1

Basic Concepts

1.0 RENEWABLE, NONRENEWABLE, AND ENVIRONMENTAL RESOURCES

Economics might be defined as the study of how society allocates scarce resources. The field of *resource economics*, would then be the study of how society allocates scarce natural resources, such as stocks of fish, stands of trees, fresh water, oil, and other naturally occurring resources. A distinction is sometimes made between *resource* and *environmental economics*, where the latter field is concerned with the way wastes are disposed and the resulting quality of air, water, and soil serving as waste receptors. In addition, environmental economics is concerned with the conservation of natural environments and biodiversity.

Natural resources are often categorized as being renewable or nonrenewable. A renewable resource must display a significant rate of growth or renewal on a relevant economic time scale. An economic time scale is a time interval for which planning and management are meaningful. The notion of an economic time scale can make the classification of natural resources a bit tricky. For example, how should we classify a stand of old-growth coast redwood or an aquifer with an insignificant rate of recharge? While the redwood tree is a plant and can be grown commercially, old-growth redwoods may be 800 to 1,000 years old, and the remaining stands may be more appropriately viewed as a nonrenewable resource. While the water cycle provides precipitation that will replenish lakes and streams, the water contained in an aquifer with little or no recharge may be economically more similar to a pool of oil (a nonrenewable resource) than to a lake or reservoir that receives significant recharge from rain or melting snow.

A critical question in the allocation of natural resources is, “How much of the resource should be harvested (extracted) today?” Finding the “best” allocation of natural resources over time can be regarded as a dynamic optimization problem. What makes a problem a dynamic optimization problem? The critical variable in a dynamic optimization problem is a stock or *state variable* that requires a difference or differential equation to describe its evolution over time. The other key feature of a dynamic optimization problem is that a decision taken today, in period t, will change the amount or level of the state variable that is available in the next period, $t+1$.

In a dynamic optimization problem, it is common to try to maximize some measure of net economic value, over some future horizon, subject to the dynamics of the harvested resource and any other relevant constraints. The solution to a natural resource dynamic optimization problem would be a schedule or “time path” indicating the optimal amount to be harvested (extracted) in each period or a “policy” indicating how harvest depends on the size of the resource stock. The optimal rate of harvest or extraction in a particular time period may be zero. For example, if a fish stock has been historically mismanaged, and the current stock is below what is deemed optimal, then zero harvest (a moratorium on fishing) may be best until the stock recovers to a size where a positive level of harvest is optimal.

Aspects of natural resource allocation are illustrated in Figure 1.1. On the right-hand side (RHS) of this figure I depict a trawler harvesting tuna. The tuna stock at the beginning of period t is denoted by the variable X_t, measured in metric tons. In each period, the level of net growth is assumed to depend on the size of the fish stock and is given by the function $F(X_t)$. I will postpone a detailed discussion of the properties of $F(X_t)$ until Chapter 3. For now, simply assume that if the tuna stock is bounded by some *environmental carrying capacity*, denoted K, so that $K \geq X_t \geq 0$, then $F(X_t)$ might be increasing as X_t goes from a low but positive level to where $F(X_t)$ reaches a maximum, at $X_t = X_{\mathrm{MSY}}$, and then $F(X_t)$ declines as X_t goes from X_{MSY} to K.

Let Y_t denote the harvest of tuna in period t, also measured in metric tons, and assume that net growth occurs before harvest. Then the change in the tuna stock, going from period t to period $t+1$, is the

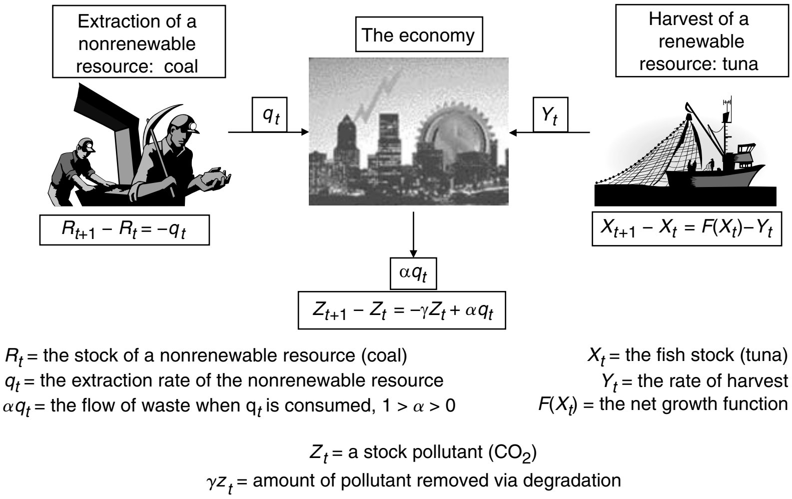


Figure 1.1. Renewable, nonrenewable, and environmental resources.

difference $X_{t+1} - X_t$ and is given by the difference equation

$$X_{t+1} - X_t = F(X_t) - Y_t \tag{1.1}$$

If harvest exceeds net growth, $Y_t > F(X_t)$, the tuna stock declines, and $X_{t+1} - X_t < 0$. If harvest is less than net growth, $Y_t < F(X_t)$, the tuna stock increases, and $X_{t+1} - X_t > 0$.

We might rewrite Equation (1.1) in *iterative form* as $X_{t+1} = X_t - Y_t + F(X_t)$. As we will see, the iterative form is often used in spreadsheets and computer programs. During period t, harvest Y_t flows to the economy, where it yields a net benefit to various firms and individuals. The portion of the stock that is not harvested, $X_t - Y_t \geq 0$, is sometimes referred to as *escapement*. Escapement plus net growth $F(X_t)$ determine the inventory of tuna at the start of period $t + 1$. The stock X_t also conveys a benefit to the economy because it provides the basis for growth, and it is often the case that larger stocks will lower the cost of harvest in period t. Thus, implicit in the harvest decision is a balancing of current net benefit from Y_t and future benefit from a slightly larger stock X_{t+1}.

In some fishery models, growth depends on escapement, where escapement is calculated as $S_t = X_t - Y_t \geq 0$. The fish stock available for harvest in period $t+1$ is determined according to the equation $X_{t+1} = S_t + F(S_t)$. Given an initial stock level X_0 and a harvest schedule or harvest policy (where Y_t depends on X_t), it is relatively simple to use a spreadsheet to *simulate* the dynamics of our tuna stock.

On the left-hand side (LHS) of Figure 1.1 I depict miners extracting a nonrenewable resource, say, coal. The remaining reserves of coal in period t are denoted by R_t, and the current rate of extraction is denoted by q_t. With no growth or renewal, the change in the stock is the negative of the amount extracted in period t, so

$$R_{t+1} - R_t = -q_t \tag{1.2}$$

In iterative form, we might write $R_{t+1} = R_t - q_t$.

The amount of coal extracted also flows into the economy, where it generates net benefits, but in contrast to harvest from the tuna stock, consumption of the nonrenewable resource generates a residual waste flow αq_t, say, CO_2, assumed to be proportional to the rate of extraction $(1 > \alpha > 0)$.

This residual waste can accumulate as a *stock pollutant*, denoted Z_t. The change in the stock pollutant might depend on the relative magnitudes of the waste flow and the rate at which the stock pollutant is assimilated into the environment, say, carbon sequestration by plants. Let the stock pollutant be reduced by an amount given by the term $-\gamma Z_t$, where the parameter γ is called the *assimilation* or *degradation coefficient,* and it is usually assumed that $1 > \gamma > 0$. The change in the stock pollutant then would be given by the difference equation

$$Z_{t+1} - Z_t = -\gamma Z_t + \alpha q_t \tag{1.3}$$

If the waste flow exceeds the amount degraded, $Z_{t+1} - Z_t > 0$. If the amount degraded exceeds the waste flow, $Z_{t+1} - Z_t < 0$. In iterative form, this equation might be written as $Z_{t+1} = (1 - \gamma)Z_t + \alpha q_t$.

Not shown in Figure 1.1 are the consequences of different levels of Z_t. Presumably, there would be some *social* or *external cost* imposed on the economy (society). This is sometimes represented through a damage function $D(Z_t)$. Damage functions will be discussed in greater detail in Chapter 6.

If the economy is represented by the cityscape in Figure 1.1, then the natural environment, surrounding the economy, can be thought of as providing a flow of renewable and nonrenewable resources, as well as various media for the disposal of unwanted (negatively valued) wastes. Missing from Figure 1.1, however, is one additional service, usually referred to as *amenity value.* A wilderness, a pristine stretch of beach, or a lake with "swimmable" water quality provides individuals in the economy with places for observing flora and fauna, relaxation, and recreation that are fundamentally different from comparable services provided at a city zoo, an exclusive beach hotel, or a backyard swimming pool. The amenity value provided by various natural environments may depend on the location of economic activities (including the harvest and extraction of resources) and the disposal of wastes. Thus the optimal rates of harvest, extraction, and disposal should take into account any reduction in amenity values. In general, current net benefit from, say, Y_t or q_t must be balanced with the discounted future costs from reduced resource stocks X_{t+1} and R_{t+1} *and* any reduction in amenity values caused by harvest, extraction, or disposal of associated wastes.

1.1 POPULATION DYNAMICS: SIMULATION, STEADY STATE, AND LOCAL STABILITY

In this section I will illustrate the use of the iterative form of Equation (1.1) to simulate the dynamics of a fish stock. I will use a spreadsheet to perform the simulation, and in the process, I will define what is meant by a *steady-state equilibrium* and the *local stability* for such equilibria for a single, first-order difference equation. A steady state is said to be locally stable if neighboring states are attracted to it and unstable if the converse is true.

In iterative form, Equation (1.1) was written as $X_{t+1} = X_t + F(X_t) - Y_t$. To make things more concrete, suppose that the net-growth function takes the form $F(X_t) = rX_t(1 - X_t/K)$, where I will call $r > 0$ the *intrinsic growth rate* and $K > 0$ the *environmental carrying capacity*. This net-growth function is drawn as the concave (from below) symmetric curve in Figure 1.2.

We will assume that fishery managers have a simple rule for determining *total allowable catch* Y_t, where the harvest rule takes the form

$$Y_t = \alpha X_t \tag{1.4}$$

We will assume that $r > \alpha > 0$ for reasons that will become apparent in a moment. Equations that express harvest or total allowable catch

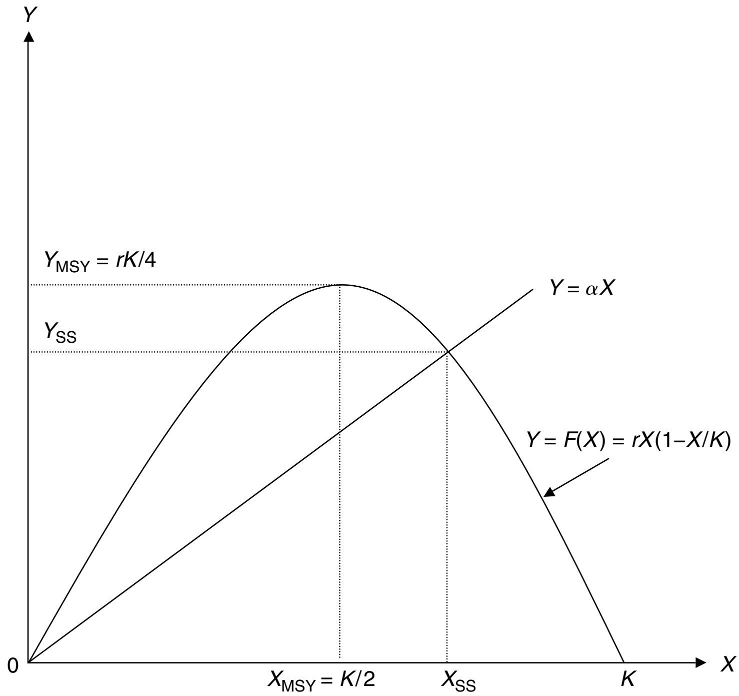


Figure 1.2. Steady-state equilibrium for Equation (1.6).

as a function of stock size are also called *feedback harvest policies*. This particular harvest policy is simply a line through the origin and is also drawn in Figure 1.2. Substituting the specific forms for our net-growth function and the feedback harvest policy into the iterative form of Equation (1.1) yields

$$X_{t+1} = X_t + rX_t(1 - X_t/K) - \alpha X_t = (1 + r - \alpha - rX_t/K)X_t \quad (1.5)$$

We can see that Equation (1.5) is almost begging for a spreadsheet. If we had *parameter values* for r, α, and K and an *initial condition* X_0, we could program Equation (1.5) and have the spreadsheet calculate X_1. With the same parameters and X_1, we could then calculate X_2, and so on. The fill-down feature on a spreadsheet means that we need only type Equation (1.5) once.

Before constructing our spreadsheet and doing some simulations, we might ask the following question: "Would it ever be the case that the feedback harvest policy would lead to a steady-state equilibrium where $X_{t+1} = X_t = X_{SS}$ and $Y_{t+1} = Y_t = Y_{SS}$?" The unknowns X_{SS} and Y_{SS} are *constant* levels for the fish stock and harvest rate, respectively, that are sustainable *ad infinitum*. The short answer to our question is, "Maybe." If the steady-state equilibrium is locally stable, and if X_0 is in what is called the *basin of attraction*, then, over time, $X_t \to X_{SS}$.

We might rewrite Equation (1.5) one last time as

$$X_{t+1} = (1 + r - \alpha - rX_t/K)X_t = G(X_t)$$

For equations such as $X_{t+1} = G(X_t)$, steady-state equilibria, also called *fixed points*, must satisfy $X = G(X)$. For our net-growth function and harvest policy, there will be a single (unique) steady-state equilibrium at

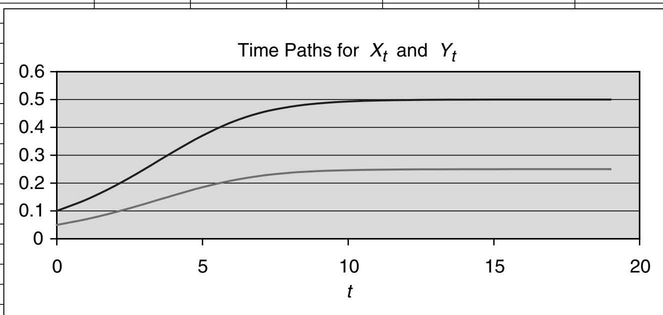
$$X_{SS} = \frac{K(r - \alpha)}{r} \tag{1.6}$$

For $X_{SS} > 0$, we need $r > \alpha > 0$. Graphically, X_{SS} occurs at the intersection of $Y = \alpha X$ and $Y = rX(1 - X/K)$ in Figure 1.2. It can be shown that X_{SS} will be locally stable *if and only if* $\left|G'(X_{SS})\right| < 1$. We refer to Equation (1.6) as an *analytic expression* for X_{SS} because our algebra allowed us to obtain an expression where X_{SS} is isolated on the LHS, whereas on the RHS we have nothing but parameters (K, r, and α).

In Figure 1.2 I also have included the reference values $X_{\text{MSY}} = K/2$ and $Y_{\text{MSY}} = rX_{\text{MSY}}(1 - X_{\text{MSY}}/K) = rK/4$. $X_{\text{MSY}} = K/2$ is called the stock level that supports *maximum sustainable yield*. When $X_{\text{MSY}} = K/2$ is substituted into the net-growth function and the expression is simplified, it will imply that the maximum sustainable yield is $Y_{\text{MSY}} = rK/4$.

We are almost ready to build our spreadsheet to simulate a fish population whose dynamics are described by Equation (1.5). Knowing Equation (1.6) and the necessary and sufficient condition for local stability will allow us to calculate X_{SS} and know whether $X_t \to X_{SS}$ if X_0 is in the basin of attraction. With $G(X_t) = (1 + r - \alpha - rX_t/K)X_t$, we can take a derivative and show that $G'(X_t) = 1 + r - \alpha - 2rX_t/K$. With $X_{SS} = K(r - \alpha)/r$, we then can show that $G'(X_{SS}) = 1 - r + \alpha$. Recall that the local stability of X_{SS} required that $\left|G'(X_{SS})\right| < 1$, so if $|1 - r + \alpha| < 1$, we would expect that $X_t \to X_{SS}$.

	A	B	C	D	E	F	G	H
1	Spreadsheet S1.1							
2								
3	r =	1						
4	K =	1						
5	α =	0.5						
6	X_{ss} =	0.5						
7	Y_{ss} =	0.25						
8	$\|G'(X_{ss})\|$ =	0.5						
9	X_o =	0.1						
10								
11								
12	t	X_t	Y_t					
13	0	0.1	0.05					
14	1	0.14	0.07					
15	2	0.1904	0.0952					
16	3	0.24934784	0.12467392					
17	4	0.311847415	0.155923707					
18	5	0.370522312	0.185261156					
19	6	0.418496684	0.209248342					
20	7	0.452605552	0.226302776					
21	8	0.474056542	0.237028271					
22	9	0.486355208	0.243177604					
23	10	0.492991424	0.246495712					
24	11	0.496446592	0.248223296					
25	12	0.498210669	0.249105335					
26	13	0.499102133	0.249551066					
27	14	0.49955026	0.24977513					
28	15	0.499774928	0.249887464					
29	16	0.499887413	0.249943707					
30	17	0.499943694	0.249971847					
31	18	0.499971844	0.249985922					
32	19	0.499985921	0.249992961					
33								
34								



Spreadsheet S1.1

In Spreadsheet S1.1, I have set $r = 1$, $K = 1$, and $\alpha = 0.5$. For consistency throughout this text, I will put parameter labels in column A of the spreadsheets and their specific values in the same row but in column B. In cells A6, A7, A8, and A9, I also enter labels for X_{SS}, Y_{SS}, $|G'(X_{SS})|$, and the initial condition X_0. In cell B6, I program = B4 ∗ (B3 − B5)/B3. In cell B7, I program = B3 ∗ B6 ∗

(1 − \$B\$6/\$B\$4). In cell B8, I program = ABS(1 − \$B\$3 + \$B\$5). In cell B9, I simply enter the number 0.1. With the carrying capacity normalized so that $K = 1$, an $X_0 = 0.1$ might be symptomatic of a bad case of *overfishing* that has resulted in a stock level that is only 1/10 its carrying capacity, which would be the stock level in the unexploited fishery.

In row 12, columns A, B, and C, I type the labels for t, X_t, and Y_t. In cell A13, I enter 0 and do a series fill-downs ending in 19 in cell A32. In cell B13, I type = \$B\$9. In cell C13, I type = \$B\$5 ∗ B13. *Note:* I omit the dollar sign (\$) when I want a variable to iterate when using a fill-down or a fill-across. This allows me to program an iterative equation only once. In cell B14, I program = (1 + \$B\$3 − \$B\$5 − \$B\$3 ∗ B13/\$B\$4) ∗ B13. I then do a one-cell fill-down from cell C13 to cell C14, and then I do a fill-down from B14:C14 to B32:C32. If you do this correctly, you should get = (1 + \$B\$3 − \$B\$5 − \$B\$3 ∗ B31/\$B\$4) ∗ B31 in cell B32 and = \$B\$5 ∗ B32 in cell C32. I finally select cells A13:C13 through A32:C32, click on the "Chart Wizard," and select the scatter diagram with lines. After entering the chart title and placing t on the x axis, we end up with the chart in the lower right-hand corner of Spreadsheet S1.1. We see from our previous calculations for X_{SS}, Y_{SS}, and $|G'(X_{SS})|$ that when we simulate the fish population from $X_0 = 0.1$, X_t in fact converges to $X_{SS} = 0.5$, whereas Y_t converges to $Y_{SS} = 0.25$.

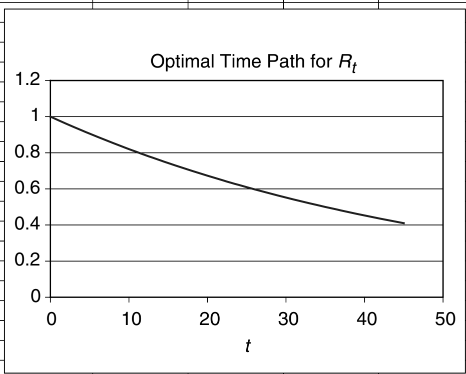
One of the great things about setting up the spreadsheet in this manner is that we can change a parameter, and the spreadsheet instantly recomputes and replots variables and charts. What happens if we change r to $r = 2.6$? In this case, the spreadsheet computes $X_{SS} = 0.80769231$ and $Y_{SS} = 0.40384615$, but the local stability condition is *not* satisfied because $\left|G'(X_{SS})\right| = 1.1 > 1$. From the plot of X_t and Y_t we see that we will never converge to the steady-state equilibrium. Looking at the numerical values for X_t and Y_t, we see that they have locked into what is called a *two-point cycle*. We will see that a single nonlinear difference equation or two or more nonlinear difference equations (called a *dynamical system*) are capable of complex dynamic behavior, including *deterministic chaos*, where the steady-state equilibrium, calculated in advance of simulation, is never reached.

1.2 EXTRACTION OF A NONRENEWABLE RESOURCE

In Section 1.5 I will present a nonrenewable-resource model where the optimal extraction policy, in feedback form, is given by the equation

$$q_t^* = [\delta/(1+\delta)]R_t \tag{1.7}$$

	A	B	C
1	Spreadsheet S1.2		
2			
3	δ =	0.02	
4	R_0 =	1	
5			
6	t	R_t	q_t
7	0	1	0.019607843
8	1	0.980392157	0.019223376
9	2	0.961168781	0.018846447
10	3	0.942322335	0.018476909
11	4	0.923845426	0.018114616
12	5	0.90573081	0.017759428
13	6	0.887971382	0.017411204
14	7	0.870560179	0.017069807
15	8	0.853490371	0.016735105
16	9	0.836755266	0.016406966
17	10	0.8203483	0.016085261
18	11	0.804263039	0.015769864
19	12	0.788493176	0.015460651
20	13	0.773032525	0.0151575
21	14	0.757875025	0.014860295
22	15	0.74301473	0.014568916
23	16	0.728445814	0.014283251
24	17	0.714162562	0.014003187
25	18	0.700159375	0.013728615
26	19	0.68643076	0.013459427
27	20	0.672971333	0.013195516
28	21	0.659775817	0.012936781
29	22	0.646839036	0.012683118
30	23	0.634155918	0.01243443
31	24	0.621721488	0.012190617
32	25	0.609530871	0.011951586
33	26	0.597579285	0.011717241
34	27	0.585862044	0.011487491
35	28	0.574374553	0.011262246
36	29	0.563112307	0.011041418
37	30	0.552070889	0.010824919
38	31	0.54124597	0.010612666
39	32	0.530633304	0.010404575
40	33	0.520228729	0.010200563
41	34	0.510028166	0.010000552
42	35	0.500027613	0.009804463
43	36	0.49022315	0.009612219
44	37	0.480610932	0.009423744
45	38	0.471187188	0.009238964
46	39	0.461948223	0.009057808
47	40	0.452890415	0.008880204
48	41	0.444010211	0.008706083
49	42	0.435304128	0.008535375
50	43	0.426768753	0.008368015
51	44	0.418400739	0.008203936
52	45	0.410196803	0.008043075



Spreadsheet S1.2