

EARTH DYNAMICS

Deformations and Oscillations of the Rotating Earth

The Earth is a dynamic system. It has a fluid, mobile atmosphere, a continually changing global distribution of ice, snow and water, a fluid core, a thermally convecting mantle and mobile tectonic plates. Internal dynamic processes, together with external gravitational forces of the Sun, Moon and planets, exert torques on the solid Earth or displace its mass, affecting the Earth's shape, rotation and gravitational field.

D. E. Smylie provides a rigorous overview of the dynamical behaviour of the solid Earth, explaining the theory and presenting methods for numerical implementation. Topics include advanced digital analysis, earthquake displacement fields, free core nutations observed by the very long baseline interferometric technique, translational modes of the solid inner core observed by the superconducting gravimeters and dynamics of the fluid outer core.

This book is fully supported by open source computer code, available online for students to explore and test the theory. Also online are a suite of graphics generated from the numerical analysis, which combine with 100 graphical worked examples in the book to make this an ideal tool for researchers and graduate students in the fields of geodesy, seismology and solid Earth geophysics.

DOUGLAS SMYLIE is a Professor of Geophysics at York University, Toronto. He has conducted research on earthquake displacement fields, the rotation of the Earth and the dynamics of the deep interior while lecturing in geophysics at the University of Toronto, the University of Western Ontario, the University of British Columbia and York University in Toronto. In 2002 he was awarded the John Tuzo Wilson Medal by the Canadian Geophysical Union for his achievements. Professor Smylie is a Fellow of the Royal Astronomical Society, a member of the American Geophysical Union and has served as Founder and President of the Canadian Geophysical Union.

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Dedication

This book is dedicated to my wife Susan, my children Diane, Janet, Hugh and Andrea and my grandchildren Grace, Gavin, Ben, Ella, Jay, Quinn, Andreas, Alan, Lance, Bob and Ashley.

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Preface and acknowledgments

The study of Earth's dynamics, from near surface earthquake displacement fields to the translational modes of the solid inner core, has long been a fascination for me. In the present work I have been influenced by *Numerical Recipes*, published by Cambridge University Press, to include computer code so often omitted in scientific publications. I have gone one step further to include, on the website www.cambridge.org/smylie, open source downloadable software through the Oracle Virtual Machine, allowing a full Fedora Linux operating system to be installed on users' machines along with the TRIUMF graphics system, giving full access to Fortran, LaTeX and TeX as well as codes from the book itself and possible updates.

Throughout the writing of this book I have been very ably assisted by Dr Gary Henderson in every aspect. While I take full responsibility for any remaining errors, his high skills in English, theory and Fortran have been very much appreciated.

I have been fortunate to have benefited from many teachers and professors in my studies. Reg Daniels kindled my interest in mathematics in high school, Fraser Grant introduced me to mathematical geophysics, while Tuzo Wilson hosted the geophysics laboratory at 49 St. George St. in the University of Toronto.

I have also been fortunate to have worked with many colleagues and students. My doctoral thesis supervisor, Michael Rochester, continued in research with me for many years. Dr Xianhua Jiang through his variational calculations discovered the prograde free core nutation and his thesis won the Canadian Association of Graduate Schools Doctoral Dissertation Prize for all fields in Canada in 1994 and the 1995 Annual Dissertation Award of the Northeastern U.S. Association of Graduate Schools. While he was unable to detect the prograde free core nutation definitively in the spectra of the VLBI nutation record at the time, this task was left to Dr Andrew Palmer. Dr Palmer, using singular value decomposition, found both the prograde and retrograde nutations in the unequally spaced VLBI observations and from the ring down of the two modes was able to measure the viscosity at the

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top of the fluid outer core. Another brilliant doctoral student, Dr Hong Ma, won the Governor General's Gold Medal as the best graduating student at York University in all fields in 1996. Other brilliant doctoral students I should mention are Dr Ian Johnson and Dr James B. Merriam.

Finally, I thank my wife Susan and all my family for their support in all my endeavours.

Doug Smylie
June 25, 2012

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