

## Elements of Crustal Geomechanics

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This key textbook considers the mechanics of geomaterials at a wide range of scales, both in time and space. It provides detailed introduction to the study of crustal geomechanics, focusing specifically on the seismogenic crust.

Following an introduction to the necessary fundamentals of structural geology and material science, the book demonstrates how the application of continuum mechanics principles can provide efficient solutions to geomechanics problems at various scales, taking into account the multiphase characteristics of the geomaterials as well as discontinuities such as fractures and faults. It shows how field and laboratory observations can be combined with basic mathematical theory to build solutions with known levels of uncertainty. Particular consideration is given to the use of microseismicity in constraining geomechanical models – especially those involving fluid–rock interactions. Case studies are provided that illustrate how *in situ* stress determinations at very different scales provide unique constraints on the rheological characteristics of the seismogenic crust, and practical results from numerical modeling are used to illustrate the applicability and limitations of current theories.

*Elements of Crustal Geomechanics* introduces students to the common basic principles used in solving geomechanics problems ranging from exploitation of geothermal energy and long-term storage of nuclear waste to mitigating the impacts of volcanic eruptions. Accessible explanations of the mathematical formulations, convenient summaries of the key equations, and exercises that encourage students to put their learning into practice make this a valuable reference for students and researchers in geomechanics, geophysics, structural geology and engineering.

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Cambridge University Press  
978-0-521-87578-3 - Elements of Crustal Geomechanics  
François Henri Cornet  
Frontmatter  
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CAMBRIDGE  
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9780521875783](http://www.cambridge.org/9780521875783)

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First published 2015

Printed in the United Kingdom by TJ International Ltd., Padstow, Cornwall

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*

Cornet, F. H.

Elements of crustal geomechanics / François Henri Cornet.

pages cm

ISBN 978-0-521-87578-3

1. Geodynamics. 2. Geology, Structural. I. Title.

QE501.C634 2015

551.1—dc23

2014037526

ISBN 978-0-52187578-3 Hardback

Additional resources for this publication at [www.cambridge.org/cornet](http://www.cambridge.org/cornet)

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L'observation scientifique est toujours une  
observation polémique; elle confirme ou  
infirme une thèse antérieure, un schéma  
préalable.

Gaston Bachelard, *Le nouvel esprit scientifique*

(Scientific observation is always polemical;  
it confirms or contradicts a previous thesis,  
an earlier sketch.)

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- worked solutions to the exercises
- field-based datasets
- MATLAB codes

## Preface

Geomechanics refers to the mechanics of geomaterials, i.e. to the deformation and flow processes that affect the materials which make up the planet earth.

Geomechanics issues are encountered in a great variety of situations with very different scales, both in space and time. Generally, in engineering applications, time scales vary from a few days to a few tens of years and the volumes under consideration vary from a few hundreds of cubic meters to a few cubic kilometers. In earth science, however, time scales range from seconds to tens of millions of years and volumes vary from a few cubic kilometers to that of the entire planet. Accordingly, each domain of application has developed its own appropriation of the geomechanics concept, given that engineers have to deal mostly with perturbations of an existing system, with particular concern for safety issues and production or construction efficiency, while earth scientists are trying to understand natural phenomena such as fault motion, mountain building and sedimentary basin evolution.

For the last 30 years engineers have been confronted with much longer time scales and much greater volumes. For example the development of a repository for nuclear waste must be proved to be safe for up to a million years. The exploitation of geothermal energy or the filling of dams must not reactivate large faults and so trigger destructive earthquakes. Similarly, earth scientists must come up with precise seismic risk analysis, which requires an accurate description of the expected ground motion at specific locations. They must analyze, in real time, deformation fields on volcanoes in order to mitigate the hazards associated with eruption.

Today, geengineers and geoscientists dealing with the mechanics of earth materials need to speak the same language. The objective of this text book is to introduce the basic principles of mechanics that earth scientists and mining, petroleum, civil and environmental engineers need to apply for solving problems in geomechanics. The only materials which are considered here are crustal geomaterials. The only paradigm considered for describing the deformation and flow processes of these geomaterials is that of continuum mechanics, but the limits of this paradigm are pointed out occasionally.

The aim of this book is to introduce the material for a two-semester class on geomechanics for upper undergraduate and first-year graduate students in earth sciences. It is based on notes prepared for my classes and inspired by notes from P. R. Fosdick's continuum mechanics classes at the University of Minnesota.

In the first part of the book (chapters 1 to 7) the basic concepts of solid and fluid mechanics necessary for understanding the mechanical behavior of geomaterials are introduced. The second part of the book (chapters 8 to 12) discusses various specificities of geomechanics that result from the complexity of geomaterials. Special attention is given

to dynamic phenomena (such as microseismicity) as well as to solid–fluid interactions. In the last part of the book (chapters 13 and 14) various *in situ* stress determination methods are introduced and practical examples at various scales illustrate how a sound evaluation of the stress field helps a better understanding of the various mechanical processes at work in the seismogenic crust.

The first chapter introduces the concept of equivalent geomaterials and a description of their discontinuities (fractures and faults). The second chapter presents various unidirectional rheological models that help one to understand the basic concepts of elasticity, viscosity, plasticity and friction. The third and fourth chapters discuss the concepts of stress, strain and deformation. In the fifth chapter the behavior of linearly elastic solids is discussed and problems frequently encountered in geomechanics are solved. The sixth chapter introduces some basic elements of continuum mechanics with application to the laminar flow of incompressible materials. The seventh chapter presents basic principles of linear fracture mechanics. With chapter 8, our attention turns more specifically to geomaterials, and the results of laboratory investigations are presented. Chapter 9 addresses the application of continuum mechanics principles to geomechanics, and chapter 10 introduces specific characteristics of fractures and faults. In chapter 11 we describe the various types of wave observed in seismology and then we discuss more specifically seismic sources. Chapter 12 addresses various aspects of solid–fluid interactions, including linear poroelasticity, thermoelasticity and the nonlinear effects associated with failure processes (hydraulic fracturing and fluid induced shear fractures). Chapter 13, on *in situ* stress determination methods, gives practical applications of the various concepts that have been introduced throughout the book. In the final chapter these methods are illustrated through examples that concern the design of an underground hydroelectric power scheme ( $\text{km}^3$  scale), the design of a nuclear waste repository ( $100 \text{ km}^3$  scale) and the stress fields in the upper Rhine graben ( $1000 \text{ km}^3$  scale) and the west-central European lithosphere ( $10^6 \text{ km}^3$  scale).

I would like to thank very sincerely Susan Francis from Cambridge University Press, who suggested that I should take the time to write up my lecture notes. She did not anticipate that I would be so slow in doing so, however! I also thank her two assistants, Laura Clark and Zoe Pruce, for their help during the various preparatory phases, as well as Susan Parkinson for her thorough copyediting of the manuscript.

My sincere gratitude goes to Marco Calo, and to my son Jan, for their help in preparing most of the figures. The manuscript has also greatly benefitted from the help of my colleagues Patrick Baud, Daniel Billaux, Dominique Bruehl, Michel Cara, Mai Linh Doan, Emmanuel Detournay, Emmanuel Gaucher, Georges Jobert, Sophie Lambotte, Olivier Langline, Vincent Magnenet, Romain Prioul, Daniel Quesada and Jean Schmittbuhl for reading early versions of some chapters. They pointed out a multitude of typing errors and contributed significant improvements. But I bear the entire responsibility for all the errors that are still left in the present document.

Finally my sincere gratitude to my wife, Basia, who has helped me through all these years and kept my morale up especially during the last, never-ending, phase of this project.

F. H. Cornet