

Cambridge University Press

978-0-521-87558-5 - Biomechanics: Concepts and Computation

Cees Oomens, Marcel Brekelmans and Frank Baaijens

Frontmatter

[More information](#)

Biomechanics: Concepts and Computation

This quantitative approach integrates the classical concepts of mechanics and computational modelling techniques, in a logical progression through a wide range of fundamental biomechanics principles. Online MATLAB-based software, along with examples and problems using biomedical applications, will motivate undergraduate biomedical engineering students to practise and test their skills. The book covers topics such as kinematics, equilibrium, stresses and strains, and also focuses on large deformations and rotations and non-linear constitutive equations, including visco-elastic behaviour and the behaviour of long slender fibre-like structures. This is the first textbook that integrates both general and specific topics, theoretical background and biomedical engineering applications, as well as analytical and numerical approaches. This is the definitive textbook for students.

Cees Oomens is Associate Professor in Biomechanics and Continuum Mechanics at the Eindhoven University of Technology, the Netherlands. He has lectured many different courses ranging from basic courses in continuum mechanics at bachelor level, to courses on mechanical properties of materials and advanced courses in computational modelling at masters and postgraduate level. His current research focuses on damage and adaptation of soft biological tissues, with emphasis on skeletal muscle tissue and skin.

Marcel Brekelmans is Associate Professor in Continuum Mechanics at the Eindhoven University of Technology. Since 1998 he has also lectured in the Biomedical Engineering Faculty at the University; here his teaching addresses continuum mechanics, basic level and numerical analysis. He has published a considerable number of papers in well-known journals, and his research interests in continuum mechanics include the modelling of history-dependent material behaviour (plasticity, damage and fracture) in forming processes.

Frank Baaijens is Full Professor in Soft Tissue Biomechanics and Tissue Engineering at the Eindhoven University of Technology, where he has also been a part-time Professor in the Polymer Group of the Division of Computational and Experimental Mechanics since 1990. He is currently Scientific Director of the national research program on BioMedical Materials (BMM), and his research focuses on soft tissue biomechanics and tissue engineering.

Cambridge University Press
978-0-521-87558-5 - Biomechanics: Concepts and Computation
Cees Oomens, Marcel Brekelmans and Frank Baaijens
Frontmatter
[More information](#)

CAMBRIDGE TEXTS IN BIOMEDICAL ENGINEERING

Series Editors

W. Mark Saltzman *Yale University*
Shu Chien *University of California, San Diego*

Series Advisors

William Hendee *Medical College of Wisconsin*
Roger Kamm *Massachusetts Institute of Technology*
Robert Malkin *Duke University*
Alison Noble *Oxford University*
Bernhard Palsson *University of California, San Diego*
Nicholas Peppas *University of Texas at Austin*
Michael Sefton *University of Toronto*
George Truskey *Duke University*
Cheng Zhu *Georgia Institute of Technology*

Cambridge Texts in Biomedical Engineering provides a forum for high-quality accessible textbooks targeted at undergraduate and graduate courses in biomedical engineering. It will cover a broad range of biomedical engineering topics from introductory texts to advanced topics including, but not limited to, biomechanics, physiology, biomedical instrumentation, imaging, signals and systems, cell engineering, and bioinformatics. The series blends theory and practice, aimed primarily at biomedical engineering students, it also suits broader courses in engineering, the life sciences and medicine.

Cambridge University Press

978-0-521-87558-5 - Biomechanics: Concepts and Computation

Cees Oomens, Marcel Brekelmans and Frank Baaijens

Frontmatter

[More information](#)

Biomechanics

Concepts and Computation

Cees Oomens, Marcel Brekelmans, Frank Baaijens

Eindhoven University of Technology

Department of Biomedical Engineering

Tissue Biomechanics & Engineering



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-0-521-87558-5 - Biomechanics: Concepts and Computation
Cees Oomens, Marcel Brekelmans and Frank Baaijens
Frontmatter
[More information](#)

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi
Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521875585

© C. Oomens, M. Brekelmans and F. Baaijens 2009

This publication is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without
the written permission of Cambridge University Press.

First published 2009

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-87558-5 hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or
third-party internet websites referred to in this publication, and does not guarantee that any content on such
websites is, or will remain, accurate or appropriate.

Contents

<i>About the cover</i>	<i>page xi</i>
<i>Preface</i>	<i>xiii</i>
1 Vector calculus	1
1.1 Introduction	1
1.2 Definition of a vector	1
1.3 Vector operations	1
1.4 Decomposition of a vector with respect to a basis	5
Exercises	8
2 The concepts of force and moment	10
2.1 Introduction	10
2.2 Definition of a force vector	10
2.3 Newton’s Laws	12
2.4 Vector operations on the force vector	13
2.5 Force decomposition	14
2.6 Representation of a vector with respect to a vector basis	17
2.7 Column notation	21
2.8 Drawing convention	24
2.9 The concept of moment	25
2.10 Definition of the moment vector	26
2.11 The two-dimensional case	29
2.12 Drawing convention of moments in three dimensions	32
Exercises	33
3 Static equilibrium	37
3.1 Introduction	37
3.2 Static equilibrium conditions	37
3.3 Free body diagram	40
Exercises	47

vi	Contents
4	The mechanical behaviour of fibres 50
4.1	Introduction 50
4.2	Elastic fibres in one dimension 50
4.3	A simple one-dimensional model of a skeletal muscle 53
4.4	Elastic fibres in three dimensions 55
4.5	Small fibre stretches 61
	Exercises 66
5	Fibres: time-dependent behaviour 69
5.1	Introduction 69
5.2	Viscous behaviour 71
5.2.1	Small stretches: linearization 73
5.3	Linear visco-elastic behaviour 74
5.3.1	Continuous and discrete time models 74
5.3.2	Visco-elastic models based on springs and dashpots: Maxwell model 78
5.3.3	Visco-elastic models based on springs and dashpots: Kelvin–Voigt model 82
5.4	Harmonic excitation of visco-elastic materials 83
5.4.1	The Storage and the Loss Modulus 83
5.4.2	The Complex Modulus 85
5.4.3	The standard linear model 87
5.5	Appendix: Laplace and Fourier transforms 92
	Exercises 94
6	Analysis of a one-dimensional continuous elastic medium 99
6.1	Introduction 99
6.2	Equilibrium in a subsection of a slender structure 99
6.3	Stress and strain 101
6.4	Elastic stress–strain relation 104
6.5	Deformation of an inhomogeneous bar 104
	Exercises 111
7	Biological materials and continuum mechanics 114
7.1	Introduction 114
7.2	Orientation in space 115
7.3	Mass within the volume V 117
7.4	Scalar fields 120
7.5	Vector fields 122
7.6	Rigid body rotation 125

7.7	Some mathematical preliminaries on second-order tensors	127
	Exercises	130
8	Stress in three-dimensional continuous media	132
8.1	Stress vector	132
8.2	From stress to force	133
8.3	Equilibrium	134
8.4	Stress tensor	142
8.5	Principal stresses and principal stress directions	146
8.6	Mohr’s circles for the stress state	149
8.7	Hydrostatic pressure and deviatoric stress	150
8.8	Equivalent stress	150
	Exercises	152
9	Motion: the time as an extra dimension	156
9.1	Introduction	156
9.2	Geometrical description of the material configuration	156
9.3	Lagrangian and Eulerian description	158
9.4	The relation between the material and spatial time derivative	159
9.5	The displacement vector	161
9.6	The gradient operator	162
9.7	Extra displacement as a rigid body	164
9.8	Fluid flow	166
	Exercises	167
10	Deformation and rotation, deformation rate and spin	170
10.1	Introduction	170
10.2	A material line segment in the reference and current configuration	170
10.3	The stretch ratio and rotation	173
10.4	Strain measures and strain tensors and matrices	176
10.5	The volume change factor	180
10.6	Deformation rate and rotation velocity	180
	Exercises	183
11	Local balance of mass, momentum and energy	186
11.1	Introduction	186
11.2	The local balance of mass	186
11.3	The local balance of momentum	187

11.4	The local balance of mechanical power	189
11.5	Lagrangian and Eulerian description of the balance equations	190
	Exercises	192
12	Constitutive modelling of solids and fluids	194
12.1	Introduction	194
12.2	Elastic behaviour at small deformations and rotations	195
12.3	The stored internal energy	198
12.4	Elastic behaviour at large deformations and/or large rotations	200
12.5	Constitutive modelling of viscous fluids	203
12.6	Newtonian fluids	204
12.7	Non-Newtonian fluids	205
12.8	Diffusion and filtration	205
	Exercises	206
13	Solution strategies for solid and fluid mechanics problems	210
13.1	Introduction	210
13.2	Solution strategies for deforming solids	210
13.2.1	General formulation for solid mechanics problems	211
13.2.2	Geometrical linearity	212
13.2.3	Linear elasticity theory, dynamic	213
13.2.4	Linear elasticity theory, static	213
13.2.5	Linear plane stress theory, static	214
13.2.6	Boundary conditions	218
13.3	Solution strategies for viscous fluids	220
13.3.1	General equations for viscous flow	221
13.3.2	The equations for a Newtonian fluid	221
13.3.3	Stationary flow of an incompressible Newtonian fluid	222
13.3.4	Boundary conditions	223
13.3.5	Elementary analytical solutions	223
13.4	Diffusion and filtration	225
	Exercises	227
14	Solution of the one-dimensional diffusion equation by means of the Finite Element Method	232
14.1	Introduction	232
14.2	The diffusion equation	233
14.3	Method of weighted residuals and weak form of the model problem	235
14.4	Polynomial interpolation	237

ix	Contents	
	14.5	Galerkin approximation 239
	14.6	Solution of the discrete set of equations 246
	14.7	Isoparametric elements and numerical integration 246
	14.8	Basic structure of a finite element program 250
	14.9	Example 253
		Exercises 256
	15	Solution of the one-dimensional convection-diffusion equation by means of the Finite Element Method 264
	15.1	Introduction 264
	15.2	The convection-diffusion equation 264
	15.3	Temporal discretization 266
	15.4	Spatial discretization 269
		Exercises 273
	16	Solution of the three-dimensional convection-diffusion equation by means of the Finite Element Method 277
	16.1	Introduction 277
	16.2	Diffusion equation 278
	16.3	Divergence theorem and integration by parts 279
	16.4	Weak form 280
	16.5	Galerkin discretization 280
	16.6	Convection-diffusion equation 283
	16.7	Isoparametric elements and numerical integration 284
	16.8	Example 288
		Exercises 291
	17	Shape functions and numerical integration 295
	17.1	Introduction 295
	17.2	Isoparametric, bilinear quadrilateral element 297
	17.3	Linear triangular element 299
	17.4	Lagrangian and Serendipity elements 302
		17.4.1 Lagrangian elements 303
		17.4.2 Serendipity elements 304
	17.5	Numerical integration 305
		Exercises 309
	18	Infinitesimal strain elasticity problems 313
	18.1	Introduction 313
	18.2	Linear elasticity 313

Cambridge University Press
978-0-521-87558-5 - Biomechanics: Concepts and Computation
Cees Oomens, Marcel Brekelmans and Frank Baaijens
Frontmatter
[More information](#)

x	Contents	
18.3	Weak formulation	315
18.4	Galerkin discretization	316
18.5	Solution	322
18.6	Example	322
	Exercises	324
	<i>References</i>	329
	<i>Index</i>	331



About the cover

The cover contains images reflecting biomechanics research topics at the Eindhoven University of Technology. An important aspect of mechanics is experimental work to determine material properties and to validate models. The application field ranges from microscopic structures at the level of cells to larger organs like the heart. The core of biomechanics is constituted by models formulated in terms of partial differential equations and computer models to derive approximate solutions.

- *Main image:* Myogenic precursor cells have the ability to differentiate and fuse to form multinucleated myotubes. This differentiation process can be influenced by means of mechanical as well as biochemical stimuli. To monitor this process of early differentiation, immunohistochemical analyses are performed to provide information concerning morphology and localization of characteristic structural proteins of muscle cells. In the illustration, the sarcomeric proteins actin (red), and myosin (green) are shown. Nuclei are stained blue. Image courtesy of Mrs Marloes Langelaan.
- *Left top:* To study the effect of a mechanical load on the damage evolution of skeletal tissue an in-vitro model system using tissue engineered muscle was developed. The image shows this muscle construct in a set-up on a confocal microscope. In the device the construct can be mechanically deformed by means of an indenter. Fluorescent identification of both necrotic and apoptotic cells can be established using different staining techniques Image courtesy of Mrs Debby Gawlitta.
- *Left middle:* A three-dimensional finite element mesh of the human heart ventricles is shown. This mesh is used to solve the equations of motion for the beating heart. The model was used to study the effect of depolarization waves and mechanics in the paced heart. Image courtesy of Mr Roy Kerckhoffs.
- *Left bottom:* The equilibrium equations are derived from Newton's laws and describe (quasi-)static force equilibrium in a three-dimensional continuum. Chapter 9 of the present book.



Preface

In September 1997 an educational programme in Biomedical Engineering, unique in the Netherlands, started at the Eindhoven University of Technology, together with the University of Maastricht, as a logical step after almost two decades of research collaboration between both universities. This development culminated in the foundation of the Department of Biomedical Engineering in April 1999 and the creation of a graduate programme (MSc) in Biomedical Engineering in 2000 and Medical Engineering in 2002.

Already at the start of this educational programme, it was decided that a comprehensive course in biomechanics had to be part of the curriculum and that this course had to start right at the beginning of the Bachelor phase. A search for suitable material for this purpose showed that excellent biomechanics textbooks exist. But many of these books are very specialized to certain aspects of biomechanics. The more general textbooks are addressing mechanical or civil engineers or physicists who wish to specialize in biomechanics, so these books include chapters or sections on biology and physiology. Almost all books that were found are at Masters or post-graduate level, requiring basic to sophisticated knowledge of mechanics and mathematics. At a more fundamental level only books could be found that were written for mechanical and civil engineers.

We decided to write our own course material for the basic training in mechanics appropriate for our candidate biomedical engineers at Bachelor level, starting with the basic concepts of mechanics and ending with numerical solution procedures, based on the Finite Element Method. The course material assembled in the current book, comprises three courses for our biomedical engineers curriculum, distributed over the three years of their Bachelor studies. Chapters 1 to 6 mostly treat the basic concepts of forces, moments and equilibrium in a discrete context in the first year. Chapters 7 to 13 in the second year discuss the basis of continuum mechanics and Chapters 14 to 18 in the third year are focussed on solving the field equations of mechanics using the Finite Element Method.

What makes this book different from other basic mechanics or biomechanics treatises? Of course there is the usual attention, as in standard books, focussed on kinematics, equilibrium, stresses and strains. But several topics are discussed that are normally not found in one single textbook or only described briefly.

- Much attention is given to large deformations and rotations and non-linear constitutive equations (see Chapters 4, 9 and 10).
- A separate chapter is devoted to one-dimensional visco-elastic behaviour (Chapter 5).
- There is special attention to long slender fibre-like structures (Chapter 4).
- The similarities and differences in describing the behaviour of solids and fluids and aspects of diffusion and filtration are discussed (Chapters 12 to 16).
- Basic concepts of mechanics and numerical solution strategies for partial differential equations are integrated in one single textbook (Chapters 14 to 18).

Because of the usually rather complex geometries (and non-linear aspects) found in biomechanical problems hardly any relevant analytical solutions can be derived for the field equations and approximate solutions have to be constructed. It is the opinion of the authors that at Bachelor level at least the basis for these numerical techniques has to be addressed.

In Chapters 14 to 18 extensive use is made of a finite element code written in Matlab by one of the authors, which is especially developed as a tool for students. Applying this code requires that the user has a licence for the use of Matlab, which can be obtained via MathWorks (www.mathworks.com). The finite element code, which is a set of Matlab scripts, including manuals, is freely available and can be downloaded from the website: www.mate.tue.nl/biomechanicsbook.