QUANTUM FIELD THEORY OF NON-EQUILIBRIUM STATES

This text introduces the real-time approach to non-equilibrium statistical mechanics and the quantum field theory of non-equilibrium states in general. After a lucid introduction to quantum field theory and Green's functions, Schwinger's closed time path technique is developed, followed eventually by the real-time formulation and its Feynman diagram technique. The formalism is employed to derive quantum kinetic equations by using the quasi-classical Green's function technique, and is applied to study renormalization effects, non-equilibrium superconductivity, and quantum effects in disordered conductors.

The book offers two ways of learning how to study non-equilibrium states of many-body systems: the mathematical, canonical way, and an intuitive way using Feynman diagrams. The latter provides an easy introduction to the powerful functional methods of field theory. The usefulness of Feynman diagrams, even in a classical context, is shown by studies of classical stochastic dynamics such as vortex dynamics in disordered superconductors. The book demonstrates that quantum fields and Feynman diagrams are the universal language for studying fluctuations, be they of quantum or thermal origin, or even purely statistical.

Complete with numerous exercises to aid self-study, this textbook is suitable for graduate students in statistical mechanics, condensed matter physics, and quantum field theory in general.

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Preface

The purpose of this book is to provide an introduction to the applications of quantum field theoretic methods to systems out of equilibrium. The reason for adding a book on the subject of quantum field theory is two-fold: the presentation is, to my knowledge, the first to extensively present and apply to non-equilibrium phenomena the real-time approach originally developed by Schwinger, and subsequently applied by Keldysh and others to derive transport equations. Secondly, the aim is to show the universality of the method by applying it to a broad range of phenomena. The book should thus not just be of interest to condensed matter physicists, but to physicists in general as the method is of general interest with applications ranging the whole scale from high-energy to soft condensed matter physics. The universality of the method, as testified by the range of topics covered, reveals that the language of quantum fields is the universal description of fluctuations, be they of quantum nature, thermal or classical stochastic. The book is thus intended as a contribution to unifying the languages used in separate fields of physics, providing a universal tool for describing non-equilibrium states.

Chapter 1 introduces the basic notions of quantum field theory, the bose and fermi quantum fields operating on the multi-particle state spaces. In Chapter 2, operators on the multi-particle space representing physical quantities of a many-body system are constructed. The detailed exposition in these two chapters is intended to ensure the book is self-contained. In Chapter 3, the quantum dynamics of a many-body system is described in terms of its quantum fields and their correlation functions, the Green's functions. In Chapter 4, the key formal tool to describe non-equilibrium states is introduced: Schwinger's closed time path formulation of non-equilibrium quantum field theory, quantum statistical mechanics. Perturbation theory for non-equilibrium states is constructed starting from the canonical operator formalism presented in the previous chapters. In Chapter 5 we develop the real-time formalism necessary to deal with non-equilibrium states; first in terms of matrices and eventually in terms of two different types of Green's functions. The diagram representation of non-equilibrium perturbation theory is constructed in a way that the different aspects of spectral and quantum kinetic properties appear in a physically transparent and important fashion for non-equilibrium states. The equivalence of the real-time and imaginary-time formalisms are discussed in detail. In Chapter 6 we consider the coexistence regime between equilibrium and non-equilibrium states, the linear response regime. In Chapter 7 we develop and apply the quantum kinetic equation approach to the normal state, and in particular consider electrons in metals and semiconductors. As applications we consider the Boltzmann limit, and then phenomena beyond the Boltzmann theory, such as renormalization of transport coefficients due to interactions. In Chapter 8 we consider non-equilibrium superconductivity. In particular we introduce the quasi-classical Green's function technique so efficient for the description of superfluids. We derive the quantum kinetic equation describing elastic and inelastic scattering in superconductors. The time-dependent Ginzburg–Landau equation is obtained for a dirty superconductor. As an application of the quasi-classical theory, we consider the phenomena of conversion of normal currents to supercurrents and the corresponding charge imbalance.

Unlike Schwinger, not stooping to the paganism of using diagrams, we shall, like the boys in the basement, take heavy advantage of using Feynman diagrams. By introducing Feynman diagrams, the most developed of our senses can become functional in the pursuit of understanding quantum dynamics, an addition that shall make its pursuit easier also for non-equilibrium situations. Though the picture of reality that the representation of perturbation theory in terms of Feynman diagrams inspires might be a figment of the imagination, its usefulness for developing physical intuition has amply proved its value, as witnessed first in elementary particle physics. We develop the diagrammatics for non-equilibrium states, and show that the additional rules for the universal vertex display the two important features of quantum statistics and spectral properties of the interacting particles in an explicit fashion. In Chapter 9 we shall take the stand of formulating the laws of physics in terms of propagators and vertices and their Feynman diagrams representing probability amplitudes as dictated by the superposition principle. In fact, we take the Shakespearian approach and construct quantum dynamics in terms of Feynman diagrams by invoking the only two options for a particle: *to act or not to interact*. From this diagrammatic starting point, and employing the intuitive appeal of diagrammatic arguments, we then construct the formalism of non-equilibrium quantum field theory in terms of the powerful functional methods; first in terms of the generating functional and functional differentiation technique. In Chapter 10 we then introduce the final tool in the functional arsenal: functional integration, and arrive at the effective action description of general non-equilibrium states. As an application of the effective action approach we consider the dilute Bose gas, and the case of a trapped Bose–Einstein condensate. In Chapter 11 we consider quantum transport properties of disordered conductors, weak localization and interaction effects. In particular we show how the quasi-classical Green's function technique used in describing non-equilibrium properties of a dirty superconductor can be utilized to describe the destruction of phase coherence in the normal state due to non-equilibrium effects and interactions. Finally, in Chapter 12, we consider the classical limit of the developed general non-equilibrium quantum field theory. We consider classical stochastic dynamics and show that field theoretic methods and diagrammatics are useful tools even in the classical context. As an example we consider the flux flow properties of the Abrikosov lattice in a type-II superconductor. We thus demonstrate the fact that quantum field theory, through its diagrammatics and functional formulations, is the universal language for describing fluctuations whatever their nature.

Readers' guide. Firstly, readers bothered by the old-fashioned habit of footnotes can simply skip them; they are either quick reminders or serve the purpose of providing a general perspective. The book can be read chronologically but, like any fox hole, it has two entrances. For the reader whose interest is the general structure of quantum field theories, the book offers the possibility to jump directly to Chapter 9 where a quantum field theory is defined in terms of its propagators and vertices and their resulting Feynman diagrams as dictated by the superposition principle. The powerful methods of generating functionals are then constructed from the diagrammatics. However, the reader acquainted with Chapter 4 will then have at hand the general quantum field theory applicable to non-equilibrium states.

The scope of the book is not so much to dwell on a detailed application of the non-equilibrium theory to a single topic, but rather to show the versatility and universality of the method by applying it to a broad range of core topics of physics. One purpose of the book is to demonstrate the utility of Feynman diagrams in non-equilibrium quantum statistical mechanics using an approach appealing to physical intuition. The real-time description of non-equilibrium quantum statistical mechanics is therefore adopted, and the diagrammatic technique for systems out of equilibrium is developed systematically, and a representation most appealing to physical intuition applied. Though most examples are taken from condensed matter physics, the book is intended to contribute to the cross-fertilization between all the fields of physics studying the influence of fluctuations, be they quantum or thermal or purely statistical, and to establish that the convenient technique to use is in fact that of non-equilibrium quantum field theory. The book should therefore be of interest to a wide audience of physicists; in particular the book is intended to be self-contained so that students of physics and physicists in general can benefit from its detailed expositions. It is even contended that the method is of importance for other fields such as chemistry, and of course useful for electrical engineers.

A complete allocation of the credit for the progress in developing and applying the real-time description of non-equilibrium states has not been attempted. However, the references, in particular the cited review articles, should make it possible for the interested reader to trace this information.

The book is intended to be sufficiently broad to serve as a text for a one- or two-semester graduate course on non-equilibrium statistical mechanics or condensed matter theory. It is also hoped that the book can serve as a useful reference book for courses on quantum field theory, physics of disordered systems, and quantum transport in general. It is hoped that this attempt to make the exposition as lucid as possible will be successful to the point that the book can be read by students with only elementary knowledge of quantum and statistical mechanics, and read with benefit on its own. Exercises have been provided in order to aid self-instruction.

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Jørgen Rammer