In this book, which arose from an MSRI research workshop cosponsored by the Clay Mathematical Institute, leading experts give an overview of several areas of dynamical systems that have recently experienced substantial progress.

In symplectic geometry, a fast-growing field having its roots in classical mechanics, Cieliebak, Hofer, Latschev and Schlenk give a definitive survey of quantitative techniques and symplectic capacities, which have become a central research tool. Fisher’s survey on local rigidity of group actions is a broad and up-to-date account of a flourishing subject built on the fact that for actions of noncyclic groups, topological conjugacy commonly implies smooth conjugacy.

Other articles by Eigen, Feres, Kochergin, Krieger, Navarro, Pinto, Prasad, Rand and Robinson cover subjects in hyperbolic, parabolic and symbolic dynamics as well as ergodic theory. Among the specific areas of interest are random walks and billiards, diffeomorphisms and flows on surfaces, amenability and tilings.

The articles are complemented by a fifty-page commented problem list, compiled by the editor with the help of numerous specialists. Several sections of this list focus on problems beyond the areas covered in the surveys, and all are sure to inspire and guide further research.
Mathematical Sciences Research Institute Publications

1 Freed/Uhlenbeck: Instantons and Four-Manifolds, second edition
2 Chern (ed.): Seminar on Nonlinear Partial Differential Equations
3 Lepowsky/Mandelstam/Singer (eds.): Vertex Operators in Mathematics and Physics
4 Kac (ed.): Infinite Dimensional Groups with Applications
5 Blackadar: K-Theory for Operator Algebras, second edition
6 Moore (ed.): Group Representations, Ergodic Theory, Operator Algebras, and Mathematical Physics
7 Chorin/Majda (eds.): Wave Motion: Theory, Modelling, and Computation
8 Gersten (ed.): Essays in Group Theory
9 Moore/Schochet: Global Analysis on Foliated Spaces, second edition
10–11 Ni/Pedeltier/Serrin (eds.): Nonlinear Diffusion Equations and Their Equilibrium States
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21 Ratiu (ed.): The Geometry of Hamiltonian Systems
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53 Hasselblatt (ed.): Dynamics, Ergodic Theory, and Geometry

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and Geometry

Dedicated to Anatole Katok

Edited by

Boris Hasselblatt
Tufts University
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Foreword

This volume owes its existence to the Clay Mathematics Institute / Mathematical Sciences Research Institute Workshop on “Recent Progress in Dynamics”, held at the Mathematical Sciences Research Institute for a week in late September to early October 2004. This is not a proceedings volume, but most authors were participants of the workshop, and the two lead surveys reflect a good deal of what David Fisher and Helmut Hofer presented during their workshop talks.

The workshop represented a broad array of dynamical systems, not least in order to reflect the breadth of taste exhibited by Anatole Katok, whose sixtieth birthday was observed during the workshop. This was possible because we were able to invite a great number of participants from near and far, and the essential ingredient in making this possible was the generous financial support from the Clay Mathematics Institute and the Mathematical Sciences Research Institute (which is in turn supported by the National Science Foundation), as well as from the Pennsylvania State University and Tufts University. As the host of the workshop, the Mathematical Sciences Research Institute also provided administrative support for the organizers and participants. It is a pleasure to acknowledge this support.

Funding alone does not produce a successful workshop, and I want to thank my fellow workshop organizers Michael Brin, Gregory Margulis, Yakov Pesin, Peter Sarnak, Klaus Schmidt, Ralf Spatzier and Robert Zimmer. Foremost among these was Yakov Pesin, whose involvement was constant and most valuable. And a successful workshop does not by itself lead to written works of interest, so I wish to thank the authors of the articles in this volume for their contributions. Thanks also go to Silvio Levy for his smooth handling of the entire production process, and to Kathleen Hasselblatt for her support.

The aim of the workshop and this volume is to impact the development of dynamical systems, and to that end we paid some attention to making it possible for younger participants to attend the workshop, and the surveys in this volume may attract young mathematicians to those subject areas. The Mathematical Sciences Research Institute adds to the impact of the event by maintaining streaming video of the lectures given at the workshop, which enables
everyone to view these lectures (see http://www.msri.org/calendar/workshops/WorkshopInfo/267/show_workshop). Finally, a problem list in this volume is hoped to inspire research into subjects that posed challenges at the time of the workshop. I hope that the readers will find this volume interesting, useful and inspiring.

I am writing these lines on the sixty-second birthday of Anatole Katok and wish to dedicate this volume to him.

Boris Hasselblatt
Somerville (MA), August 2006