena.

Key features:

- Provides a consistent framework for understanding dynamic economic models.
- Introduces key concepts in finance in a discrete time setting.
- Develops a simple recursive approach for analyzing a variety of problems in a dynamic, stochastic environment.
- Sequentially builds up the analysis of consumption, production, and investment models to study their implications for allocations and asset prices.
- Reviews business cycle analysis and the business cycle implications of monetary and international models.
- Covers latest research on asset pricing in overlapping generations models and on models with borrowing constraints and transaction costs.
- Includes end-of-chapter exercises allowing readers to monitor their understanding of each topic.

Online resources available at [www.cambridge.org/altug\\_labadie](http://www.cambridge.org/altug_labadie)

SUMRU ALTUG is Professor of Economics at Koç University, Istanbul, and Research Fellow on the International Macroeconomics Programme at the Centre for Economic Policy Research (CEPR), London.

PAMELA LABADIE is Professor of Economics at George Washington University, Washington DC.

Cambridge University Press

978-0-521-87585-1 - Asset Pricing for Dynamic Economies

Sumru Altug and Pamela Labadie

Frontmatter

[More information](#)

---

# ASSET PRICING FOR DYNAMIC ECONOMIES

SUMRU ALTUG AND PAMELA LABADIE



**CAMBRIDGE**  
UNIVERSITY PRESS

Cambridge University Press  
 978-0-521-87585-1 - Asset Pricing for Dynamic Economies  
 Sumru Altug and Pamela Labadie  
 Frontmatter  
[More information](#)

CAMBRIDGE UNIVERSITY PRESS  
 Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi  
 Cambridge University Press  
 The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
 Information on this title: [www.cambridge.org/9780521699143](http://www.cambridge.org/9780521699143)

© Sumru Altug and Pamela Labadie 2008

This publication is in copyright. Subject to statutory exception  
 and to the provisions of relevant collective licensing agreements,  
 no reproduction of any part may take place without  
 the written permission of Cambridge University Press.

First published 2008

Printed in the United Kingdom at the University Press, Cambridge

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*  
 Altug, Sumru.

Asset pricing for dynamic economics / Sumru Altug and Pamela Labadie.

p. cm.

ISBN 978-0-521-87585-1

I. Capital assets pricing model. I. Labadie, Pamela, 1953– II. Title.

HG4636.A448 2008

332.6–dc22

2008024826

ISBN-13 978-0-521-87585-1 hardback

ISBN-13 978-0-521-69914-3 paperback

Cambridge University Press has no responsibility for  
 the persistence or accuracy of URLs for external or  
 third-party internet websites referred to in this publication,  
 and does not guarantee that any content on such  
 websites is, or will remain, accurate or appropriate.

## *Contents*

<i>List of figures</i>	<i>page xi</i>
<i>List of tables</i>	<i>xii</i>
<i>Preface</i>	<i>xiii</i>
I BASIC CONCEPTS	I
1 Complete contingent claims	3
1.1. A one-period model	3
1.1.1. Contingent claims equilibrium	5
1.1.2. Computing the equilibrium	6
1.1.3. Pareto optimal allocations	11
1.2. Security market equilibrium	12
1.2.1. Definition	12
1.2.2. Attaining a CCE by an SME	16
1.2.3. The Pareto optimum and the representative consumer	20
1.3. Conclusions	22
1.4. Exercises	22
2 Arbitrage and asset valuation	25
2.1. Absence of arbitrage: some definitions	25
2.1.1. The law of one price	27
2.1.2. Arbitrage opportunities	30
2.2. Existence of a state-price vector	32
2.2.1. Risk-free asset	34
2.2.2. Risk-neutral pricing	35
2.2.3. The stochastic discount factor	37
2.3. Binomial security markets	38
2.3.1. An economy with two dates	39
2.3.2. A multi-period economy	41
2.4. Conclusions	47
2.5. Exercises	47
3 Expected utility	51
3.1. Expected utility preferences	51
3.1.1. Some definitions	51
3.2. Risk aversion	54
3.3. One-period expected utility analysis	56
3.3.1. The risk premium	58

vi	<i>Contents</i>	
	3.3.2. Measures of risk aversion	58
	3.3.3. Risk aversion in a portfolio choice problem	61
	3.4. Measures of increasing risk	63
	3.5. Conclusions	67
	3.6. Exercises	67
4	CAPM and APT	72
	4.1. The capital asset-pricing model	72
	4.1.1. The discount factor	72
	4.1.2. Expected utility maximization	74
	4.1.3. Alternative derivations	77
	4.2. Arbitrage pricing theory	80
	4.3. Conclusions	83
	4.4. Exercises	83
5	Consumption and saving	86
	5.1. A deterministic economy	86
	5.1.1. Properties of the saving function	88
	5.1.2. Optimal consumption over time	90
	5.2. Portfolio choice under uncertainty	94
	5.3. A more general problem	95
	5.3.1. Precautionary saving	100
	5.4. Conclusions	103
	5.5. Exercises	104
II	RECURSIVE MODELS	107
6	Dynamic programming	109
	6.1. A deterministic growth problem	109
	6.1.1. Guess-and-verify	111
	6.1.2. Finite horizon economies	113
	6.2. Mathematical preliminaries	115
	6.2.1. Markov processes	116
	6.2.2. Vector space methods	118
	6.2.3. Contraction mapping theorem	122
	6.3. A consumption-saving problem under uncertainty	126
	6.4. Exercises	129
7	Intertemporal risk sharing	133
	7.1. Multi-period contingent claims	133
	7.1.1. Aggregate uncertainty	134
	7.1.2. Central planning problem	139
	7.1.3. Sequential trading	140
	7.1.4. Implications for pricing assets	145
	7.2. Idiosyncratic endowment risk	146
	7.2.1. Notation	147
	7.2.2. The economy	148
	7.2.3. Complete contingent claims	149
	7.2.4. Dynamic programming	151

<i>Contents</i>		vii
7.3.	Risk sharing with idiosyncratic and aggregate risk	153
7.3.1.	First-best solution	154
7.4.	Conclusions	159
7.5.	Exercises	160
<b>8</b>	<b>Consumption and asset pricing</b>	<b>162</b>
8.1.	The consumption-based CAPM	162
8.1.1.	Recursive competitive equilibrium	164
8.1.2.	Asset-pricing functions	167
8.1.3.	Risk premia	171
8.1.4.	Volatility bounds for intertemporal MRSs	175
8.1.5.	The “equity premium puzzle”	178
8.2.	Pricing alternative assets	180
8.2.1.	Discount bonds and the yield curve	180
8.2.2.	Pricing derivative instruments	186
8.2.3.	The Black-Scholes options pricing formula	188
8.3.	A growing economy	191
8.3.1.	Cointegration in asset-pricing relations	195
8.4.	Conclusions	198
8.5.	Exercises	199
<b>9</b>	<b>Non-separable preferences</b>	<b>202</b>
9.1.	Non-time-additive preferences	202
9.1.1.	Habit persistence and consumption durability	203
9.1.2.	A more general specification	204
9.1.3.	A recursive framework	206
9.1.4.	Pricing durable consumption goods	209
9.1.5.	Asset-pricing relations	210
9.1.6.	Log-linear asset-pricing formulas	213
9.2.	Non-expected utility	215
9.2.1.	Recursive preferences under certainty	215
9.2.2.	The role of temporal lotteries	217
9.2.3.	Properties of non-expected utility preferences	220
9.2.4.	Optimal consumption and portfolio choices	223
9.3.	Tests of asset-pricing relations	228
9.4.	A model with an external habit	231
9.5.	Conclusions	235
9.6.	Exercises	235
<b>10</b>	<b>Economies with production</b>	<b>239</b>
10.1.	Recursive competitive equilibrium with production	240
10.1.1.	Households own the capital stock	241
10.1.2.	Households lease capital to firms	246
10.2.	Extensions	248
10.2.1.	Economies with distortions	248
10.2.2.	The role of expectations	252
10.3.	Solving models with production	255
10.3.1.	A parametric model	256
10.3.2.	The stationary distribution	260
10.4.	Financial structure of a firm	262

10.4.1.	The irrelevance of debt versus equity financing	266
10.4.2.	The equity price and the equity premium	267
10.4.3.	Empirical implications	269
10.4.4.	Taxes and the debt-equity ratio	273
10.5.	Conclusions	277
	Appendix: The invariant distribution	278
10.6.	Exercises	282
<b>11</b>	<b>Investment</b>	<b>285</b>
11.1.	The neoclassical model of investment	286
11.2.	The $Q$ theory adjustment-cost model of investment	288
11.2.1.	The $Q$ theory of investment	288
11.2.2.	Adjustment costs	288
11.2.3.	The social planner's problem	289
11.2.4.	The market economy	291
11.2.5.	Asset-pricing relations	295
11.3.	Irreversible investment	296
11.3.1.	A model with partial irreversibility and expandability	297
11.3.2.	A model of irreversible investment	305
11.4.	An asset-pricing model with irreversible investment	307
11.4.1.	The model	307
11.4.2.	The social planner's problem	308
11.4.3.	The competitive equilibrium	314
11.4.4.	The value of the firm and $Q$	319
11.4.5.	The relation among stock returns, investment, and $Q$	321
11.5.	Conclusions	323
11.6.	Exercises	323
<b>12</b>	<b>Business cycles</b>	<b>326</b>
12.1.	Business cycle facts	327
12.2.	Shocks and propagation mechanisms	331
12.3.	Real business cycle models	333
12.3.1.	An RBC model	335
12.3.2.	A model with indivisible labor supply	338
12.3.3.	Other "puzzles"	342
12.4.	Solving business cycle models	346
12.4.1.	Quadratic approximation	346
12.5.	Business cycle empirics	352
12.5.1.	Dynamic factor analysis	353
12.5.2.	ML and GMM estimation approaches	357
12.5.3.	A New Keynesian critique	360
12.6.	Conclusions	366
12.7.	Exercises	367
<b>III</b>	<b>MONETARY AND INTERNATIONAL MODELS</b>	<b>371</b>
<b>13</b>	<b>Models with cash-in-advance constraints</b>	<b>373</b>
13.1.	"Evil is the root of all money"	374
13.2.	The basic cash-in-advance model	376

<i>Contents</i>		ix
13.2.1.	Solution for velocity	383
13.2.2.	Empirical results	385
13.2.3.	Inflation risk and the inflation premium	386
13.2.4.	Velocity shock	389
13.3.	Inflation and interest rates	390
13.4.	Transactions services model	394
13.5.	Growing economies	399
13.6.	Money and real activity	401
13.6.1.	Consumption-leisure choices	402
13.6.2.	Business cycle implications	410
13.7.	Conclusions	416
13.8.	Exercises	417
14	International asset markets	422
14.1.	A two-country model	423
14.2.	International monetary model	430
14.2.1.	The terms of trade and the exchange rate	435
14.2.2.	Pricing alternative assets	439
14.3.	Variants of the basic model	444
14.3.1.	Non-traded goods	444
14.3.2.	Exchange rates and international capital flows	448
14.4.	Conclusions	456
14.5.	Exercises	456
IV	MODELS WITH MARKET INCOMPLETENESS	459
15	Asset pricing with frictions	461
15.1.	The role of idiosyncratic risk for asset pricing	462
15.2.	Transactions costs	467
15.2.1.	A model with bid-ask spreads	469
15.3.	Volatility bounds with frictions	472
15.4.	Conclusions	475
15.5.	Exercises	476
16	Borrowing constraints	478
16.1.	Idiosyncratic risk and borrowing constraints	479
16.1.1.	The basic model	480
16.1.2.	Restrictions on markets	480
16.1.3.	Pure insurance economy	481
16.1.4.	Pure credit model	484
16.1.5.	Asset span	491
16.2.	Townsend turnpike model	492
16.2.1.	Description of the model	493
16.2.2.	Borrowing-constrained households	496
16.2.3.	Borrowing constraints as netting schemes	498
16.2.4.	Liquidity-constrained households	500
16.2.5.	Debt-constrained economies	501
16.3.	Conclusions	502
16.4.	Exercises	502

x	<i>Contents</i>	
17	Overlapping generations models	504
17.1.	The environment	505
17.1.1.	Primitives	505
17.1.2.	Autarky in the absence of an outside asset	506
17.2.	The stochastic overlapping generations model	508
17.2.1.	Central planning problem	510
17.2.2.	Equal-treatment Pareto-optimal solution	514
17.3.	Competitive equilibrium	515
17.3.1.	Deterministic economy	515
17.3.2.	Fiat money	517
17.3.3.	The stochastic economy	518
17.4.	Equity pricing in a growing economy	526
17.4.1.	Risk premia	529
17.5.	Capital accumulation and social security	533
17.5.1.	Social security	540
17.6.	Conclusions	542
17.7.	Exercises	542
V	SUPPLEMENTARY MATERIAL	547
A	Mathematical appendix	549
A.1	Stochastic processes	549
A.2	Some useful theorems	553
	<i>Bibliography</i>	558
	<i>Index</i>	581

## *List of figures*

1.1	The consumer's optimum in an economy with two states	<i>page</i> 8
2.1	Three-period binomial tree	41
3.1	Expected utility indifference curves	53
3.2	Attitudes towards risk	56
3.3	First-order stochastic dominance	64
3.4	Mean-preserving spread	66
4.1	Security market line	77
5.1	Consumer's first-order conditions	87
5.2	Alternating deterministic endowment	93
6.1	Plots of $\varphi_n(t)$ for $n = 2, 3$	121
6.2	A fixed point on the real line	123
8.1	Mean-variance frontier for MRSs	176
9.1	Early versus late resolution of uncertainty	218
10.1	Configurations of capital stocks	242
10.2	Optimal consumption and capital stocks	245
10.3	Stationary distribution for the capital stock	261
12.1	Amplitude and duration of a business cycle	328
12.2	Impulse responses to a shock in technology	353
13.1	Timing of trades in the Lucas model	379
13.2	Timing of trades in the consumption/leisure model	404
A.1	An upper hemi-continuous correspondence	555
A.2	A lower hemi-continuous correspondence	556

## *List of tables*

2.1	An economy with four dates and two states	<i>page</i> 45
10.1	Optimal savings levels as a function of $(k, z)$	259
10.2	Unconditional moments of consumption, capital stock and output	262
12.1	Cyclical properties of key variables	352

Cambridge University Press

978-0-521-87585-1 - Asset Pricing for Dynamic Economies

Sumru Altug and Pamela Labadie

Frontmatter

[More information](#)

## *Preface*

The starting point for any analysis in finance involves assigning a current price to a future stream of uncertain payoffs. This is the basic notion behind any asset-pricing model. Take, for example, the price of a share to a competitive firm. Since the share entitles the owner to claims for the future profits of the firm, a central problem is to assign a value to these future profits. Take another asset – a house. This provides housing services in all states of nature and at all dates. Consequently, the value of the house today must reflect the value of these future services. Other examples include the pricing of durable goods or investment projects based on their future expected marginal products. One approach to monetary economics also follows this basic principle – if money as an asset has value in equilibrium (in the absence of any legal restrictions), then this value must reflect the stream of services provided by this asset.

Our approach is to derive pricing relationships for different assets by specifying the economic environment at the outset. One of the earliest examples of this approach is Merton [342]. However, Merton does not relate the technological sources of uncertainty to the equilibrium prices of the risky assets. Alternatively, he assumes a given stochastic process for the returns of different types of assets and then prices them given assumptions about consumer preferences. Consequently, the supply side is not explicitly considered by Merton. The asset-pricing model of Lucas [317] is fully general equilibrium but it is an endowment economy, so that consumption and investment decisions are trivial. Brock [76] develops an asset-pricing model with both the demand and supply side fully specified and links it up to Ross's [369] arbitrage pricing model.

In this book, we will start from an explicit economic environment and deduce the implications for asset prices, and the form of the asset-pricing function from the equilibrium in these environments. To study the problem of asset pricing, we could also follow another approach: we could take a very general and abstract approach, viewing asset pricing as the valuation of a future stream of uncertain payoffs from the asset according to a general pricing function. Given a minimal set of assumptions about the set of payoffs, we could try to characterize the properties of this abstract pricing function. This is the approach taken by Ross [371],

Cambridge University Press

978-0-521-87585-1 - Asset Pricing for Dynamic Economies

Sumru Altug and Pamela Labadie

Frontmatter

[More information](#)

xiv

*Preface*

Harrison and Kreps [240], Chamberlain and Rothschild [100], amongst others. One general point to note about the relationship between the two approaches to asset pricing is that the former abstract approach acquires economic content when interpreted in terms of the equilibrium approach. In fact, the benchmark payoff in the pricing function used to price streams of uncertain payoffs turns out to be the intertemporal marginal rate of substitution function for consumption. Depending on the nature of heterogeneity among consumers, the existence of complete contingent claims markets, and the role of money for acquiring consumption goods, the form of this intertemporal marginal rate of substitution function changes.

The purpose of this book is to provide an integrated treatment of a variety of dynamic equilibrium frameworks and to examine their empirical implications. The book is organized in four main parts. In Part I, we present material that constitutes the basis for much thinking in dynamic macroeconomics and finance. We begin by describing a simple version of the Arrow-Debreu contingent claims model, which is one of the building blocks of asset pricing. We also present the basics of arbitrage and asset valuation, expected utility analysis, CAPM and APT, and consumption/savings decisions under uncertainty. In Part II, we present a more fully developed set of results for dynamic economies under uncertainty using a recursive approach. In this part, we describe a pure exchange, representative consumer economy as well as economies with production. This framework allows us to derive the form of the asset-valuation function and to examine such issues as the effects of taxation on asset returns, the optimal financial structure of a firm, and the role of uncertainty in determining asset pricing and equilibrium allocations. Part III is devoted to cash-in-advance models, which allow us to examine the effects of inflation and exchange rate risk. Part IV presents material at a slightly more advanced level. In this part, we examine questions related to market incompleteness and the effects of frictions such as transactions costs. We consider the effects of borrowing constraints on equilibrium allocations and prices in a model with consumer heterogeneity and idiosyncratic risk. The stochastic overlapping generations model has been suggested as a convenient framework for analyzing issues related to “bubbles” in asset prices and the determinants of savings decisions with intergenerational heterogeneity among consumers. We examine a variety of issues using the stochastic overlapping generations framework. In many recent empirical applications of dynamic models, numerical solution methods have been combined with simulation or estimation procedures to assess quantitatively the importance of alternative model features. In this book, we also describe how numerical dynamic programming methods and other numerical methods can be used for solving and simulating a variety of dynamic economic problems.

There are many excellent texts in macroeconomics and finance that also cover material that is presented in this text. Cochrane’s [109] text is an

excellent reference that covers all of the standard issues in finance, updated using the modern approach to asset pricing. The texts by Darell Duffie [159, 161] also present the modern general equilibrium approach to finance but they are more technical in nature and help to serve as useful references, especially for advanced graduate students. The texts by Ingersoll [261], Huang and Litzenberger [256], Jarrow [263], Copeland and Weston [127], Hull [259], amongst others, present many of the standard issues of finance at differing levels of abstraction. They are recommended for students who (i) either lack a more traditional knowledge of finance and financial markets, or (ii) wish to obtain more detailed knowledge of some of the issues that we cover in this book. There is also some overlap between the topics we consider in this book and other texts dealing with dynamic general equilibrium modeling or macroeconomics such as Sargent [384] or Ljungqvist and Sargent [325].

A unifying feature of our discussion is that many of the dynamic equilibrium models that we consider can be formulated as dynamic programming problems and solved using a contraction mapping approach. Rather than introduce explicit measure-theoretic considerations for analyzing dynamic stochastic models, we describe uncertainty in terms of Markov uncertainty in a discrete-time setting. In Chapter 6, we provide a review of some results from functional analysis that we use in later chapters. For a review of basic results from functional analysis, we refer the reader to Kreyszig [290] and Naylor and Sell [351], and to Papoulis [356] for a review of probability theory and stochastic processes.

We have provided a set of detailed exercises at the end of each chapter and their solutions as a separate file. These exercises are intended to introduce some new topics at the same time that they allow the student to apply the methods described earlier. We developed this book from our teaching of finance, graduate financial economics and macroeconomics at the University of York, Duke University, the University of Minnesota, the University of Wisconsin, and Columbia University. It reflects our desire to provide a unified treatment of material that we could not find in one place. For teaching purposes, this text can be used as the basis for a graduate macroeconomics or financial economics course. We hope that this text will also prove useful to students and practitioners in the fields of macroeconomics, finance, applied general equilibrium modeling, and structural econometrics.

Paul Soderlind gave many useful comments that helped to improve the current version of this text. We have also received helpful comments from various colleagues for the first edition of the text, including Erdem Başı, Thomas Cooley, Scott Freeman, Christian Gilles, Jeremy Greenwood, Steve LeRoy, Bruce Smith, Allan Stockman, the participants of the International Workshop at the University of Rochester, and of a series of seminars at Bilkent University in Ankara, Turkey. We thank Zhenyu Wang

Cambridge University Press

978-0-521-87585-1 - Asset Pricing for Dynamic Economies

Sumru Altug and Pamela Labadie

Frontmatter

[More information](#)

---

xvi

*Preface*

for many helpful comments and the numerical calculations. Finally, we are grateful to Irem Demirci and Muharrem Yeşilırmak from Koç University for reading through the chapters and providing editorial comments, for writing exercises and solutions to various chapters, and assisting with other aspects of the production of this manuscript.