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CHAPTER ONE

Introduction to Continuous Time Modelling

1.1 Introduction

The world's economies and many of its financial markets operate 24 hours a day and the formulation of realistic models to represent the economy and these markets is one of the major challenges facing economists and finance specialists today. Given that the economy is made up of millions of agents making decisions continuously the use of a continuous time model that allows for these interactions to be incorporated will be a more realistic description of the underlying phenomena we are trying to model. In response to this challenge, the econometric estimation of continuous time models in economics and finance has been a major ongoing development over the last 30 years. The major problem faced in such econometric modelling is that we do not have a

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continuous time record of observations and most economic data are available only at discrete time periods (annually, monthly, quarterly). In finance, data are more frequently available at a higher frequency, for example, daily, hourly and more recently tick-by-tick transactions data. The problem facing an econometric investigator, therefore, is how best to utilize such data in the estimation of an underlying continuous time system so that the fitted model can then be used in economic forecasting, policy analysis and derivative pricing.

In financial markets continuous time models have found widespread applications in financial asset and option pricing since the seminal work of Black and Scholes [1973] and Merton [1973a] and in the general area of continuous time term structure modelling and bond valuation over the last 20 years. These developments have opened up an explosion of research activity developing new continuous time models in finance and associated econometric estimation techniques for fitting continuous systems with the available discrete data.

The history of the earliest work on continuous time stochastic processes (for example, Bachelier [1900], Einstein [1906], Wiener [1923] and Ito [1946, 1951]) and continuous time econometric modelling (Bartlett [1946], Koopmans

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[1950], Phillips [1959], Durbin [1961]) has been well documented in survey papers by Bergstrom [1984a, 1988, 1990, Ch. 1, 1996]. Our aims in this chapter are to give a brief introduction to continuous time modelling and econometrics in areas that are important to economists and finance specialists working in academia, government and financial markets.

This chapter is organized as follows. Section 1.2 explains the advantages and problems of continuous time modelling. Section 1.3 introduces the general idea of a continuous time model and the estimation of its parameters with available discrete time data. Section 1.4 discusses the application of continuous time models to term structure models in finance and Section 1.5 introduces continuous time macroeconomic modelling. Section 1.6 discusses policy analysis and Section 1.7 introduces the idea of stochastic trends in econometrics. Lastly, Section 1.8 outlines the contents of the book.

1.2 Why Model in Continuous Time

In economic and financial modelling, researchers face a fundamental choice between modelling in continuous and discrete time. This choice raises the important question of why models should be formulated in continuous time rather than

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discrete time. There are a number of advantages of modelling in continuous time that we briefly summarize here (see Bergstrom [1996] for an extensive discussion). First, since the economy and financial markets are continuously operating and the underlying decision processes we are trying to model involves millions of decisions by economic agents within the recorded data observation interval, realistic models will depend on the continuous passage of time. A continuous system will therefore be sympathetic to the underlying interactions being modelled and captured in the data, whereas traditional discrete time models are inherently less flexible because they restrict the underlying decision-making lag structures to match the observation interval in the data exactly (daily, monthly, quarterly), so that at best an economy's equilibrium and disequilibrium characteristics are being captured at successive points of time. The potential for misspecification therefore seems greater in a discrete time system when the underlying phenomenon is continuous.

Secondly, economic and financial models typically comprise two types of variables. Stock variables observed at points in time (for example, the money stock, inventories, fixed capital) and flow variables (for example, consumption, exports, output and imports) observed as aggregations over

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the unit observation period. Continuous time econometric methods, as applied in this monograph, allow for the correct treatment of these different types of variables in the direct and exact estimation of the models. This solves the temporal aggregation bias that commonly occurs in using discrete time models with flow variables, where no distinction is given to stock and flow variables generally.

Thirdly, when economists in government and central banks develop econometric models for short- and long-term economic forecasting, the construction of a quarterly or monthly model is commonplace due to the availability of national accounts data of that frequency. This dependence of the formulation of the model on the observed data frequency is a major disadvantage of discrete time models. For example, the construction of a model on monthly data will be different from one based on daily data. In contrast, the specification of a continuous time model in economics and finance is independent of the available data frequency. Related to the previous point, the fourth advantage of continuous time models is in economic and financial forecasting. The use of continuous time models allows one to obtain continuous time paths of the variables that can be used to make forecasts at shorter intervals to the available data used in the estimation of the model. For example, the continuous

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time path forecasts of manufacturing production and gross domestic product in the economy would be useful to government Treasury departments and central banks in their policy setting. In the business sector, companies would find it useful to have the continuous time path of retail sales projections for setting future production levels for example. A discrete time model is less flexible where a quarterly macroeconomic model would generate forecasts of gross domestic product at quarterly intervals and if we wanted forecasts on a weekly or monthly interval would have to be interpolated.

A fifth advantage of continuous time modelling is concerned with the modelling of dynamic adjustment mechanisms in the economy where continuous systems allow for more realistic specifications of the partial adjustment processes compared to a discrete time model. Typically, continuous systems in formulating partial adjustment (error correction) mechanisms allow the dependent variable to adjust continuously in response to deviations from its partial equilibrium level, which may be set to continuously depend on other variables in the model according to some underlying economic theory. These partial adjustment equations usually take the form of first- or second-order differential equations but could, of course, be much more general. The

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main reason why economic variables adjust gradually, rather than instantaneously, to their partial equilibrium level is that there are adjustment costs, which depend on the rate of change and, possibly, the acceleration of the adjusting variable. The resulting mechanisms can be formally derived as part of the solution of a dynamic optimization problem, which takes account of the costs of adjustment, as is done in the present monograph.

The sixth advantage of continuous time models is their natural application in the development of finance theory and derivative pricing models since the pathbreaking work of Merton [1969, 1971, 1973a,b] and Black and Scholes [1973]. Sundaresan [2000] provides a major review of these developments, continuous time models in finance and their many applications. There are a number of other advantages of continuous time modelling and the reader is referred to Bergstrom [1990, Ch. 1, 1996] and Gandolfo [1981, 1993] for further discussion and motivation.

One of the disadvantages of continuous time modelling is that the estimation of these models has involved the development of complicated and sometimes specialized econometric methods that have in the past largely remained in the province of econometric theorists and finance specialists. But over the last 10 years there has been a substantial

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increase in the number of Ph.D. educated graduates in econometrics, finance, physics and mathematics who are capable of and interested in using more advanced econometric techniques in financial and macroeconomic applications. These graduates have taken up employment in government departments and the banking and financial sectors, as well as academia, and are busy working on quantitative research, risk management, asset price modelling, hedge fund research and trading floor activities, where advanced econometric methods are now being more commonly used. Secondly, the specification of a macroeconomic model in continuous time typically involves a system of structural equations derived from economic theory and detailed mathematical analysis of its long-run properties and steady state analysis. This work is very time consuming mathematically and re-specification and analysis is seldom easy to do. This compares to discrete time macroeconomic modelling in which a typical model is constructed of individually specified equations and least squares or instrumental variable estimation methods are usually employed. Re-specification and updating of estimation results as new quarterly data come available for a quarterly forecasting round can be individually done and easily achieved. On the other hand, the

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explosive development of continuous time modelling in economics, and more especially finance over the last 20 years, together with the enormous increases in computational capability has made the specification and estimation of continuous time econometric models a much more plausible practical enterprise.

1.3 Introduction to General Continuous Time Models

We present in this section a review of the general form of continuous time models and the basic ideas of econometric estimation using discrete time data. The general model considered by Bergstrom [1966a] is a first-order continuous time system specified as

$$dx(t) = \{A(\theta)x(t) + b(\theta)\}dt + \zeta(dt) \quad (t \geq 0), \tag{1.1}$$

where $x(t) = \{x_1(t), \ldots, x_n(t)\}'$ is a n-dimensional continuous time random process, $A(\theta)$ is an $n \times n$ matrix whose elements are functions of a vector $\theta = [\theta_1, \ldots, \theta_p]$ of unknown structural parameters $(p \leq n(n+1))$ and $b(\theta)$ is a vector that is a function of θ. The error term $\zeta(dt)$ is assumed to be a vector of white noise innovations (see Bergstrom

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[1984a] for a precise definition and interpretation of this system).

We assume the continuous time model generates equispaced discrete data observed as the sequence $\{x(0), x(1), \ldots\}$ and our objective is to estimate the parameters of the continuous time model. Two approaches that have been used in the past have become known as the Discrete Approximation Method and the Exact Discrete Model approach. The first approach was originally developed by Bergstrom [1966a] (see also Houthakker and Taylor [1966], Sargan [1974, 1976], Wymer [1972]). The discrete approximation is obtained by using a trapezoidal approximation that gives an approximate simultaneous equations system of the form

$$x(t) - x(t-1) = \frac{1}{2}A(\theta)\{x(t) + x(t-1)\} + b(\theta) + u_t \quad (1.2)$$
$$E(u_t) = 0, \quad E\left(u_t u_t'\right) = \Sigma,$$
$$E\left(u_s u_t'\right) = 0, \quad s \neq t, \quad s, t = 1, 2, \ldots.$$

In using this model it was shown in Bergstrom [1966a] that the estimates have a small asymptotic bias. The continuous time model can be extended to include an m-dimensional vector of exogenous variables $z(t) = \{z_1(t), \ldots, z_m(t)\}'$ with

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