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# 1 Vector calculus

## 1.1 Introduction

Before we can start with biomechanics it is necessary to introduce some basic mathematical concepts and to introduce the mathematical notation that will be used throughout the book. The present chapter is aimed at understanding some of the basics of vector calculus, which is necessary to elucidate the concepts of force and momentum that will be treated in the next chapter.

## 1.2 Definition of a vector

A **vector** is a physical entity having both a magnitude (length or size) and a direction. For a vector $\vec{a}$ it holds, see Fig. 1.1:

$$\vec{a} = a\vec{e}. \tag{1.1}$$

The **length** of the vector $\vec{a}$ is denoted by $|\vec{a}|$ and is equal to the length of the arrow. The length is equal to $a$, when $a$ is positive, and equal to $-a$ when $a$ is negative. The **direction** of $\vec{a}$ is given by the unit vector $\vec{e}$ combined with the sign of $a$. The unit vector $\vec{e}$ has length 1. The vector $\vec{0}$ has length zero.

## 1.3 Vector operations

**Multiplication** of a vector $\vec{a} = a\vec{e}$ by a positive scalar $\alpha$ yields a vector $\vec{b}$ having the same direction as $\vec{a}$ but a different magnitude $\alpha|\vec{a}|$:

$$\vec{b} = \alpha\vec{a} = \alpha a\vec{e}. \tag{1.2}$$

This makes sense: pulling twice as hard on a wire creates a force in the wire having the same orientation (the direction of the wire does not change), but with a magnitude that is twice as large.

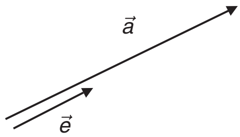


**Figure 1.1**

The vector $\vec{a} = a\vec{e}$ with $a > 0$.

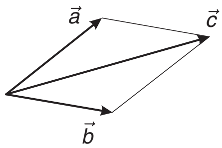


**Figure 1.2**

Graphical representation of the sum of two vectors: $\vec{c} = \vec{a} + \vec{b}$.

The **sum** of two vectors $\vec{a}$ and $\vec{b}$ is a new vector $\vec{c}$, equal to the diagonal of the parallelogram spanned by $\vec{a}$ and $\vec{b}$, see Fig. 1.2:

$$\vec{c} = \vec{a} + \vec{b}. \tag{1.3}$$

This may be interpreted as follows. Imagine two thin wires which are attached to a point P. The wires are being pulled at in two different directions according to the vectors $\vec{a}$ and $\vec{b}$. The length of each vector represents the magnitude of the pulling force. The net force vector exerted on the attachment point P is the vector sum of the two vectors $\vec{a}$ and $\vec{b}$. If the wires are aligned with each other and the pulling direction is the same, the resulting force direction is clearly coinciding with the direction of the two wires and the length of the resulting force vector is the sum of the two pulling forces. Alternatively, if the two wires are aligned but the pulling forces are in opposite directions and of equal magnitude, the resulting force exerted on point P is the zero vector $\vec{0}$.

The **inner product** or **dot product** of two vectors is a scalar quantity, defined as

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos(\phi)\,, \tag{1.4}$$

where $\phi$ is the smallest angle between $\vec{a}$ and $\vec{b}$, see Fig. 1.3. The inner product is **commutative**, i.e.

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}. \tag{1.5}$$

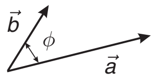


**Figure 1.3**

Definition of the angle $\phi$.

The inner product can be used to define the length of a vector, since the inner product of a vector with itself yields ($\phi = 0$):

$$\vec{a} \cdot \vec{a} = |\vec{a}||\vec{a}| \cos(0) = |\vec{a}|^2. \tag{1.6}$$

If two vectors are perpendicular to each other the inner product of these two vectors is equal to zero, since in that case $\phi = \frac{\pi}{2}$:

$$\vec{a} \cdot \vec{b} = 0, \text{ if } \phi = \frac{\pi}{2}. \tag{1.7}$$

The **cross product** or **vector product** of two vectors $\vec{a}$ and $\vec{b}$ yields a new vector $\vec{c}$ that is perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{a}$, $\vec{b}$ and $\vec{c}$ form a right-handed system. The vector $\vec{c}$ is denoted as

$$\vec{c} = \vec{a} \times \vec{b}. \tag{1.8}$$

The length of the vector $\vec{c}$ is given by

$$|\vec{c}| = |\vec{a}||\vec{b}| \sin(\phi), \tag{1.9}$$

where $\phi$ is the smallest angle between $\vec{a}$ and $\vec{b}$. The length of $\vec{c}$ equals the area of the parallelogram spanned by the vectors $\vec{a}$ and $\vec{b}$. The vector system $\vec{a}$, $\vec{b}$ and $\vec{c}$ forms a right-handed system, meaning that if a corkscrew is used rotating from $\vec{a}$ to $\vec{b}$ the corkscrew would move into the direction of $\vec{c}$.

The vector product of a vector $\vec{a}$ with itself yields the zero vector since in that case $\phi = 0$:

$$\vec{a} \times \vec{a} = \vec{0}. \tag{1.10}$$

The vector product is **not** commutative, since the vector product of $\vec{b}$ and $\vec{a}$ yields a vector that has the opposite direction of the vector product of $\vec{a}$ and $\vec{b}$:

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}. \tag{1.11}$$

The **triple product** of three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is a scalar, defined by

$$\vec{a} \times \vec{b} \cdot \vec{c} = (\vec{a} \times \vec{b}) \cdot \vec{c}. \tag{1.12}$$

So, first the vector product of $\vec{a}$ and $\vec{b}$ is determined and subsequently the inner product of the resulting vector with the third vector $\vec{c}$ is taken. If all three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-zero vectors, while the triple product is equal to zero then the vector $\vec{c}$ lies in the plane spanned by the vectors $\vec{a}$ and $\vec{b}$. This can be explained by the fact that the vector product of $\vec{a}$ and $\vec{b}$ yields a vector perpendicular to the plane spanned by $\vec{a}$ and $\vec{b}$. Reversely, this implies that if the triple product is non-zero then the three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are not in the same plane. In that case the absolute value of the triple product of the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ equals the volume of the parallelepiped spanned by $\vec{a}$, $\vec{b}$ and $\vec{c}$.

The **dyadic** or **tensor product** of two vectors $\vec{a}$ and $\vec{b}$ defines a linear transformation operator called a **dyad** $\vec{a}\vec{b}$. Application of a dyad $\vec{a}\vec{b}$ to a vector $\vec{p}$ yields a vector into the direction of $\vec{a}$, where $\vec{a}$ is multiplied by the inner product of $\vec{b}$ and $\vec{p}$:

$$\vec{a}\vec{b} \cdot \vec{p} = \vec{a}\,(\vec{b} \cdot \vec{p})\,. \tag{1.13}$$

So, application of a dyad to a vector transforms this vector into another vector. This transformation is linear, as can be seen from

$$\vec{a}\vec{b} \cdot (\alpha\vec{p} + \beta\vec{q}) = \vec{a}\vec{b} \cdot \alpha\vec{p} + \vec{a}\vec{b} \cdot \beta\vec{q} = \alpha\vec{a}\vec{b} \cdot \vec{p} + \beta\vec{a}\vec{b} \cdot \vec{q}. \tag{1.14}$$

The transpose of a dyad $(\vec{a}\vec{b})^{\mathrm{T}}$ is defined by

$$(\vec{a}\vec{b})^{\mathrm{T}} \cdot \vec{p} = \vec{b}\vec{a} \cdot \vec{p}, \tag{1.15}$$

or simply

$$(\vec{a}\vec{b})^{\mathrm{T}} = \vec{b}\vec{a}. \tag{1.16}$$

An operator $\boldsymbol{A}$ that transforms a vector $\vec{a}$ into another vector $\vec{b}$ according to

$$\vec{b} = \boldsymbol{A} \cdot \vec{a}, \tag{1.17}$$

is called a second-order tensor $\boldsymbol{A}$. This implies that the dyadic product of two vectors is a second-order tensor.
In the three-dimensional space a set of three vectors $\vec{c}_1$, $\vec{c}_2$ and $\vec{c}_3$ is called a **basis** if the triple product of the three vectors is non-zero, hence if all three vectors are non-zero vectors and if they do not lie in the same plane:

$$\vec{c}_1 \times \vec{c}_2 \cdot \vec{c}_3 \neq 0. \tag{1.18}$$

The three vectors $\vec{c}_1$, $\vec{c}_2$ and $\vec{c}_3$, composing the basis, are called basis vectors.

If the basis vectors are mutually perpendicular vectors the basis is called an **orthogonal** basis. If such basis vectors have unit length, then the basis is called **orthonormal**. A **Cartesian basis** is an orthonormal, right-handed basis with basis vectors independent of the location in the three-dimensional space. In the following we will indicate the Cartesian basis vectors with $\vec{e}_x$, $\vec{e}_y$ and $\vec{e}_z$.

## 1.4 Decomposition of a vector with respect to a basis

As stated above, a Cartesian vector basis is an orthonormal basis. Any vector can be decomposed into the sum of, at most, three vectors parallel to the three basis vectors $\vec{e}_x$, $\vec{e}_y$ and $\vec{e}_z$:

$$\vec{a} = a_x\vec{e}_x + a_y\vec{e}_y + a_z\vec{e}_z. \tag{1.19}$$

The components $a_x$, $a_y$ and $a_z$ can be found by taking the inner product of the vector $\vec{a}$ with respect to each of the basis vectors:

$$\begin{aligned} a_x &= \vec{a} \cdot \vec{e}_x \\ a_y &= \vec{a} \cdot \vec{e}_y \\ a_z &= \vec{a} \cdot \vec{e}_z, \end{aligned} \tag{1.20}$$

where use is made of the fact that the basis vectors have unit length and are mutually orthogonal, for example:

$$\vec{a} \cdot \vec{e}_x = a_x\vec{e}_x \cdot \vec{e}_x + a_y\vec{e}_y \cdot \vec{e}_x + a_z\vec{e}_z \cdot \vec{e}_x = a_x. \tag{1.21}$$

The components, say $a_x$, $a_y$ and $a_z$, of a vector $\vec{a}$ with respect to the Cartesian vector basis, may be collected in a **column**, denoted by $\underset{\sim}{a}$:

$$\underset{\sim}{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}. \tag{1.22}$$

So, with respect to a Cartesian vector basis any vector $\vec{a}$ may be decomposed in components that can be collected in a column:

$$\vec{a} \longleftrightarrow \underset{\sim}{a}\,. \tag{1.23}$$

This 'transformation' is only possible and meaningful if the vector basis with which the components of the column $\underset{\sim}{a}$ are defined has been specified. The choice of a different vector basis leads to a different column representation $\underset{\sim}{a}$ of the vector $\vec{a}$, this is illustrated in Fig. 1.4. The vector $\vec{a}$ has two different column representations, $\underset{\sim}{a}$ and $\underset{\sim}{a}^*$, depending on which vector basis is used. If, in a two-dimensional context $\{\vec{e}_x, \vec{e}_y\}$ is used as a vector basis then

$$\vec{a} \longrightarrow \underset{\sim}{a} = \begin{bmatrix} a_x \\ a_y \end{bmatrix}, \tag{1.24}$$

while, if $\{\vec{e}_x^{\,*}, \vec{e}_y^{\,*}\}$ is used as vector basis:

$$\vec{a} \longrightarrow \underset{\sim}{a}^* = \begin{bmatrix} a_x^* \\ a_y^* \end{bmatrix}. \tag{1.25}$$

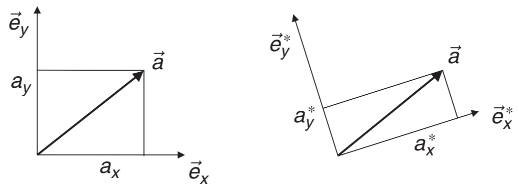


Figure 1.4

Vector $\vec{a}$ with respect to vector basis $\{\vec{e}_x, \vec{e}_y\}$ and $\{\vec{e}_x^*, \vec{e}_y^*\}$.

Consequently, with respect to a Cartesian vector basis, vector operations such as multiplication, addition, inner product and dyadic product may be rewritten as 'column' (actually matrix) operations.

Multiplication of a vector $\vec{a} = a_x\vec{e}_x + a_y\vec{e}_y + a_z\vec{e}_z$ with a scalar $\alpha$ yields a new vector, say $\vec{b}$:

$$\begin{aligned}\vec{b} = \alpha\vec{a} &= \alpha(a_x\vec{e}_x + a_y\vec{e}_y + a_z\vec{e}_z)\\ &= \alpha a_x\vec{e}_x + \alpha a_y\vec{e}_y + \alpha a_z\vec{e}_z.\end{aligned} \tag{1.26}$$

So

$$\vec{b} = \alpha\vec{a} \longrightarrow \underset{\sim}{b} = \alpha\underset{\sim}{a}. \tag{1.27}$$

The sum of two vectors $\vec{a}$ and $\vec{b}$ leads to

$$\vec{c} = \vec{a} + \vec{b} \longrightarrow \underset{\sim}{c} = \underset{\sim}{a} + \underset{\sim}{b}\,. \tag{1.28}$$

Using the fact that the Cartesian basis vectors have unit length and are mutually orthogonal, the inner product of two vectors $\vec{a}$ and $\vec{b}$ yields a scalar $c$ according to

$$\begin{aligned}c = \vec{a}\cdot\vec{b} &= (a_x\vec{e}_x + a_y\vec{e}_y + a_z\vec{e}_z)\cdot(b_x\vec{e}_x + b_y\vec{e}_y + b_z\vec{e}_z)\\ &= a_xb_x + a_yb_y + a_zb_z.\end{aligned} \tag{1.29}$$

In column notation this result is obtained via

$$c = \underset{\sim}{a}^{\mathrm{T}}\underset{\sim}{b}\,, \tag{1.30}$$

where $\underset{\sim}{a}^{\mathrm{T}}$ denotes the **transpose** of the column $\underset{\sim}{a}$ , defined as

$$\underset{\sim}{a}^{\mathrm{T}} = \left[a_x\ a_y\ a_z\right], \tag{1.31}$$

such that:

$$\underset{\sim}{a}^{\mathrm{T}}\underset{\sim}{b} = \left[a_x\ a_y\ a_z\right]\begin{bmatrix} b_x\\ b_y\\ b_z\end{bmatrix} = a_xb_x + a_yb_y + a_zb_z. \tag{1.32}$$

Using the properties of the basis vectors of the Cartesian vector basis:

$$\begin{aligned}
\vec{e}_x \times \vec{e}_x &= \vec{0} \\
\vec{e}_x \times \vec{e}_y &= \vec{e}_z \\
\vec{e}_x \times \vec{e}_z &= -\vec{e}_y \\
\vec{e}_y \times \vec{e}_x &= -\vec{e}_z \\
\vec{e}_y \times \vec{e}_y &= \vec{0} \\
\vec{e}_y \times \vec{e}_z &= \vec{e}_x \\
\vec{e}_z \times \vec{e}_x &= \vec{e}_y \\
\vec{e}_z \times \vec{e}_y &= -\vec{e}_x \\
\vec{e}_z \times \vec{e}_z &= \vec{0},
\end{aligned} \tag{1.33}$$

the vector product of a vector $\vec{a}$ and a vector $\vec{b}$ is directly computed by means of

$$\begin{aligned}
\vec{a} \times \vec{b} &= (a_x\vec{e}_x + a_y\vec{e}_y + a_z\vec{e}_z) \times (b_x\vec{e}_x + b_y\vec{e}_y + b_z\vec{e}_z) \\
&= (a_y\, b_z - a_z\, b_y)\vec{e}_x + (a_z\, b_x - a_x\, b_z)\vec{e}_y + (a_x\, b_y - a_y\, b_x)\vec{e}_z.
\end{aligned} \tag{1.34}$$

If by definition $\vec{c} = \vec{a} \times \vec{b}$, then the associated column $\underset{\sim}{c}$ can be written as:

$$\underset{\sim}{c} = \begin{bmatrix} a_y\, b_z - a_z\, b_y \\ a_z\, b_x - a_x\, b_z \\ a_x\, b_y - a_y\, b_x \end{bmatrix}. \tag{1.35}$$

The dyadic product $\vec{a}\vec{b}$ transforms another vector $\vec{c}$ into a vector $\vec{d}$, according to the definition

$$\vec{d} = \vec{a}\vec{b} \cdot \vec{c} = \boldsymbol{A} \cdot \vec{c}\,, \tag{1.36}$$

with $\boldsymbol{A}$ the second-order tensor equal to the dyadic product $\vec{a}\vec{b}$. In column notation this is equivalent to

$$\underset{\sim}{d} = \underset{\sim}{a}(\,\underset{\sim}{b}^T\underset{\sim}{c}\,) = (\,\underset{\sim}{a}\,\underset{\sim}{b}^T)\,\underset{\sim}{c}\,, \tag{1.37}$$

with $\underset{\sim}{a}\,\underset{\sim}{b}^T$ a $(3 \times 3)$ matrix given by

$$\underline{A} = \underset{\sim}{a}\,\underset{\sim}{b}^T = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \begin{bmatrix} b_x & b_y & b_z \end{bmatrix} = \begin{bmatrix} a_xb_x & a_xb_y & a_xb_z \\ a_yb_x & a_yb_y & a_yb_z \\ a_zb_x & a_zb_y & a_zb_z \end{bmatrix}, \tag{1.38}$$

or

$$\underset{\sim}{d} = \underline{A}\,\underset{\sim}{c}. \tag{1.39}$$

In this case $\underline{A}$ is called the matrix representation of the second-order tensor $\boldsymbol{A}$, as the comparison of Eqs. (1.36) and (1.39) reveals.

## Exercises

1.1 The basis $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ has a right-handed orientation and is orthonormal.
 (a) Determine $|\vec{e}_i|$ for $i = x, y, z$.
 (b) Determine $\vec{e}_i \cdot \vec{e}_j$ for $i, j = x, y, z$.
 (c) Determine $\vec{e}_x \cdot \vec{e}_y \times \vec{e}_z$.
 (d) Why is: $\vec{e}_x \times \vec{e}_y = \vec{e}_z$ ?

1.2 Let $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ be an orthonormal vector basis. The force vectors $\vec{F}_x = 3\vec{e}_x + 2\vec{e}_y + \vec{e}_z$ and $\vec{F}_y = -4\vec{e}_x + \vec{e}_y + 4\vec{e}_z$ act on point P. Calculate a vector $\vec{F}_z$ acting on P in such a way that the sum of all force vectors is the zero vector.

1.3 Let $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ be a right-handed and orthonormal vector basis. The following vectors are given: $\vec{a} = 4\vec{e}_z$, $\vec{b} = -3\vec{e}_y + 4\vec{e}_z$ and $\vec{c} = \vec{e}_x + 2\vec{e}_z$.
 (a) Write the vectors in column notation.
 (b) Determine $\vec{a} + \vec{b}$ and $3(\vec{a} + \vec{b} + \vec{c})$.
 (c) Determine $\vec{a} \cdot \vec{b}$, $\vec{b} \cdot \vec{a}$, $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$.
 (d) Determine $|\vec{a}|$, $|\vec{b}|$, $|\vec{a} \times \vec{b}|$ and $|\vec{b} \times \vec{a}|$.
 (e) Determine the smallest angle between $\vec{a}$ and $\vec{b}$.
 (f) Determine a unit normal vector on the plane defined by $\vec{a}$ and $\vec{b}$.
 (g) Determine $\vec{a} \times \vec{b} \cdot \vec{c}$ and $\vec{a} \times \vec{c} \cdot \vec{b}$.
 (h) Determine $\vec{a}\vec{b} \cdot \vec{c}$, $(\vec{a}\vec{b})^{\mathrm{T}} \cdot \vec{c}$ and $\vec{b}\vec{a} \cdot \vec{c}$.
 (i) Do the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ form a suitable vector basis? If the answer is yes, do they form an orthogonal basis? If the answer is yes, do they form an orthonormal basis?

1.4 Consider the basis $\{\vec{a}, \vec{b}, \vec{c}\}$ with $\vec{a}, \vec{b}$ and $\vec{c}$ defined as in the previous exercise. The following vectors are given: $\vec{d} = \vec{a} + 2\vec{b}$ and $\vec{e} = 2\vec{a} - 3\vec{c}$.
 (a) Determine $\vec{d} + \vec{e}$.
 (b) Determine $\vec{d} \cdot \vec{e}$.

1.5 The basis $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ is right-handed and orthonormal. The vectors $\vec{a}_x, \vec{a}_y$ and $\vec{a}_z$ are given by: $\vec{a}_x = 4\vec{e}_x + 3\vec{e}_y$ ; $\vec{a}_y = 3\vec{e}_x - 4\vec{e}_y$ and $\vec{a}_z = \vec{a}_x \times \vec{a}_y$.
 (a) Determine $\vec{a}_z$ expressed in $\vec{e}_x, \vec{e}_y$ and $\vec{e}_z$.
 (b) Determine $|\vec{a}_i|$ for $i = x, y, z$.
 (c) Determine the volume of the parallelepiped defined by $\vec{a}_x, \vec{a}_y$ and $\vec{a}_z$.
 (d) Determine the angle between the lines of action of $\vec{a}_x$ and $\vec{a}_y$.
 (e) Determine the vector $\vec{\alpha}_x$ from $\vec{a}_i = |\vec{a}_i|\vec{\alpha}_i$ for $i = x, y, z$. Is $\{\vec{\alpha}_x, \vec{\alpha}_y, \vec{\alpha}_z\}$ a right-handed, orthonormal vector basis?

(f) Consider the vector $\vec{b} = 2\vec{e}_x + 3\vec{e}_y + \vec{e}_z$. Determine the column representation of $\vec{b}$ according to the bases $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$, $\{\vec{a}_x, \vec{a}_y, \vec{a}_z\}$ and $\{\vec{\alpha}_x, \vec{\alpha}_y, \vec{\alpha}_z\}$.

(g) Show that: $\vec{a}_x \times \vec{a}_y \cdot \vec{b} = \vec{a}_x \cdot \vec{a}_y \times \vec{b} = \vec{a}_y \cdot \vec{b} \times \vec{a}_x$.

1.6 Assume $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ is an orthonormal vector basis. The following vectors are defined:

$$\begin{aligned}\vec{a} &= 4\vec{e}_x + 3\vec{e}_y - \vec{e}_z \\ \vec{b} &= 6\vec{e}_y - \vec{e}_z \\ \vec{c} &= 8\vec{e}_x - \vec{e}_z \;.\end{aligned}$$

Are $\vec{a}$, $\vec{b}$ and $\vec{c}$ linearly independent? If not, what is the relationship between the vectors?

1.7 The vector bases $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ and $\{\vec{\epsilon}_x, \vec{\epsilon}_y, \vec{\epsilon}_z\}$ are orthonormal and do not coincide:

(a) What is the effect of $\vec{e}_x\vec{\epsilon}_x + \vec{e}_y\vec{\epsilon}_y + \vec{e}_z\vec{\epsilon}_z$ acting on a vector $\vec{a}$?

(b) What is the effect of $\vec{\epsilon}_x\vec{e}_x + \vec{\epsilon}_y\vec{e}_y + \vec{\epsilon}_z\vec{e}_z$ acting on a vector $\vec{a}$?

1.8 The vector basis $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ is orthonormal. What is the effect of the following dyadic products if they are applied to a vector $\vec{a}$?

(a) $\vec{e}_x\vec{e}_x$.

(b) $\vec{e}_x\vec{e}_x + \vec{e}_y\vec{e}_y$.

(c) $\vec{e}_x\vec{e}_x + \vec{e}_y\vec{e}_y + \vec{e}_z\vec{e}_z$.

(d) $\vec{e}_x\vec{e}_y - \vec{e}_y\vec{e}_x + \vec{e}_z\vec{e}_z$.

(e) $\vec{e}_x\vec{e}_x - \vec{e}_y\vec{e}_y + \vec{e}_z\vec{e}_z$.

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# 2 The concepts of force and moment

## 2.1 Introduction

We experience the effects of force in everyday life and have an intuitive notion of force. For example, we exert a force on our body when we lift or push an object while we continuously (fortunately) feel the effect of gravitational forces, for instance while sitting, walking, etc. All parts of the human body in one way or the other are loaded by forces. Our bones provide rigidity to the body and can sustain high loads. The skin is resistant to force, simply pull on the skin to witness this. The cardiovascular system is continuously loaded dynamically due to the pulsating blood pressure. The bladder is loaded and stretched when it fills up. The intervertebral discs serve as flexible force transmitting media that give the spine its flexibility. Beside force we are using levers all the time in our daily life to increase the 'force' that we want to apply to some object, for example by opening doors with the latch, opening a bottle with a bottle-opener. We feel the effect of a lever arm when holding a weight close to our body instead of using a stretched arm. These experiences are the result of the moment that can be exerted by a force. Understanding the impact of force and moment on the human body requires us to formalize the intuitive notion of force and moment. That is the objective of this chapter.

## 2.2 Definition of a force vector

Imagine pulling on a thin wire that is attached to a wall. The pulling force exerted on the point of application is a vector with a physical meaning, it has

- a **length**: the magnitude of the pulling force
- an **orientation** in space: the direction of the wire
- a **line-of-action**, which is the line through the force vector.

The graphical representation of a force vector, denoted by $\vec{F}$, is given in Fig. 2.1. The 'shaft' of the arrow indicates the orientation in space of the force vector. The point of application of the force vector is denoted by the point P.

 